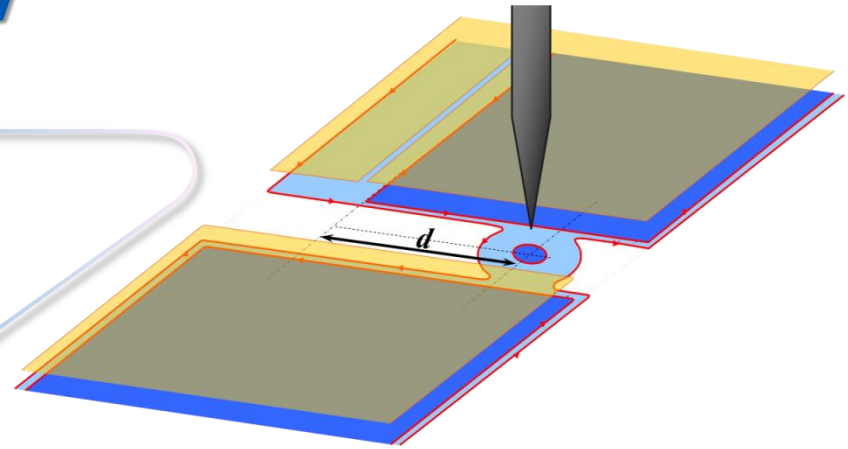


# Tomography and manipulation of quantum Hall edge channels by Scanning Gate Microscopy

**N. Paradiso**

*Scuola Normale Superiore di Pisa*

Discussione della Tesi di Perfezionamento



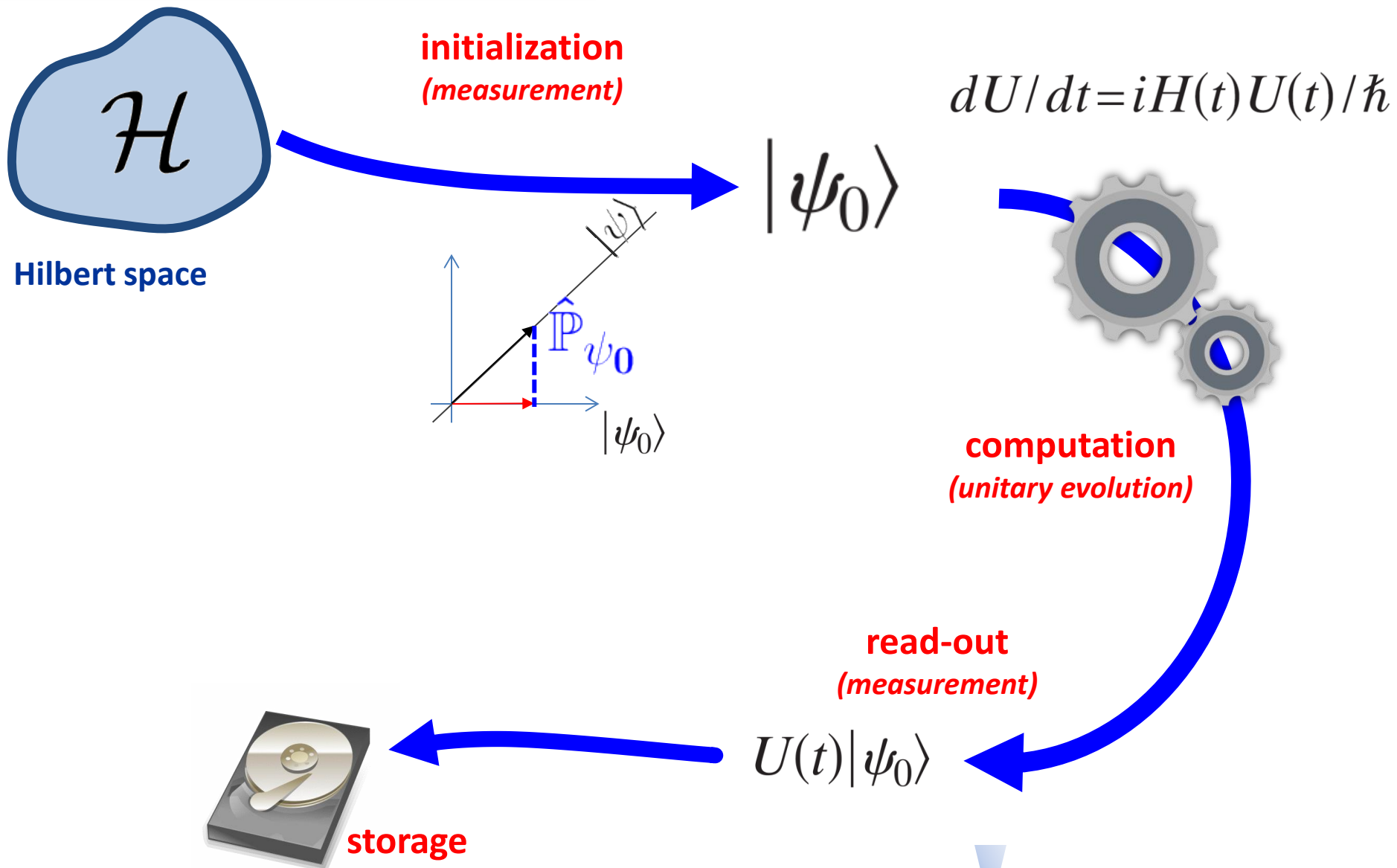
## Advisors

Prof. F. Beltram, dr. S. Heun

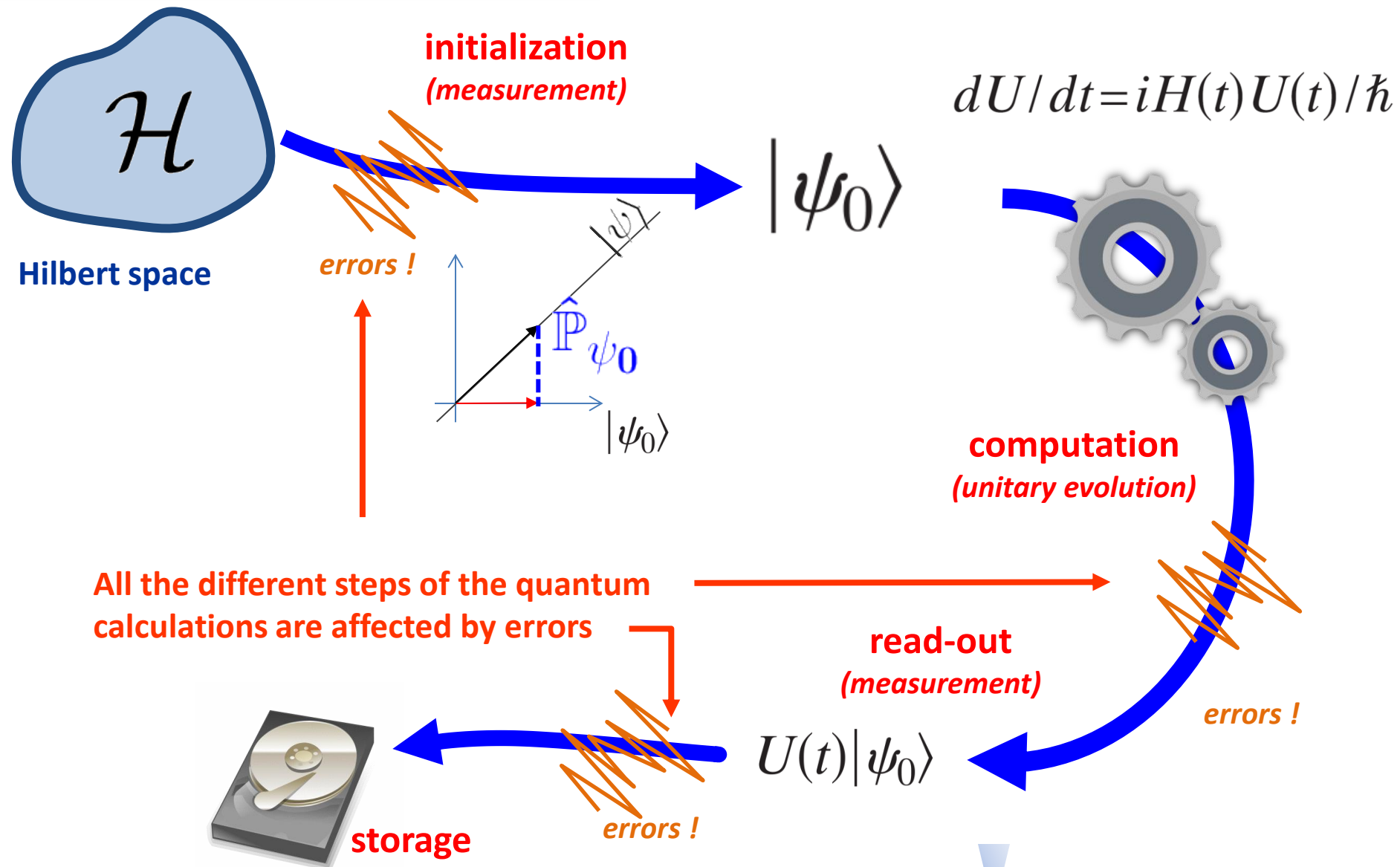
## Co-workers

- NEST, Pisa, Italy: S. Roddaro, L. Sorba
- TASC, Trieste, Italy: G. Biasiol

# The road to quantum computing



# The road to quantum computing

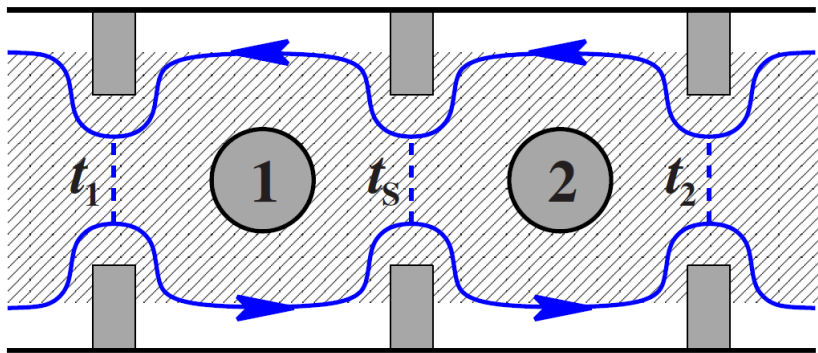


# Why a quantum Hall quantum Computer?

**Fundamental reasons:** QH liquids at peculiar filling factors ( $5/2$ ,  $12/5$ ) are expected to **exhibit non-Abelian excitations**. Since quantum operations on such objects are expected to only depend on the **topology**, they could implement **fault tolerant calculations**.  
[Nayak *et al.*, *Rev. Mod. Phys.* **80**, 1083.]

REVIEWS OF MODERN PHYSICS, VOLUME 80, JULY-SEPTEMBER 2008

## Non-Abelian anyons and topological quantum computation

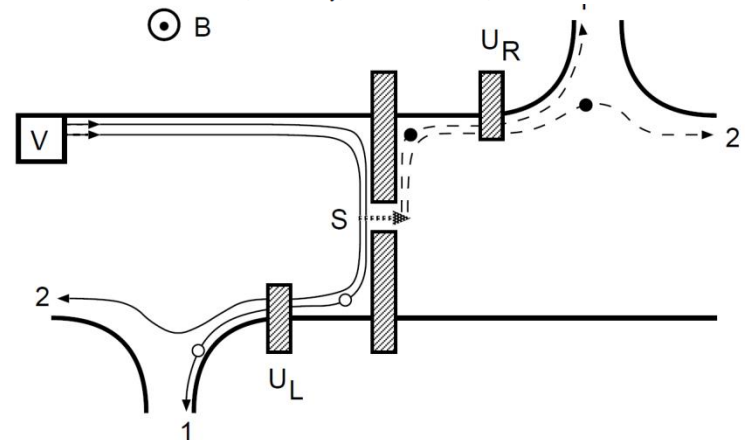


**a gate "NOT" operating with non-Abelian quasiparticles**

[Nayak *et al.*, *Rev. Mod. Phys.* **80**, 1083 (2003)]

## Proposal for Production and Detection of Entangled Electron-Hole Pairs in a Degenerate Electron Gas

C.W.J. Beenakker, C. Emary, M. Kindermann, and J.L. van Velsen



**An electron-hole entangler**

[Beenakker *et al.* *PRL* **91**, 147901 (2003)]

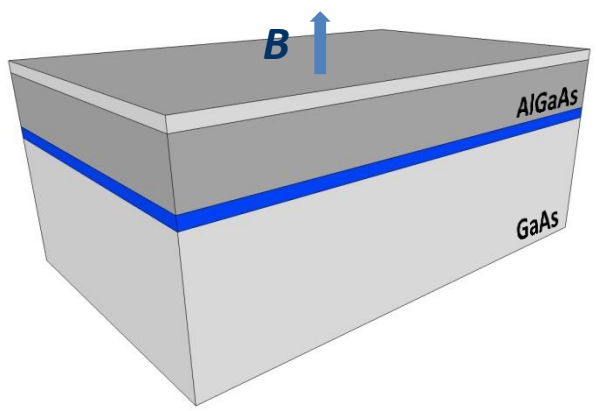
# Why a quantum Hall quantum Computer?

---

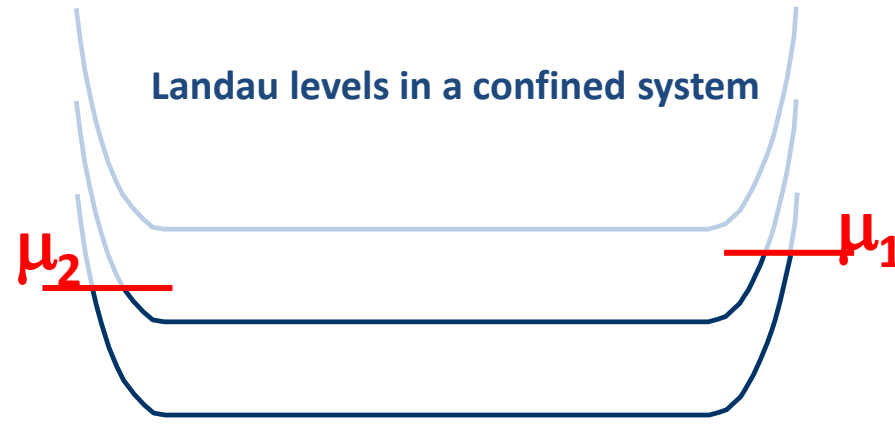
**Practical reasons:** highly **coherent**,  
**dissipationless transport** by means  
of **chiral 1D channels**

- ✓ **Solid** state devices.
- ✓ Chiral channels **insensitive to backscattering.**
- ✓ Single-fermion source
- ✓ Perfect transmission
- ✓ Accurate control of the chemical potential,  
tunnel probability, and occupation distribution of  
individual channels
- ✓ High design flexibility

# The non-interacting picture of the QH effect

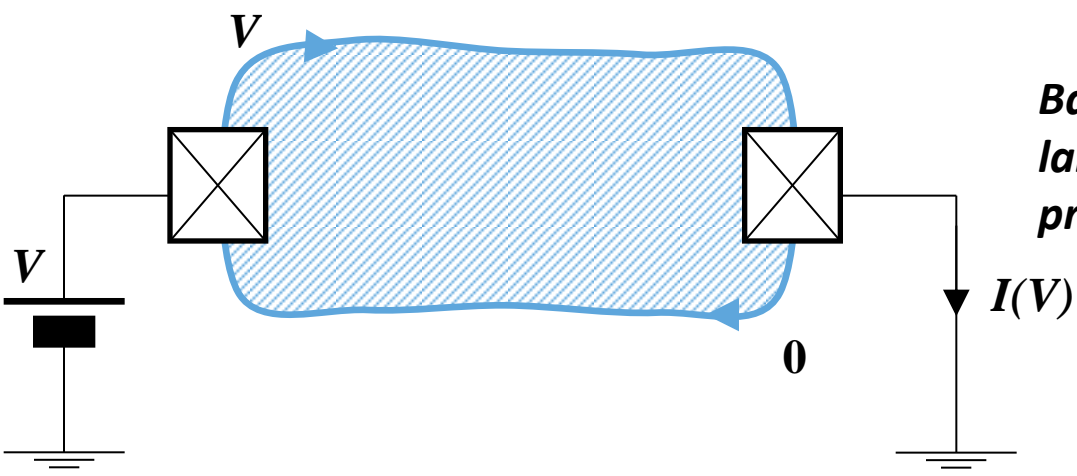


2DES  
in high field



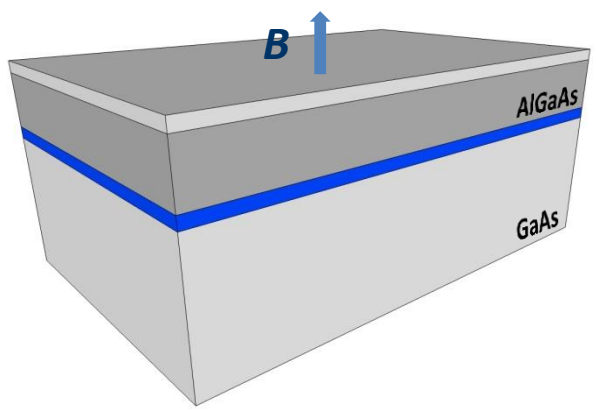
- Edge state picture:  
*current is carried by chiral 1D channels*

$$G \equiv \frac{dI}{dV} = \nu \frac{e^2}{h}$$

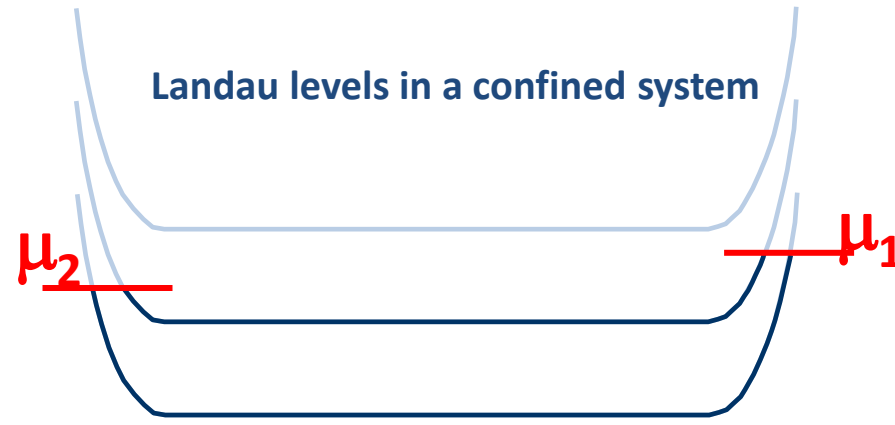


*Backscattering is suppressed due to the large spatial separation between counter-propagating channels*

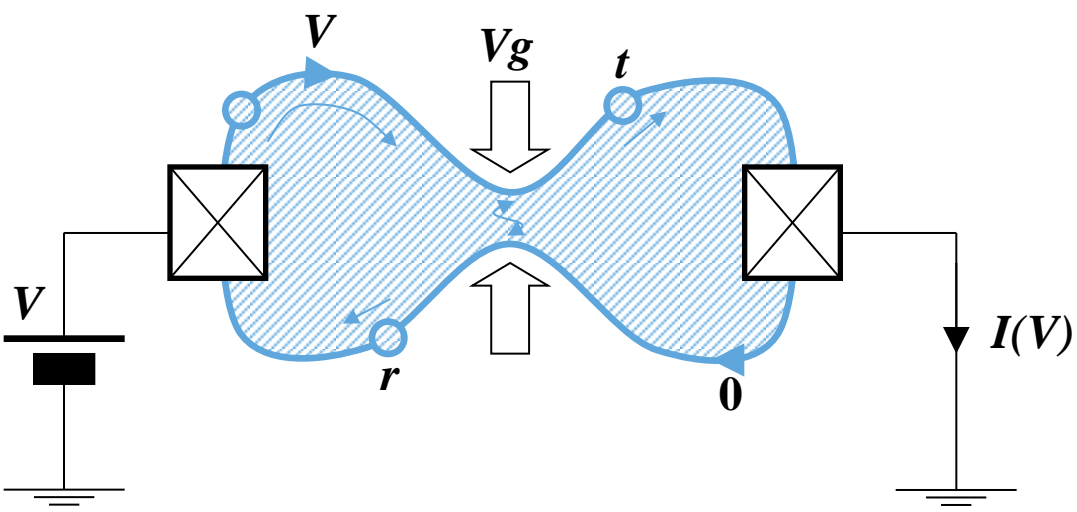
# The non-interacting picture of the QH effect



2DES  
in high field



- Edge state picture:  
*current is carried by chiral 1D channels*



*With a QPC we can intentionally induce backscattering, which provides us information about the edge properties*

Roddaro et al.: PRL **90** (2003) 046805  
 Roddaro et al.: PRL **93** (2004) 046801  
 Roddaro et al.: PRL **95** (2005) 156804  
 Roddaro, Paradiso et al.: PRL **103** (2009) 016802

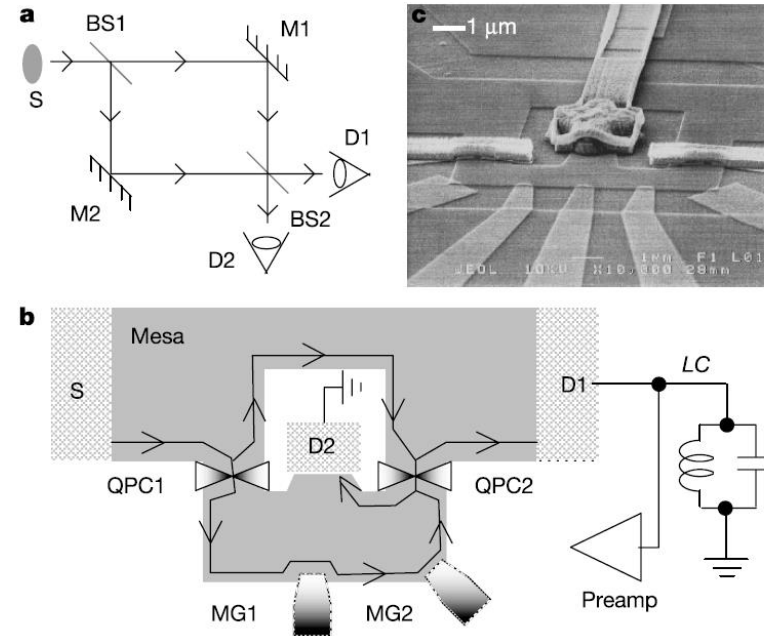
# Edge channel-based interferometers

The very large coherence length has been exploited to implement complex interferometers as the electronic Mach-Zehnder.

**Puzzle:** so far, MZI only work with electron-like excitations. The interference of **fractional** quasi-particles is inexplicably still **elusive**

## An electronic Mach-Zehnder interferometer

Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman



Ji *et al.*: Nature **422**, 415 (2003)



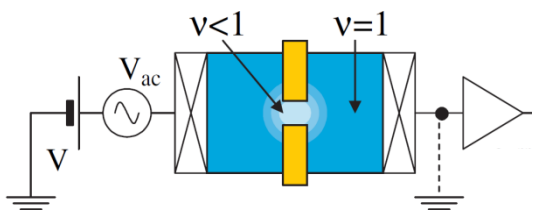
# Edge channel-based interferometers

The very large coherence length has been exploited to implement complex interferometers as the electronic Mach-Zehnder.

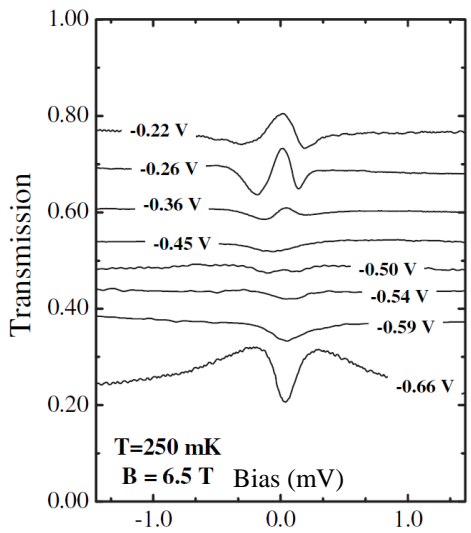
**Puzzle:** so far, MZI only work with electron-like excitations. The interference of **fractional** quasi-particles is inexplicably still **elusive**

**Role of the inner edge structure?**

Roddaro *et al.*: experiments on QPCs revealed signatures of **fractional components** in “simple” integer channels

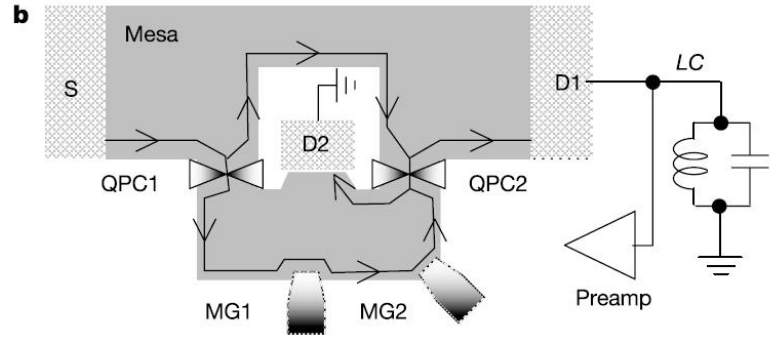
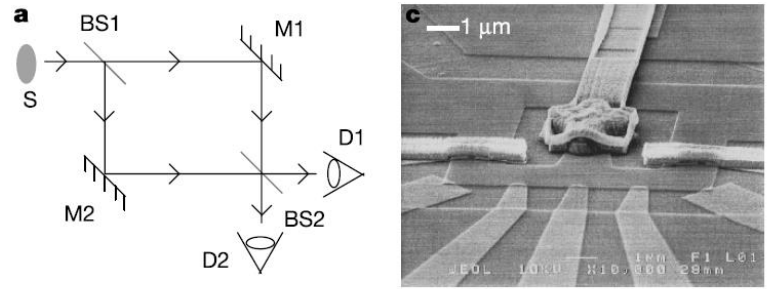


PRL 95, 156804 (2005)



## An electronic Mach-Zehnder interferometer

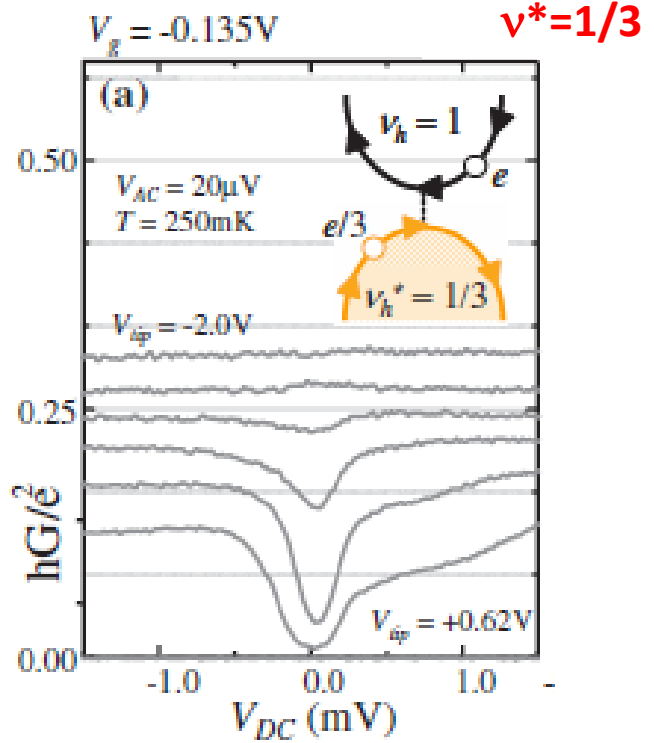
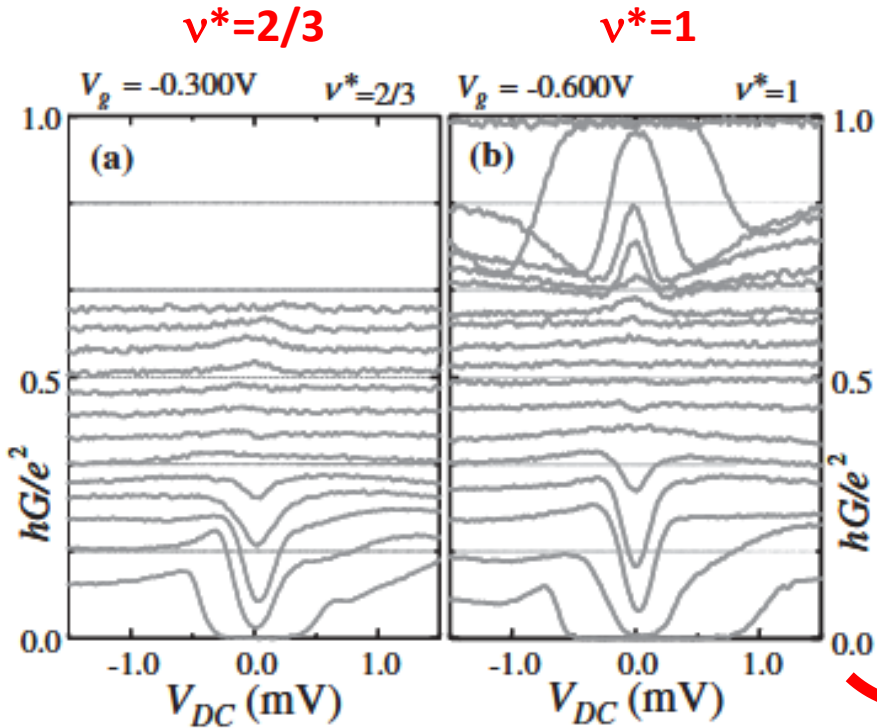
Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman



Ji *et al.*: Nature 422, 415 (2003)

# Fractional structures in integer edges

Our first transport measurements found evidences of fractional structures (Luttinger liquid-like) in a single edge (Fermi liquid).



-S. Roddaro, N. Paradiso, et al: "Tuning Nonlinear Charge Transport between Integer and Fractional Quantum Hall States"; *Phys. Rev. Lett.* **103**, (2009) 016802.

Need for **spatially** resolved measurements

# Non-interacting VS interacting picture

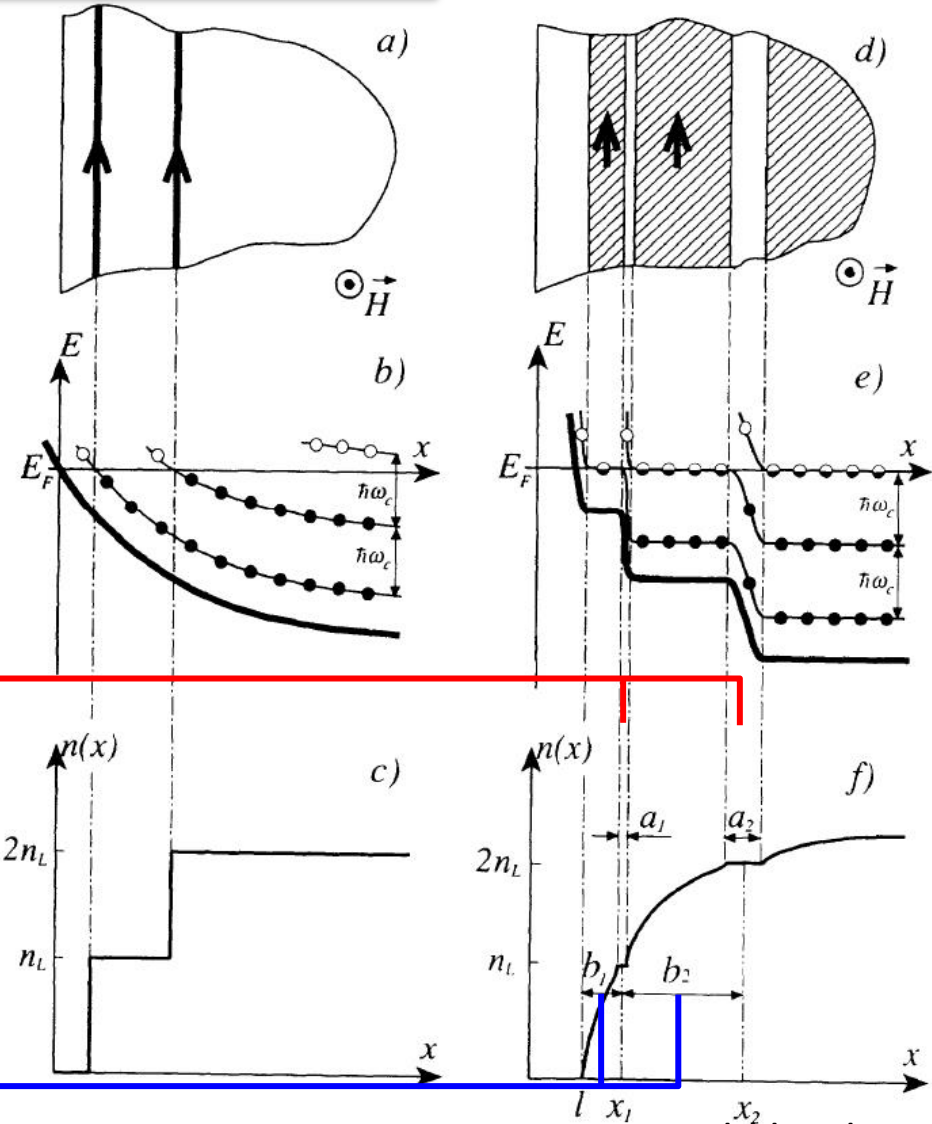
- The self consistent potential due to e-e interactions modifies the edge structure
- For any realistic potential the density goes smoothly to zero.
- Alternating compressible and incompressible stripes arise at the sample edge

## Incompressible stripes:

- The electron density is constant
- The potential has a jump

## Compressible stripes:

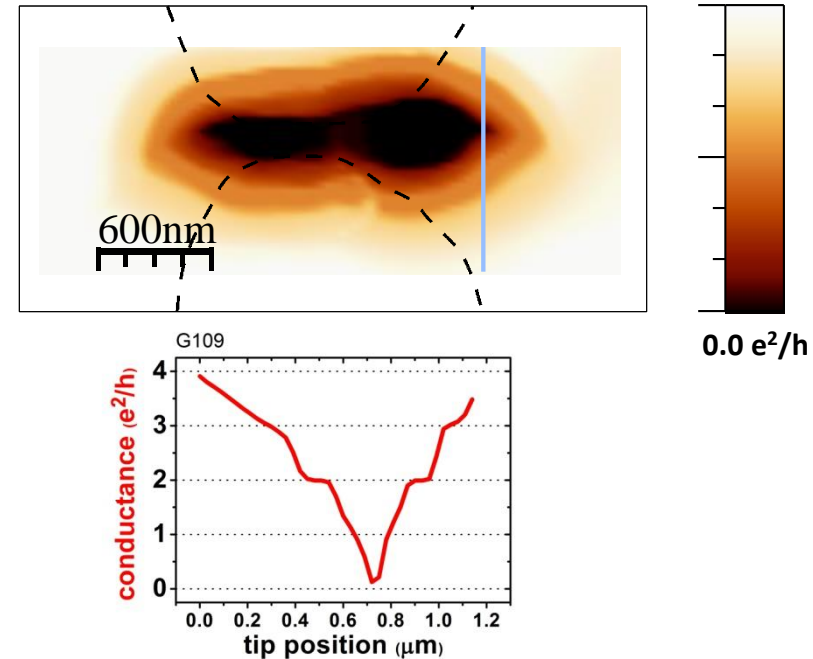
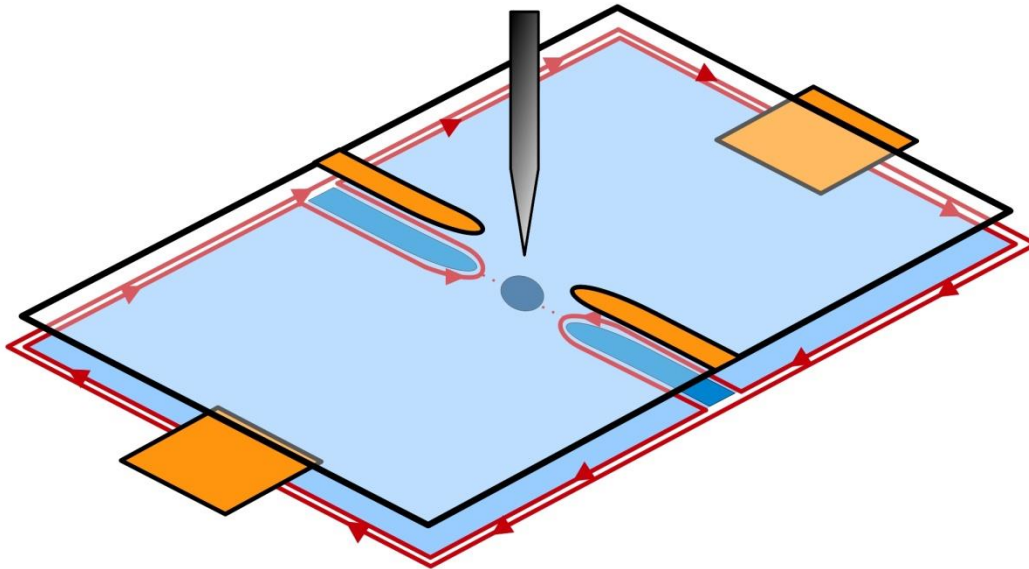
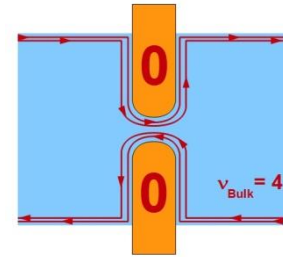
- The electron density has a jump
- The potential is constant



D. B. Chklovskii et al.:  
PRB 46 (1992) 4026.

# Edge channel tomography by SGM

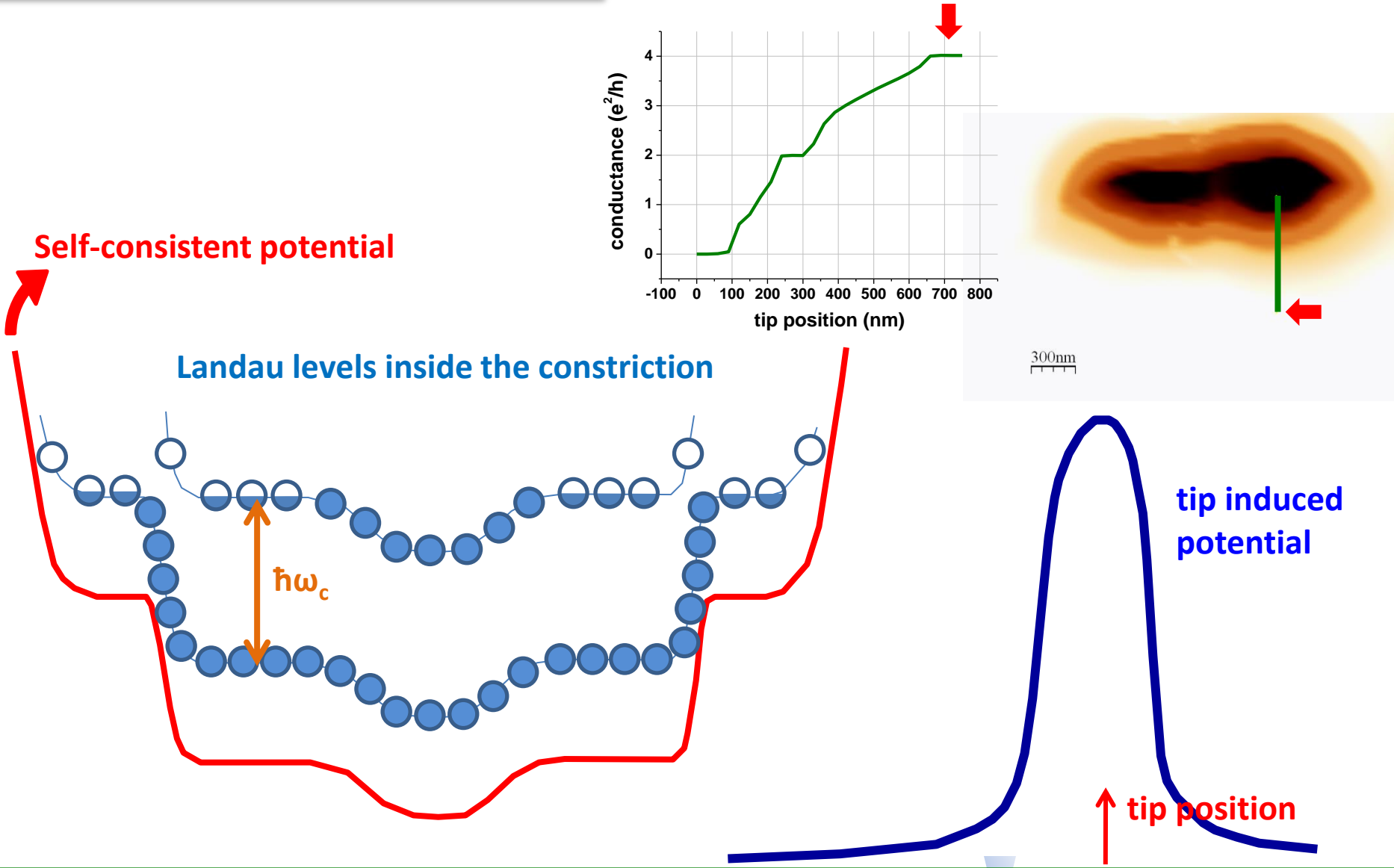
**SGM technique:** we **select** individual channels from the edge of a quantized 2DEG, we **send** them to the constriction and make them **backscatter** with the biased **SGM tip**.



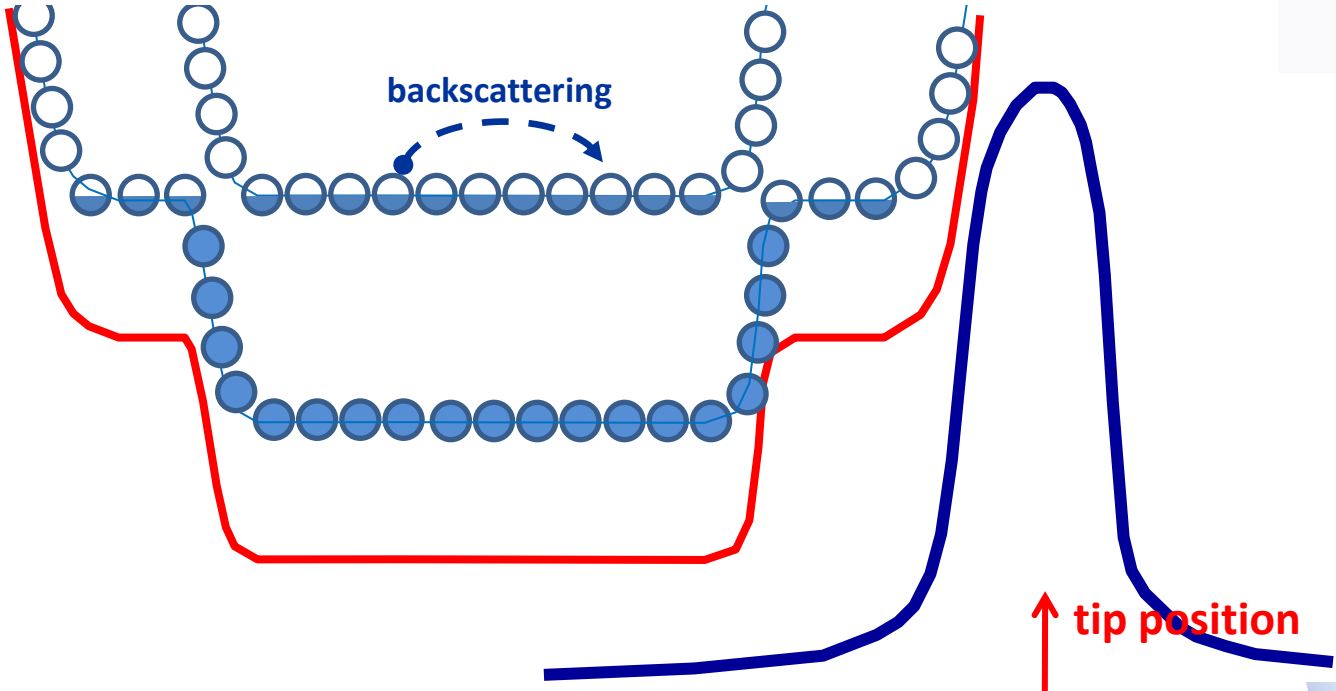
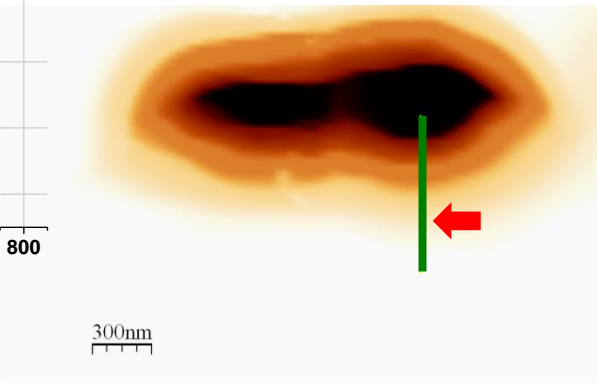
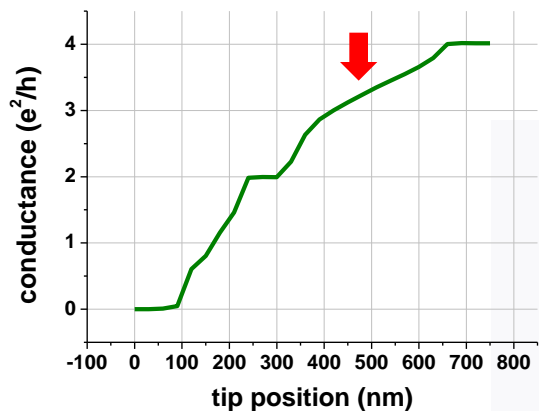
- Bulk filling factor  $\nu=4$
- $B = 3.04 \text{ T}$
- 2 spin-degenerate edge channels
- gate-region filling factors  $g_1 = g_2 = 0$

N. Paradiso *et al.*, *Physica E* 42 (2010) 1038.

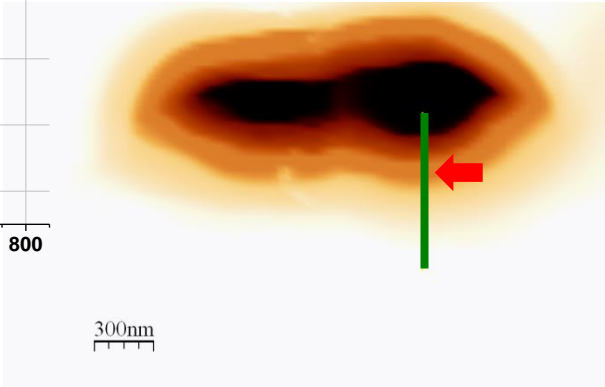
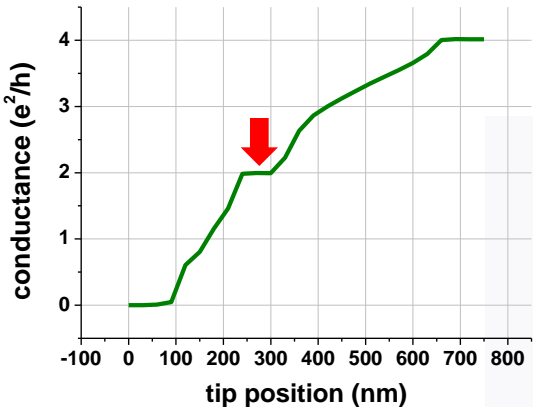
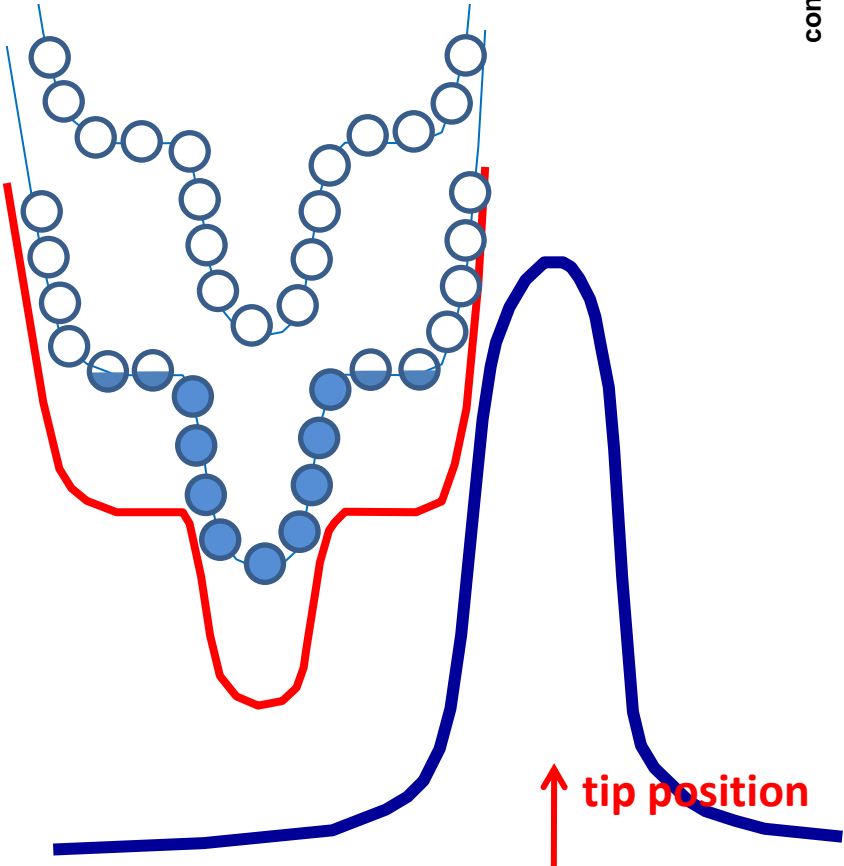
# How we probe incompressible stripes



# How we probe incompressible stripes

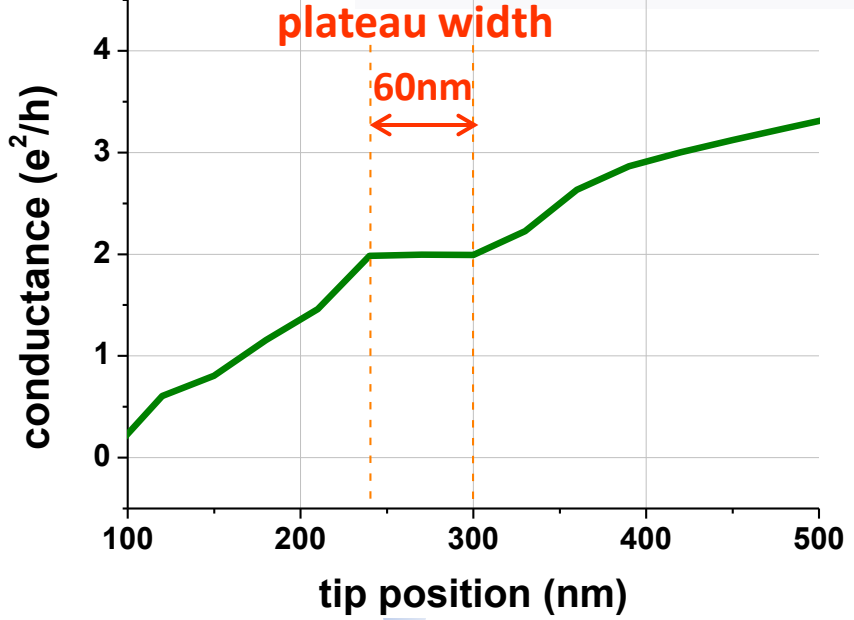
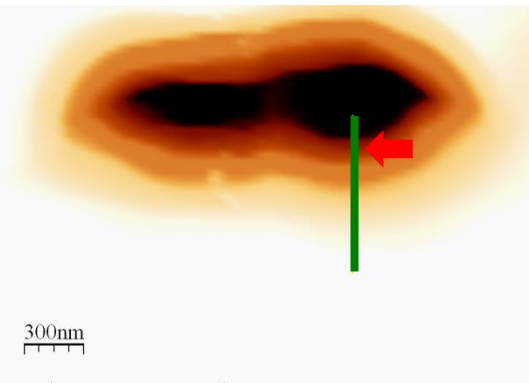
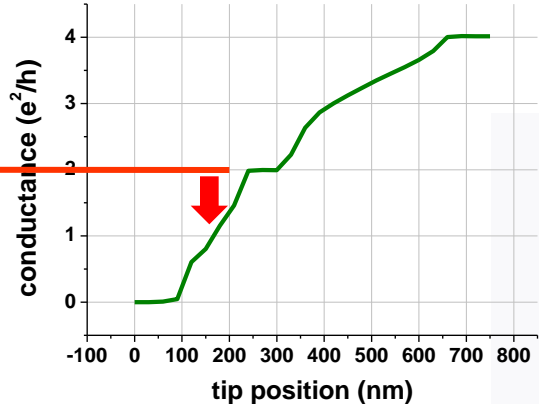
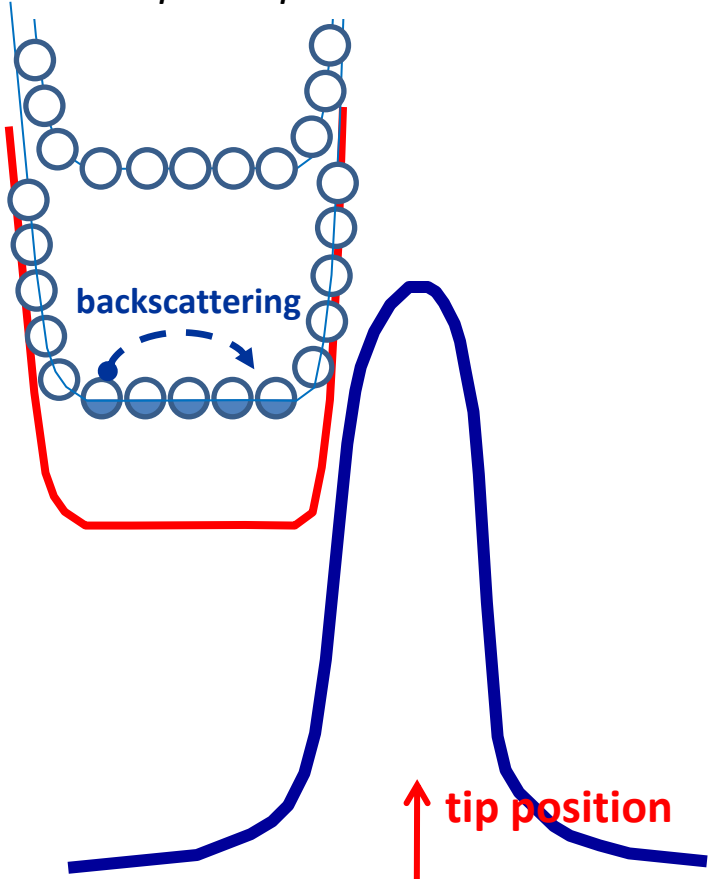


# How we probe incompressible stripes



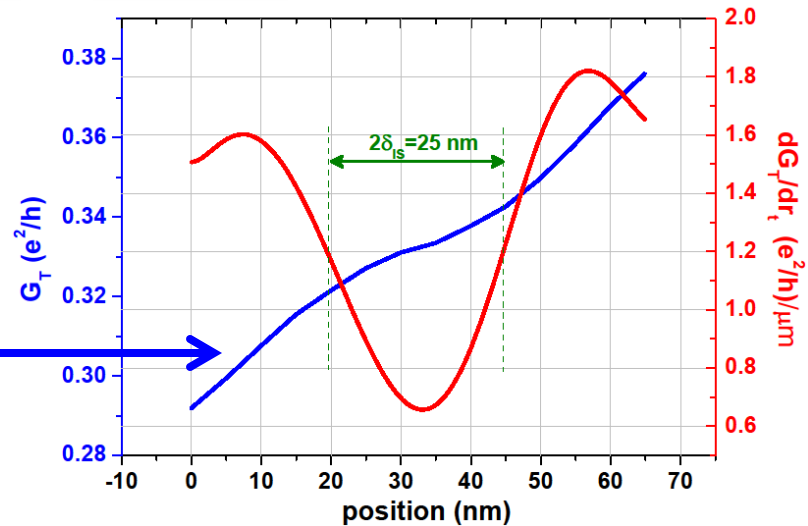
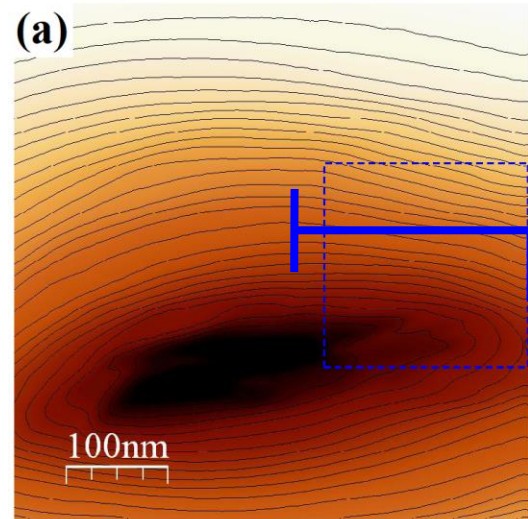
# How we probe incompressible stripes

Energy gap:  $\hbar\omega=5.7$  meV  
Plateau width: 60 nm  
Incompr. stripe width:  $\approx 30$ nm

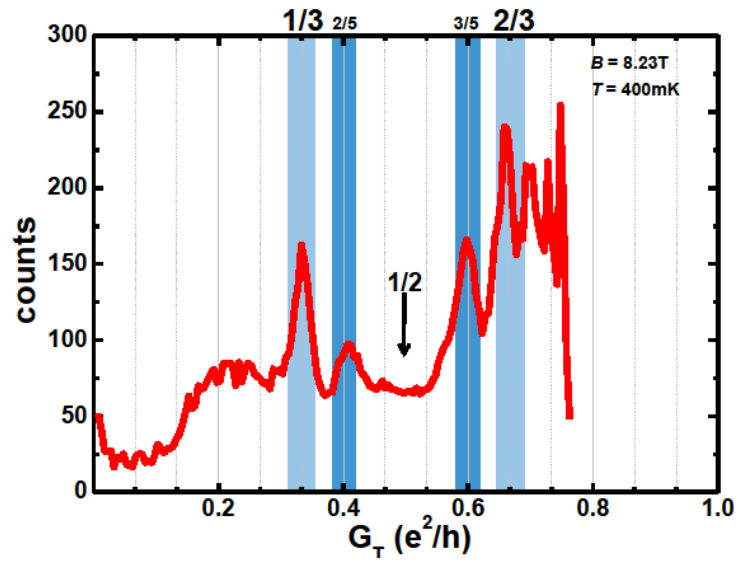
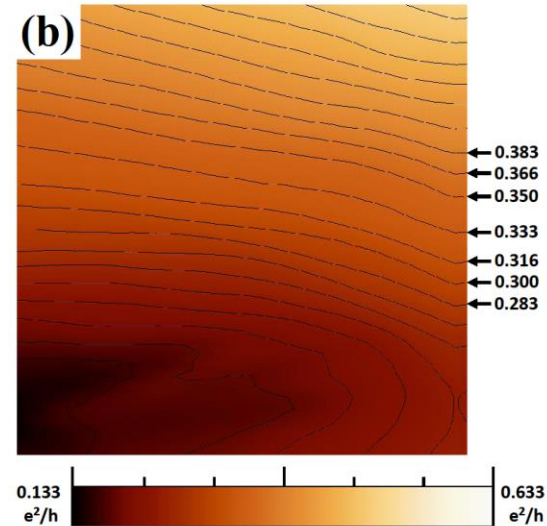




# Imaging fractional structures in integer channels

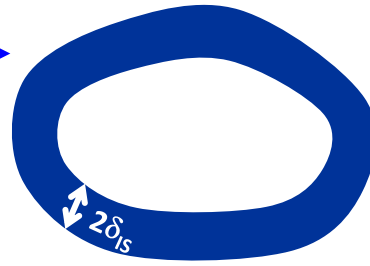
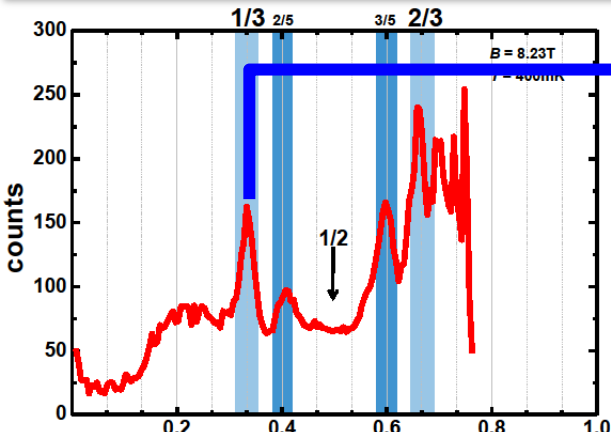


The Reconstruction Picture suggests that at the edge of a smooth **integer** edge a series of compressible/**incompressible fractional stripes** can occur. We used the SGM technique to image them.



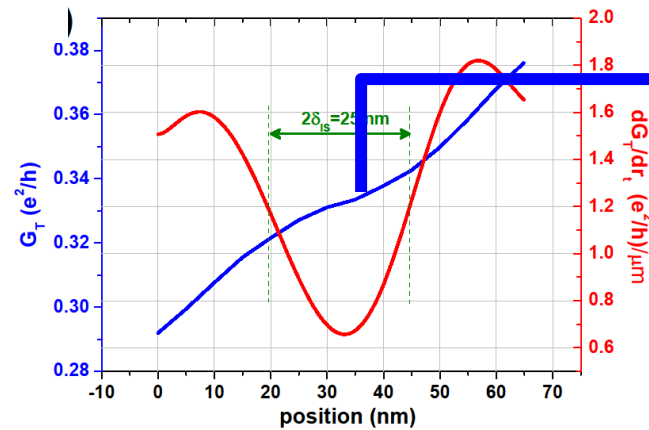
N. Paradiso *et al.*  
 Phys. Rev. Lett. **108**, 246801  
 (2012)

# Fractional edge reconstruction



the finite range in GT defines a stripe in the SGM map

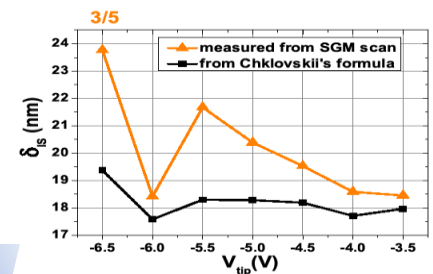
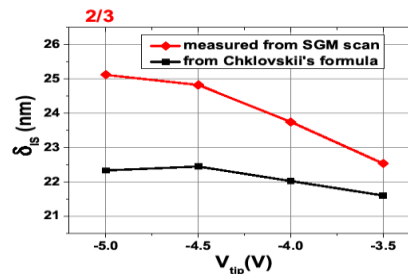
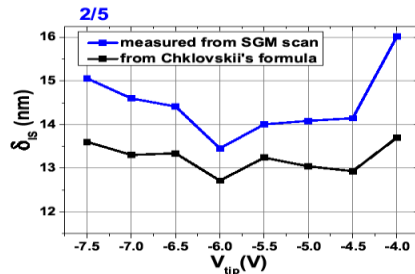
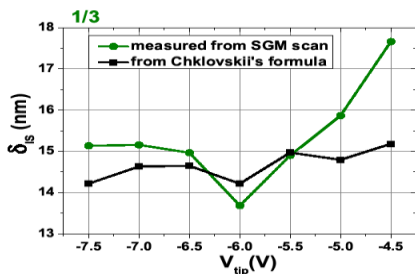
$\delta_{IS}$  determined from SGM measurements



$$\frac{dn}{dr}$$

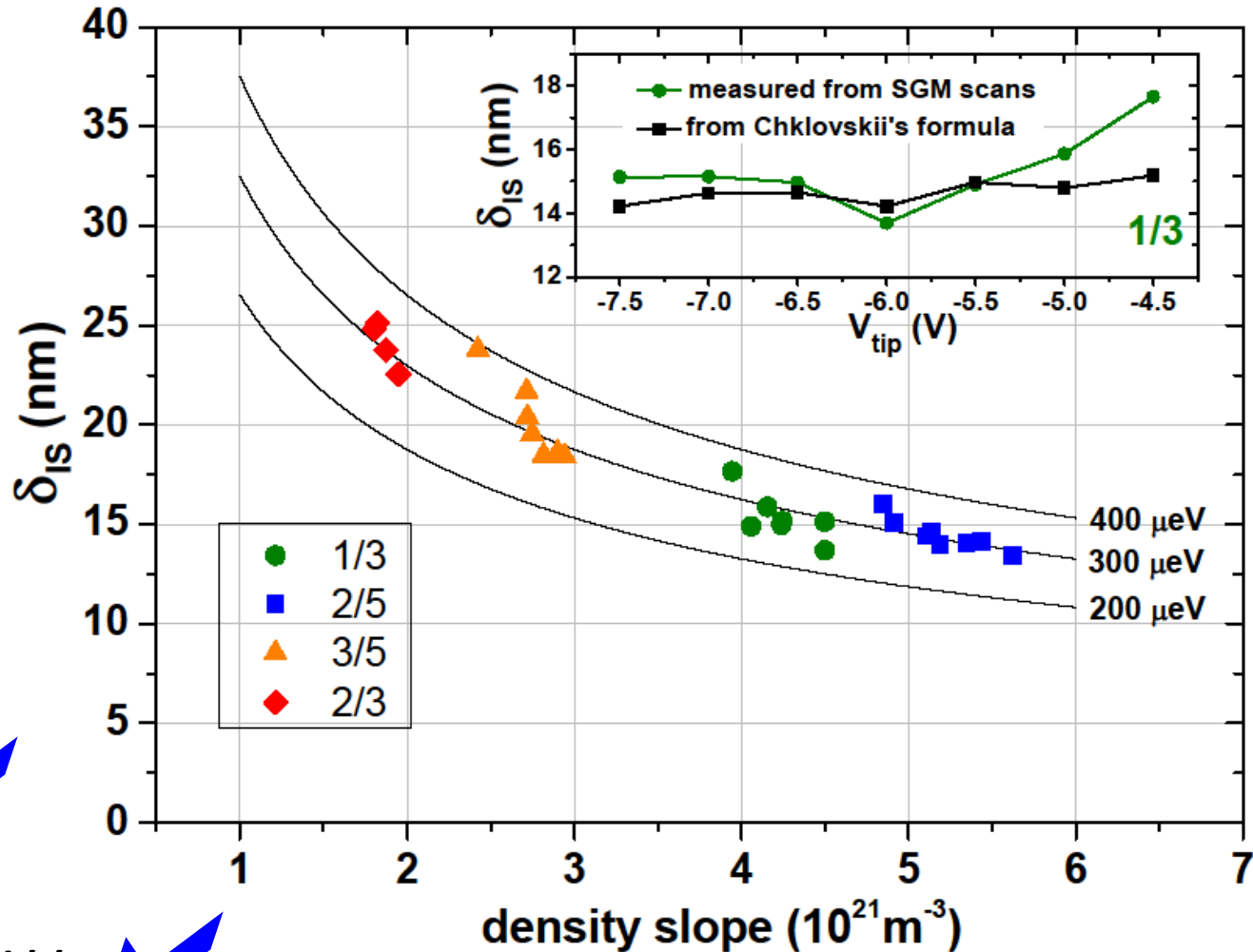
$\delta_{IS}$  determined from Chklovskii's formula

$$\delta_{IS}^2 = \frac{4\Delta\mu_f\epsilon}{\pi^2 e^2 dn/dr|_{r=r_f}}$$



# Fractional edge reconstruction

The IS width values (colored dots) obtained from SGM images compare well with the reconstruction picture predictions (black lines)

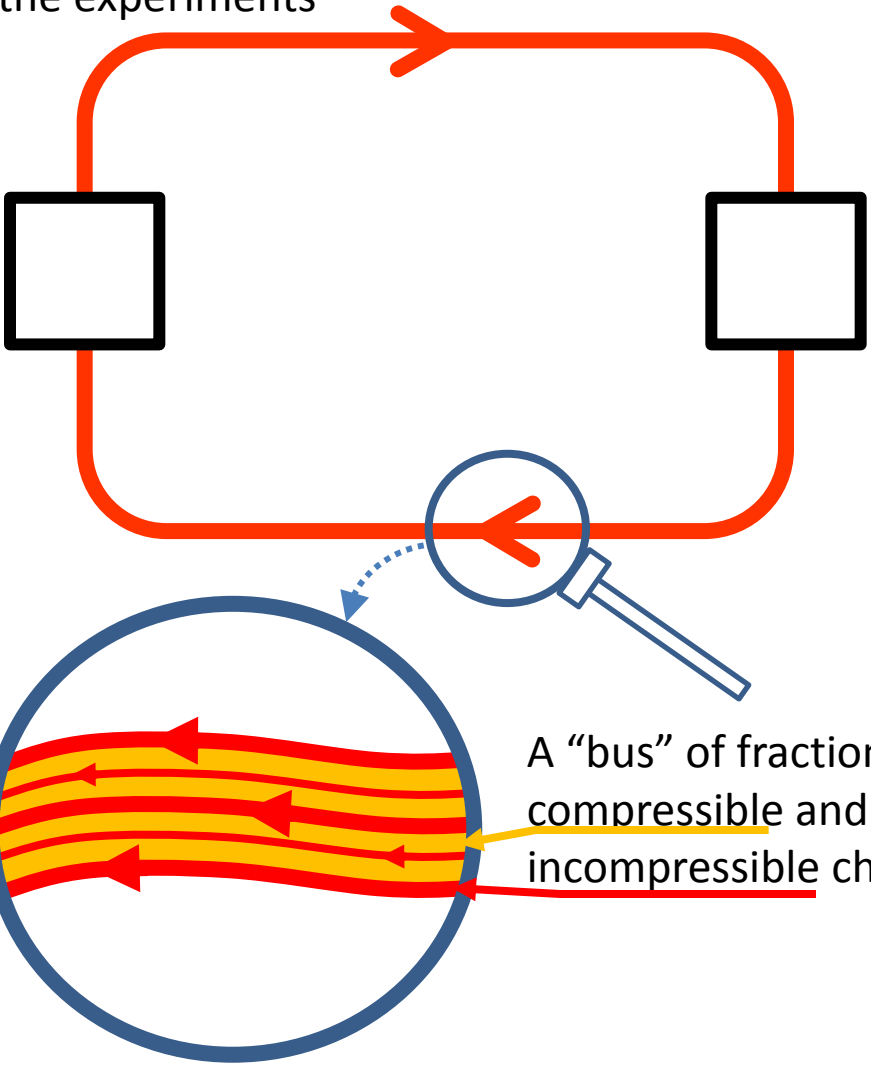


Inner edge structure demonstrated and imaged

Quantitative test of the IS width dependence on the density slope

# Can we exploit the non-trivial edge structure?

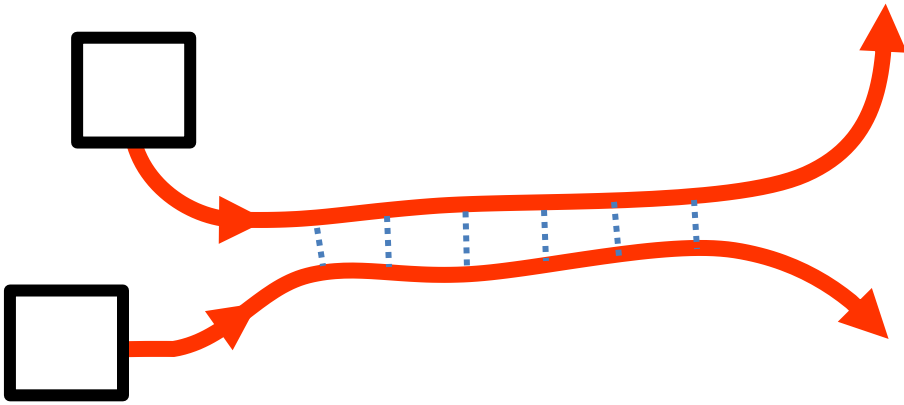
The picture of a QH device emerging from the experiments



A "bus" of fractional compressible and incompressible channels:

Charge transfer between **parallel** (i.e. *co-propagating*) edge channels

Komiyama et al. PRB **45**, 11085 (1992).  
Karmakar et al., PRL **107**, 236804 (2011)



➔ **How do these channels interact?:**

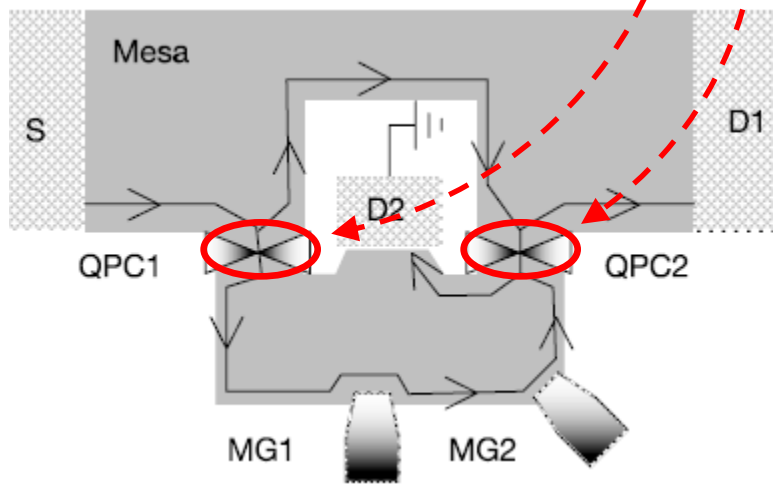
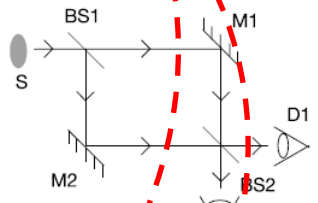
*We will exploit the SGM technique to implement a variable-geometry device to **image** the scattering processes*

# The state of the art of electronic quantum interferometry

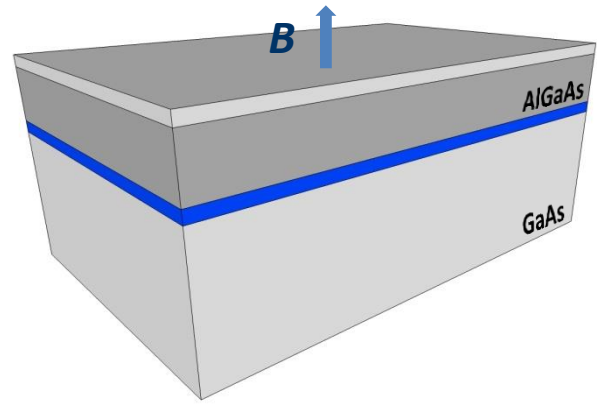
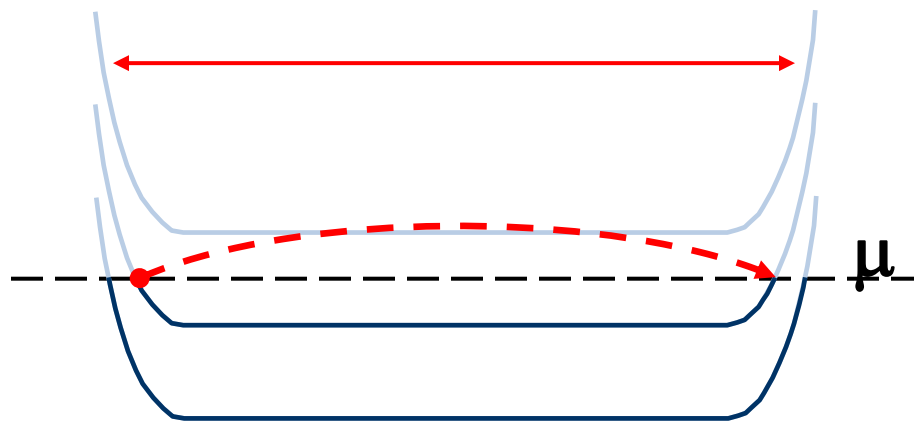
At the **beam splitters** the electrons are backscattered into the **counter-propagating edge** through two quantum point contacts (QPCs)

NATURE | VOL 422 | 27 MARCH 2003  
**An electronic Mach-Zehnder interferometer**

Yang Ji, Yunchul Chung, D. Sprinzak, M. Heiblum, D. Mahalu & Hadas Shtrikman



we induce backscattering by reducing this distance



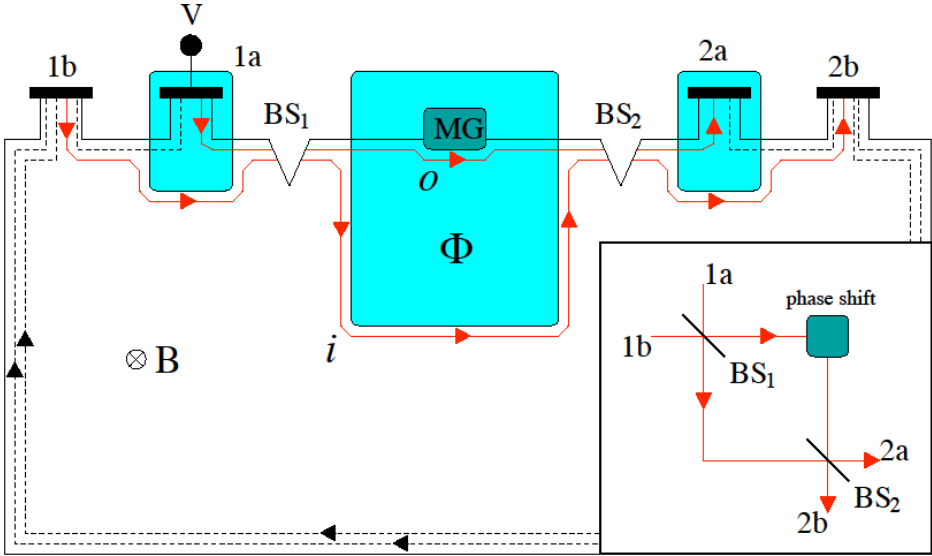
# A new architecture for QH interferometry

a simply connected QH interferometer:  
the proposal of *Giovannetti et al.*

PHYSICAL REVIEW B 77, 155320 (2008)

Multichannel architecture for electronic quantum Hall interferometry

Vittorio Giovannetti,<sup>1</sup> Fabio Taddei,<sup>1</sup> Diego Frustaglia,<sup>2</sup> and Rosario Fazio<sup>1,3</sup>



New architecture: beam splitters induce mixing between **co-propagating edge channels**

**Advantages:**

- simply connected topology (no air bridges)
- very small  $\Phi$  area, only a few flux quanta are involved
- the device is scalable: it is possible to put many devices in series

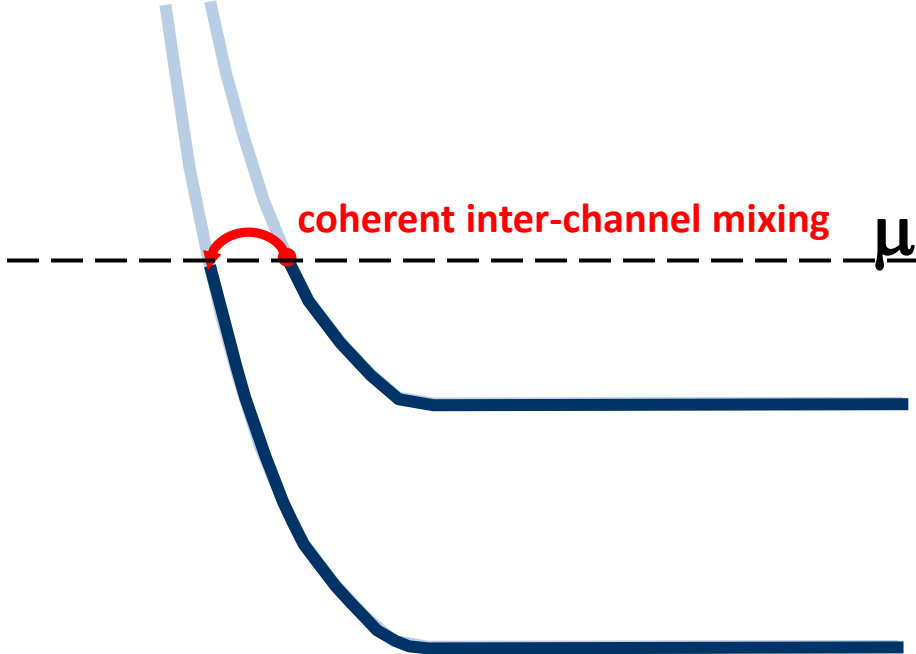
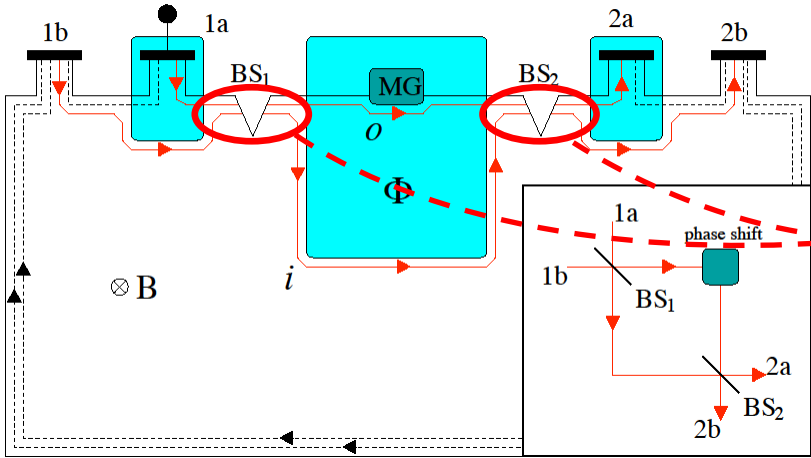
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the only elusive parts are the **beam mixers**  
between **co-propagating channels**

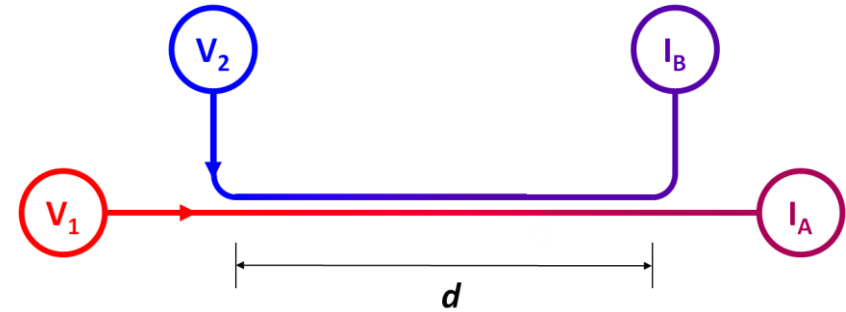
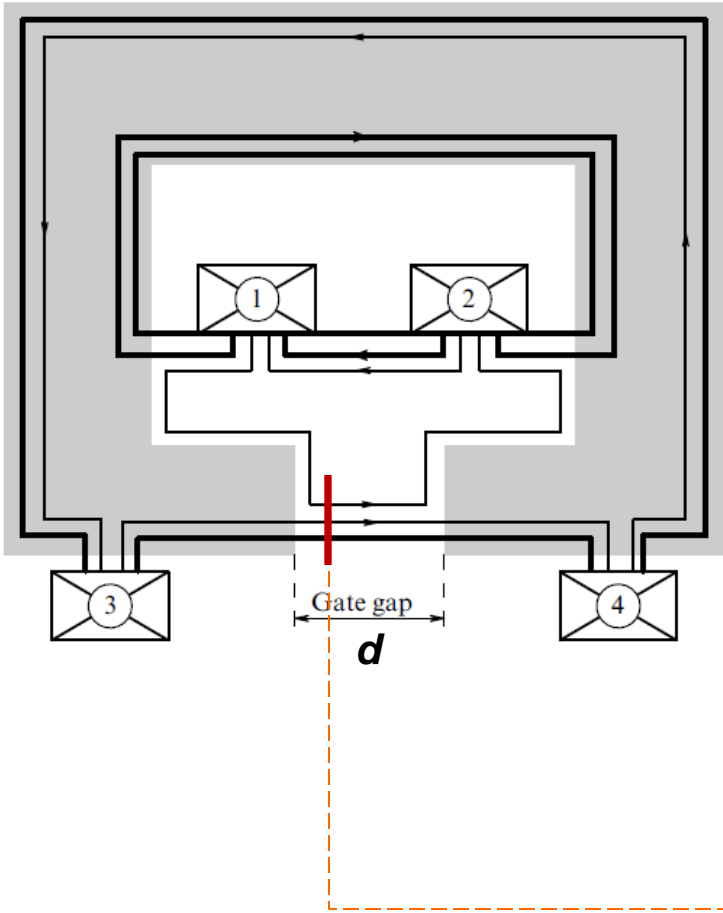
Is it possible to study and image  
the microscopic details of the  
inter-channel scattering?

# Studying the inter-channel equilibration

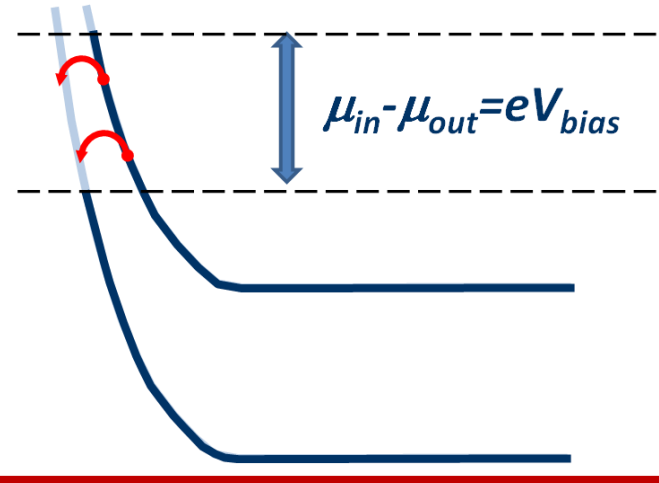
Edge states in the regimes of integer and fractional quantum Hall effects

E V Deviatov

*Physics – Uspekhi* 50 (2) 197–218 (2007)



devices with fixed interaction length  $d$ :  
elusive determination of **the microscopic details of the equilibration mechanisms**

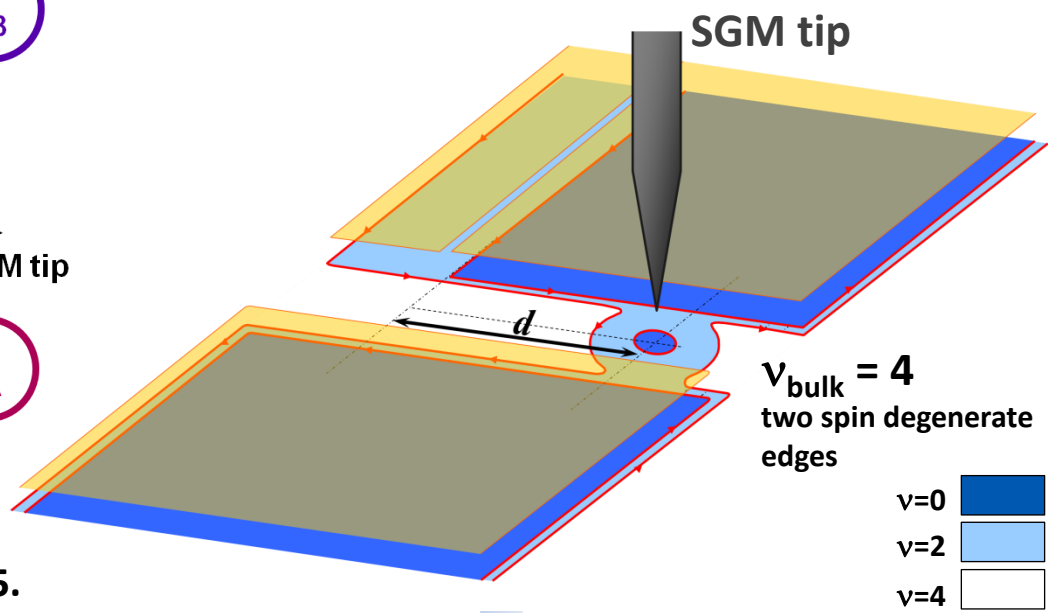
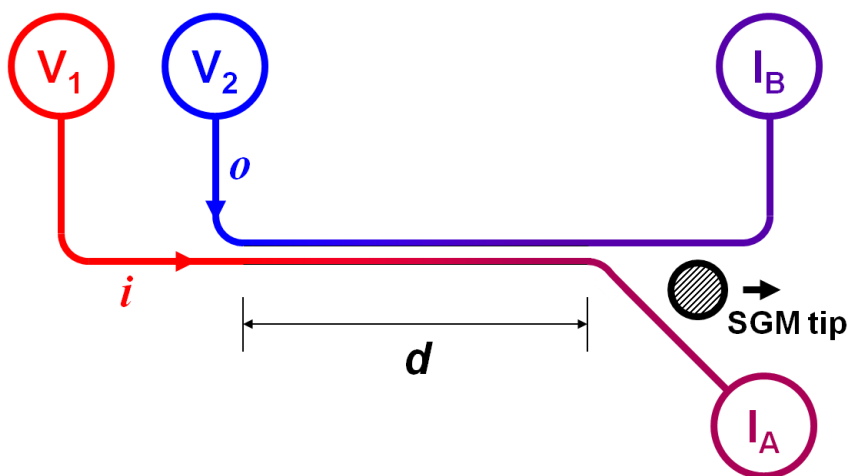




# The opportunity of the Scanning Gate Microscopy

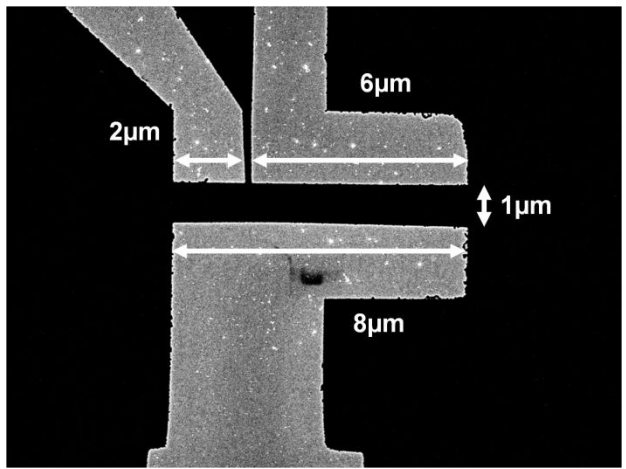
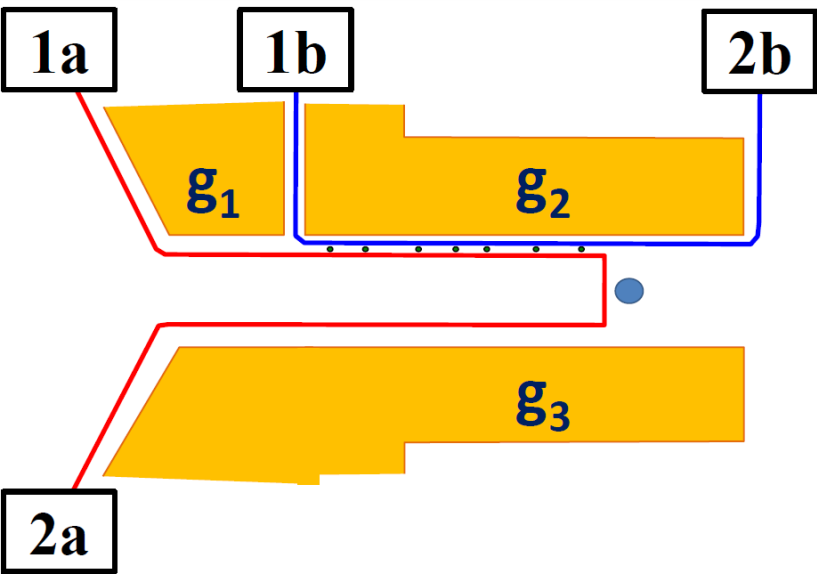
Our technique allows to selectively control the channel trajectory

**Our idea:** exploit the mobile depletion spot induced by the SGM to **continuously tune  $d$**

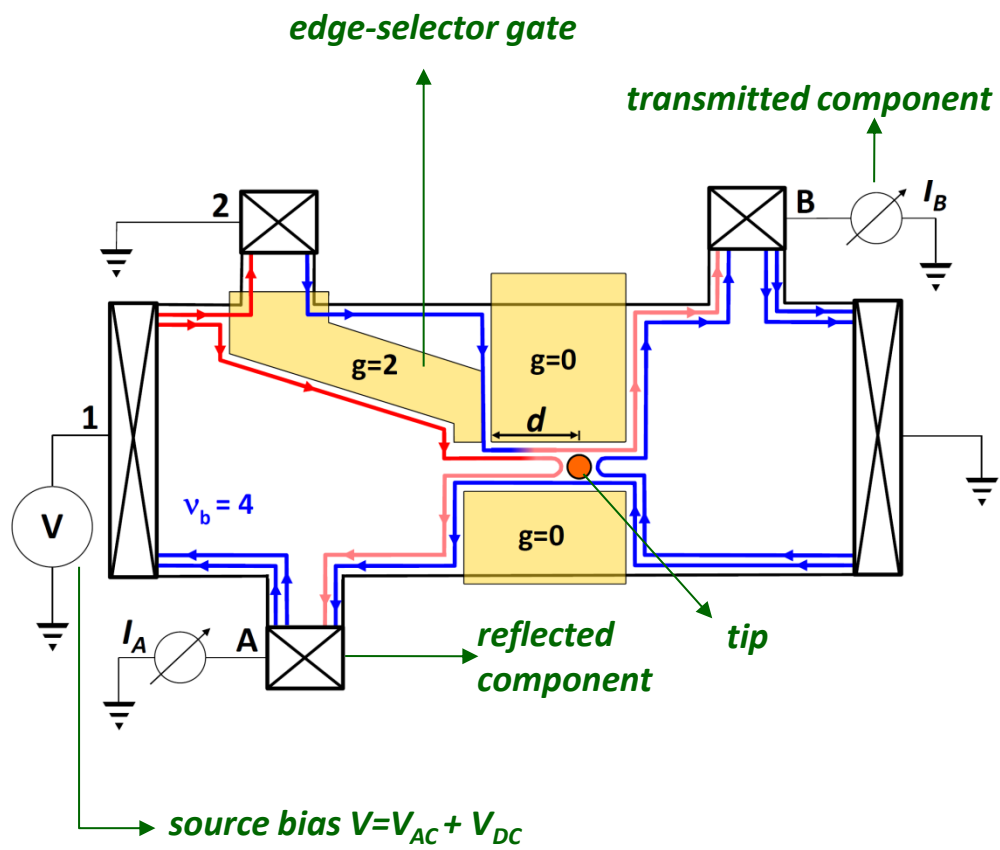


N. Paradiso *et al.* Phys. Rev. B 83 (2011) 155305.

# Experimental setup

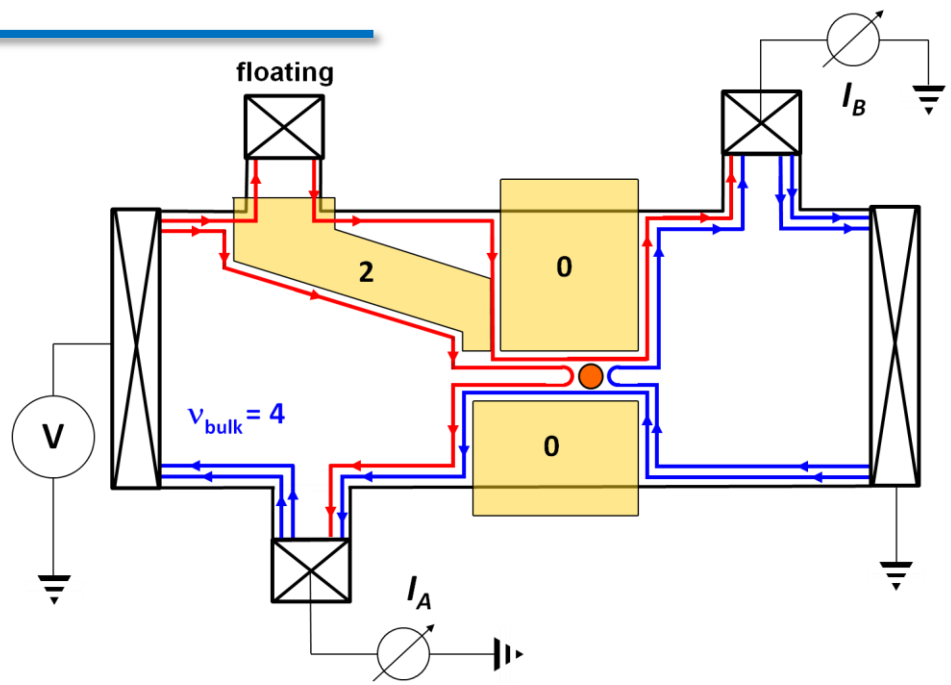
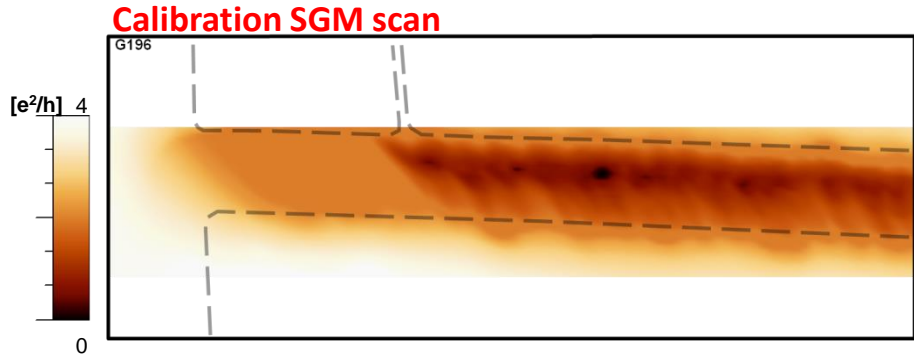
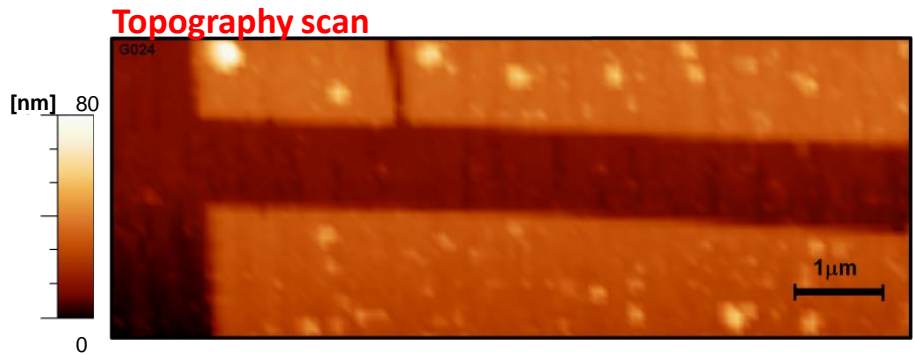


SEM micrograph of the device



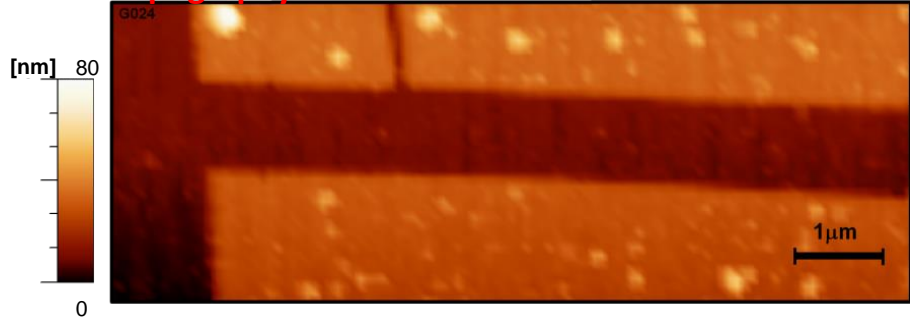
Scheme of the electronic setup

# Calibration step

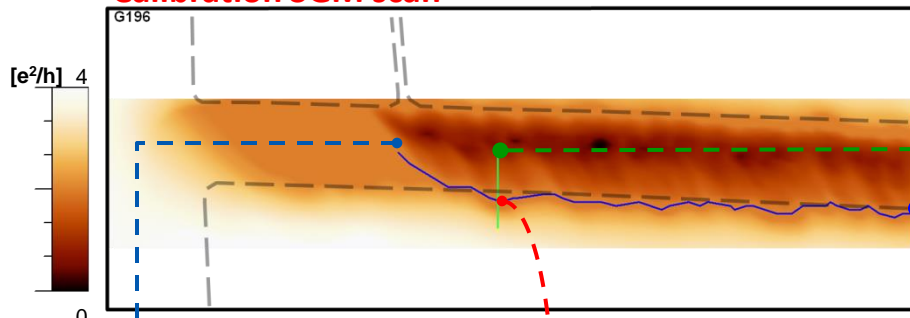


# Calibration step

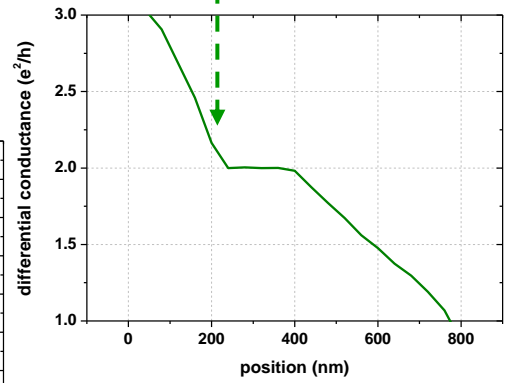
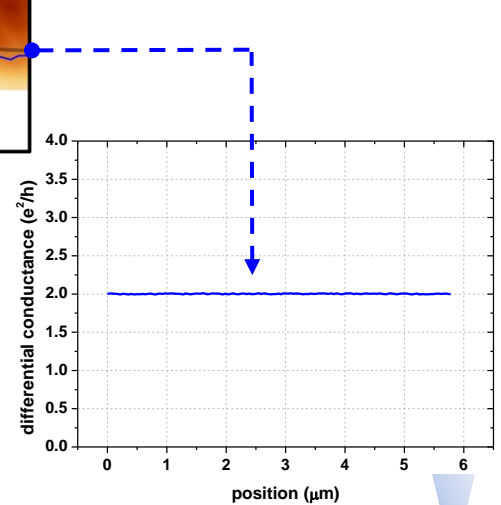
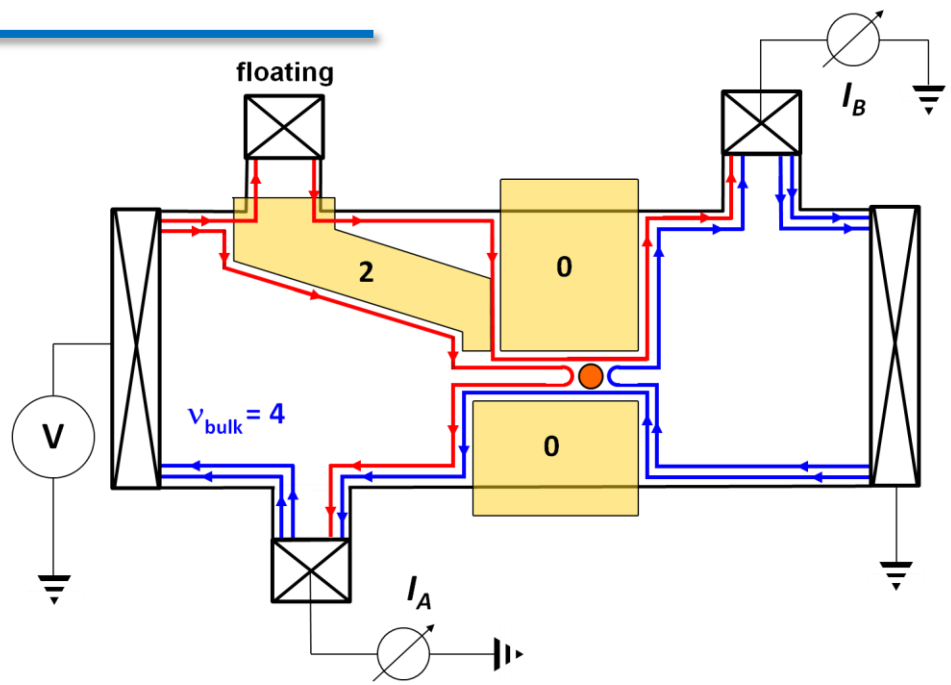
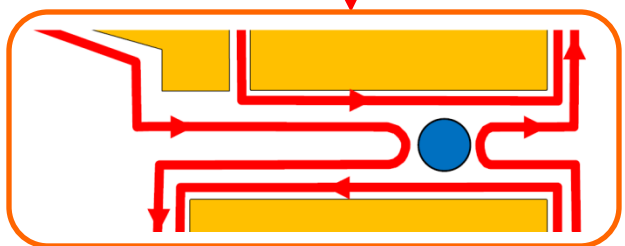
Topography scan



Calibration SGM scan

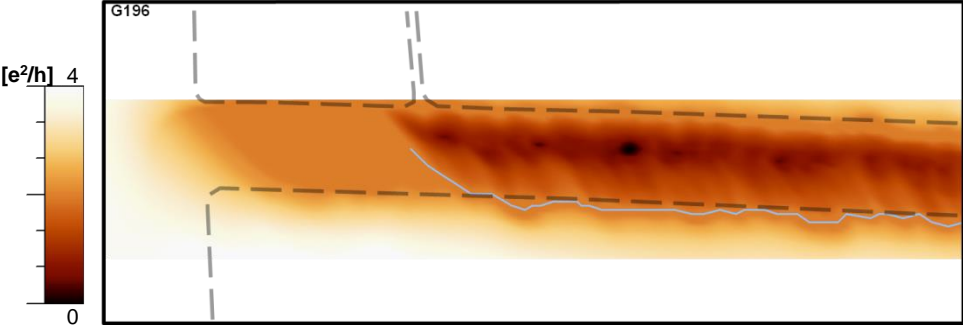


the edges meet here



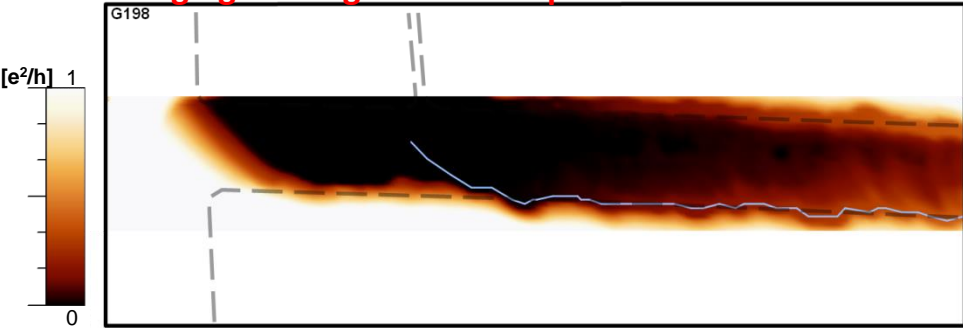
# Imaging the inter-channel equilibration

Calibration SGM scan

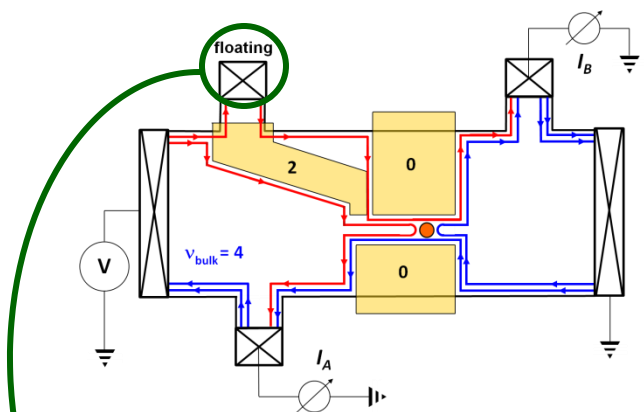


**SGM map of the  $I_B$  signal: direct imaging of the equilibration process.**

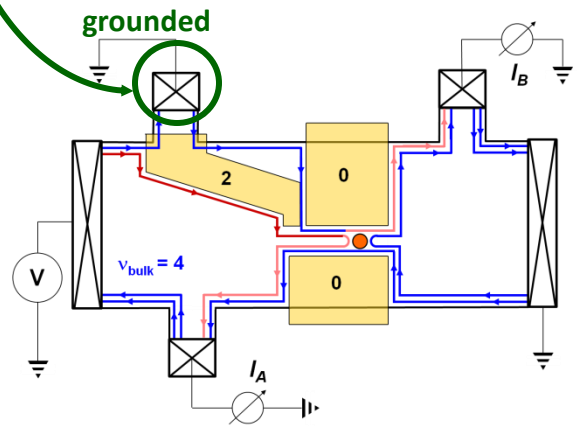
Imaging the edge channel equilibration



Source bias:  $V_{AC}=50\mu V, V_{DC}=0mV$



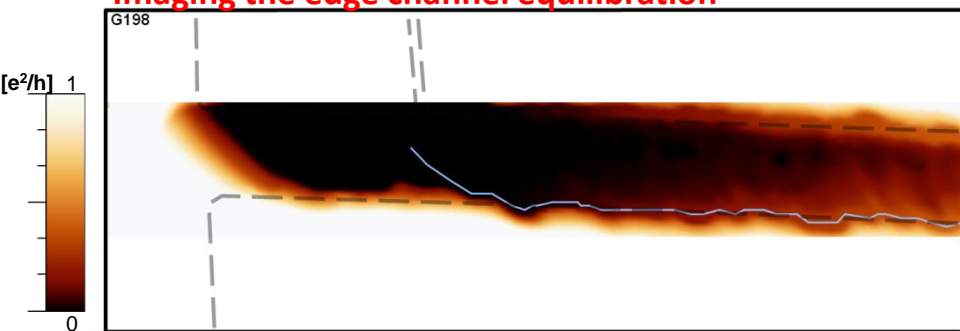
By grounding the upper contact an imbalance is established between the edges.



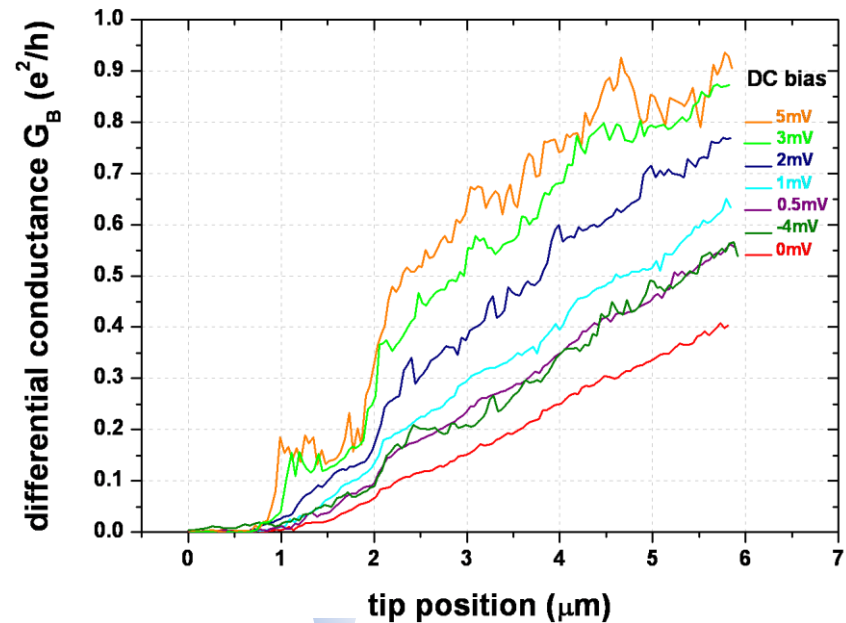
# Imaging the inter-channel equilibration

The profiles of  $G_B(d)$  along the trajectory show a strict dependance on the local details

Imaging the edge channel equilibration



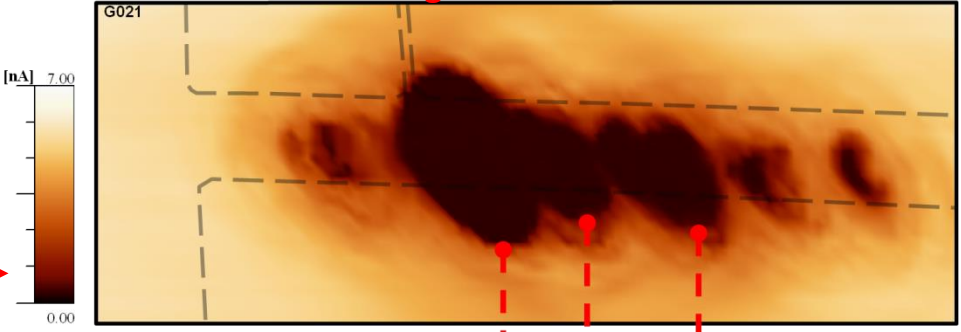
Source bias:  $V_{AC}=50\mu\text{V}$ ,  $V_{DC}=0\text{mV}$



# Imaging the inter-channel equilibration

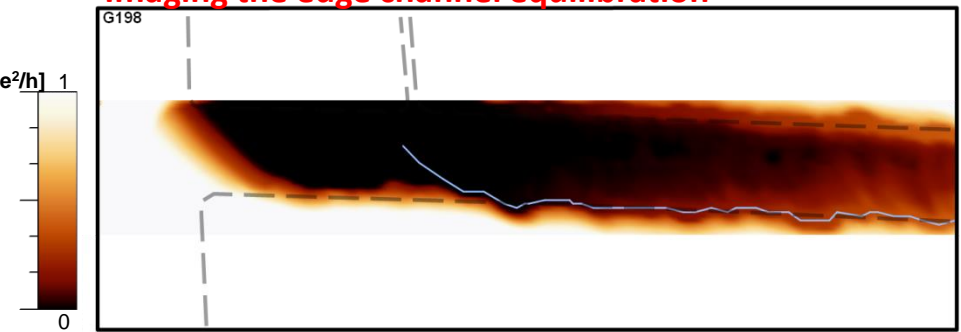
We can directly image the potential induced by the most important defects by means of a scan at zero magnetic field

SGM scan at zero magnetic field



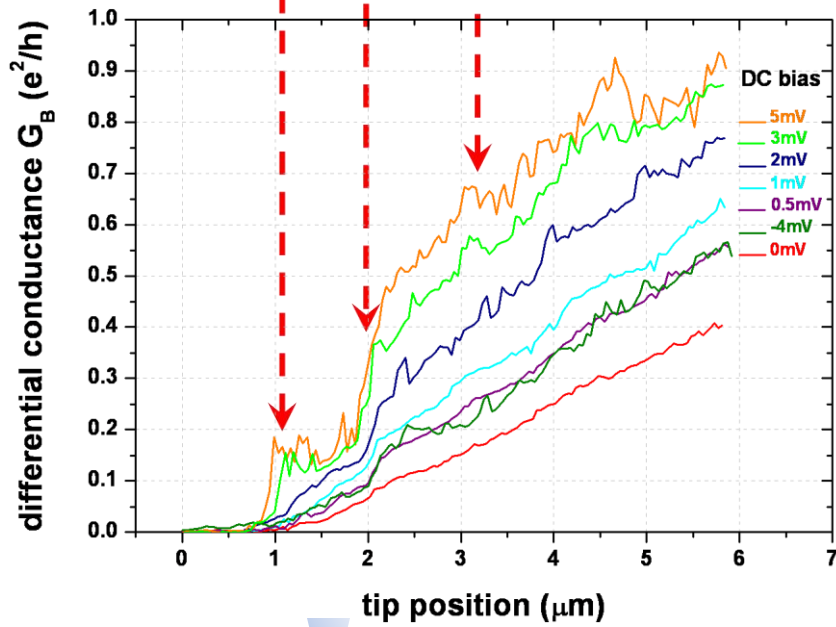
The profiles of  $G_B(d)$  along the trajectory show a strict dependence on the local details

Imaging the edge channel equilibration

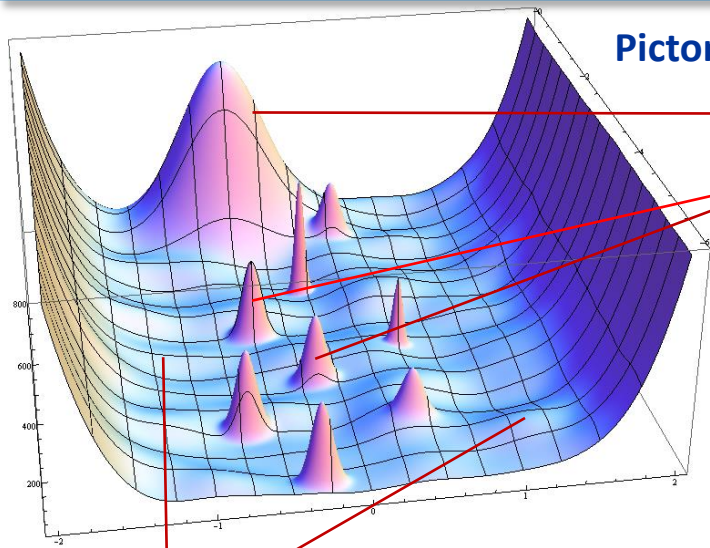


Source bias:  $V_{AC}=50\mu V, V_{DC}=0mV$

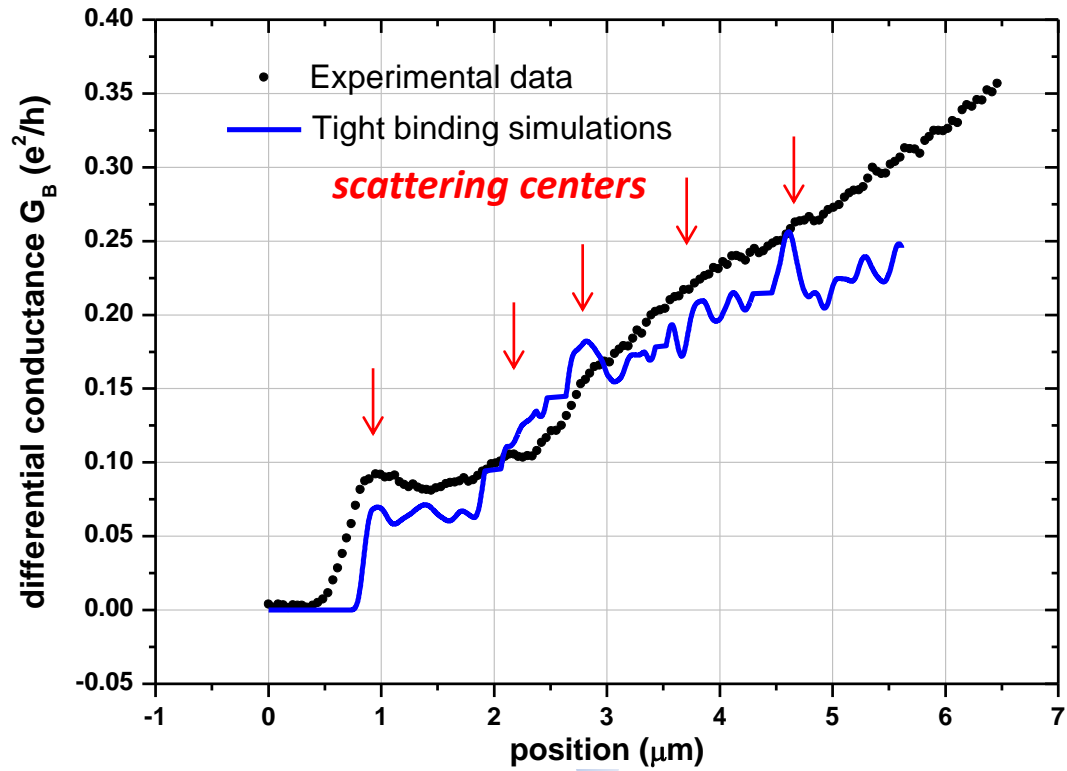
correlation found



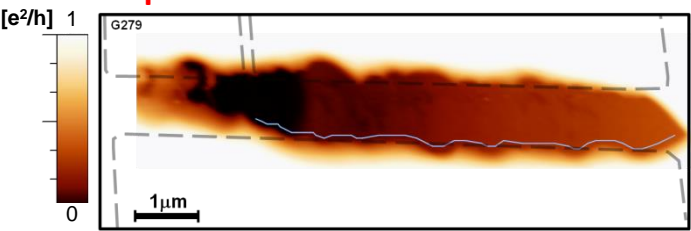
# Tight binding simulations



Simulations made by the theoretical group of  
Scuola Normale Superiore (Pisa, Italy)  
D. Venturelli, F. Taddei, V. Giovannetti and R. Fazio

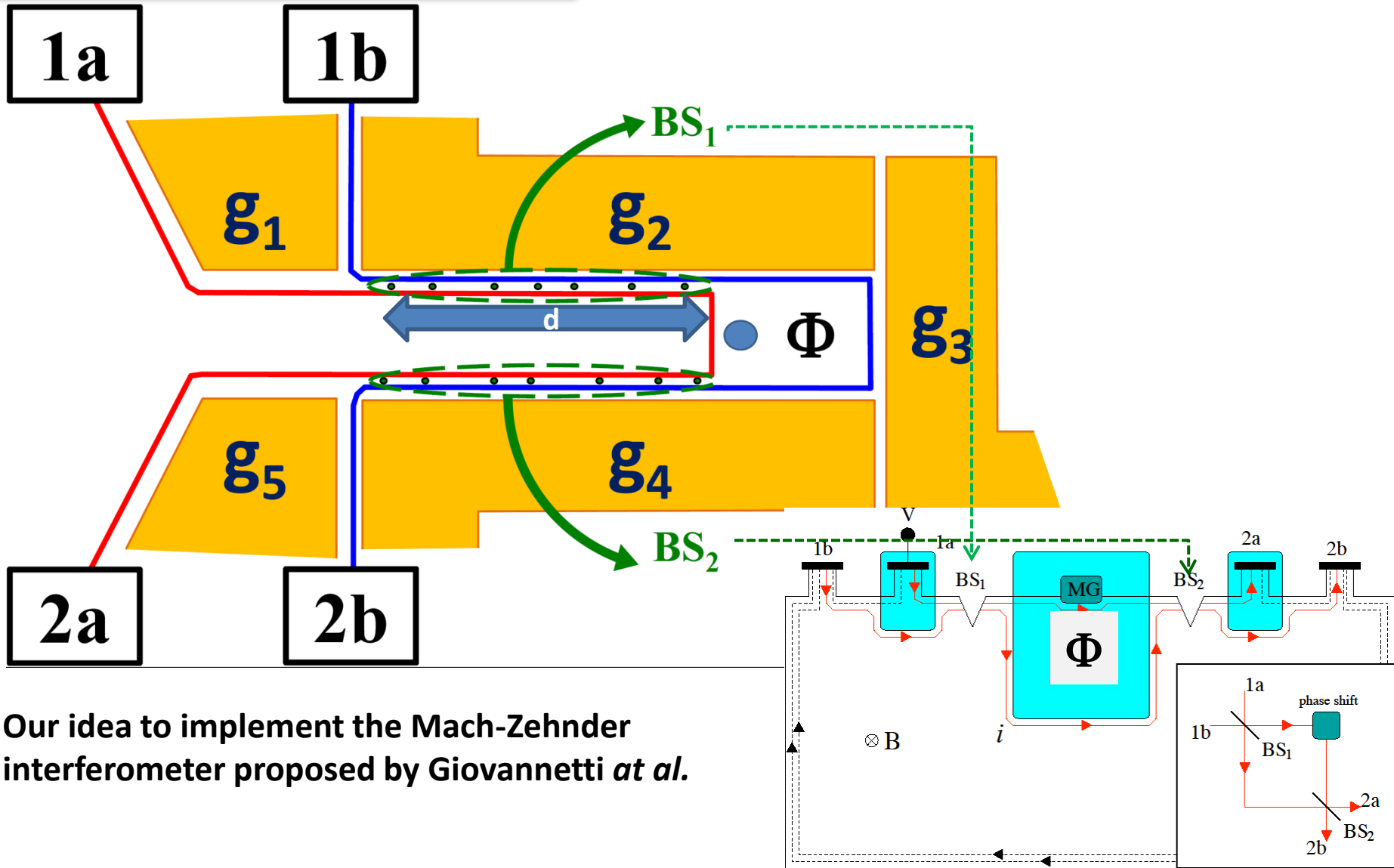


SGM map of the inter-channel equilibration in another device





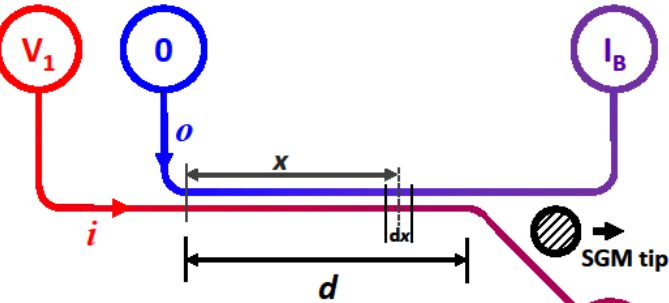
# Next step: a simply connected MZI



Our idea to implement the Mach-Zehnder interferometer proposed by Giovannetti *at al.*

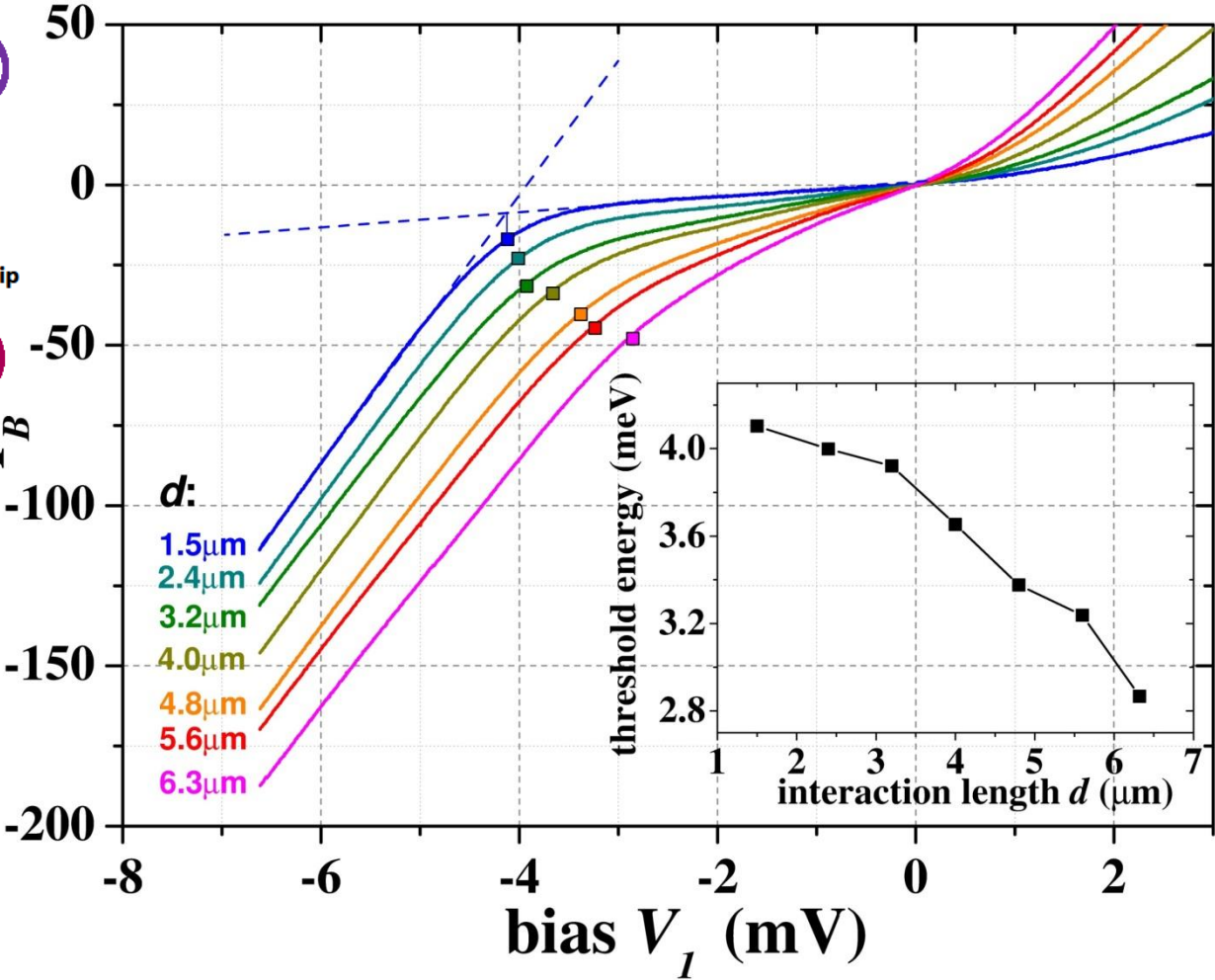
# Nonlinear regime

N. Paradiso *et al.*: Phys. Rev. B 84, 235318 (2011)

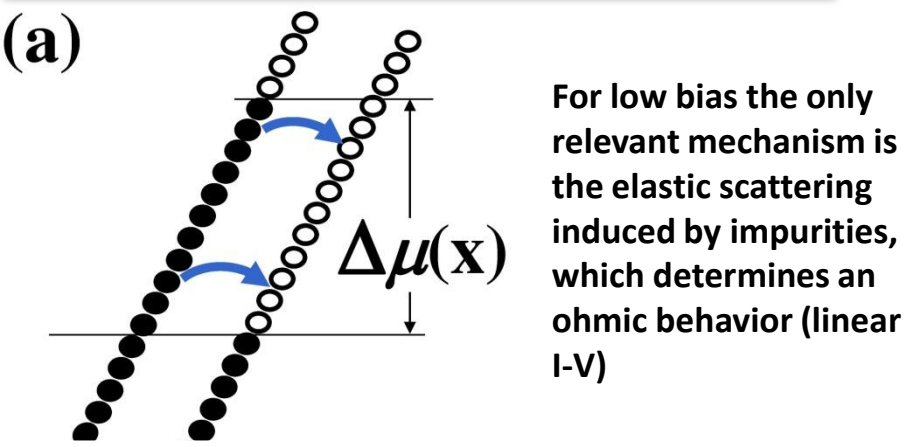


$$\frac{dI}{dx} = -\frac{e^2}{h} \frac{d}{dx} \Delta V(x) = \Phi(\Delta V(x))$$

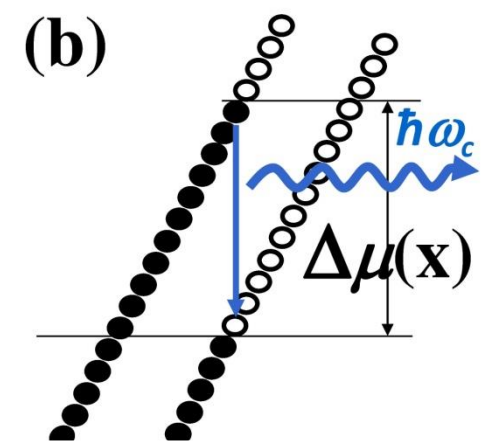
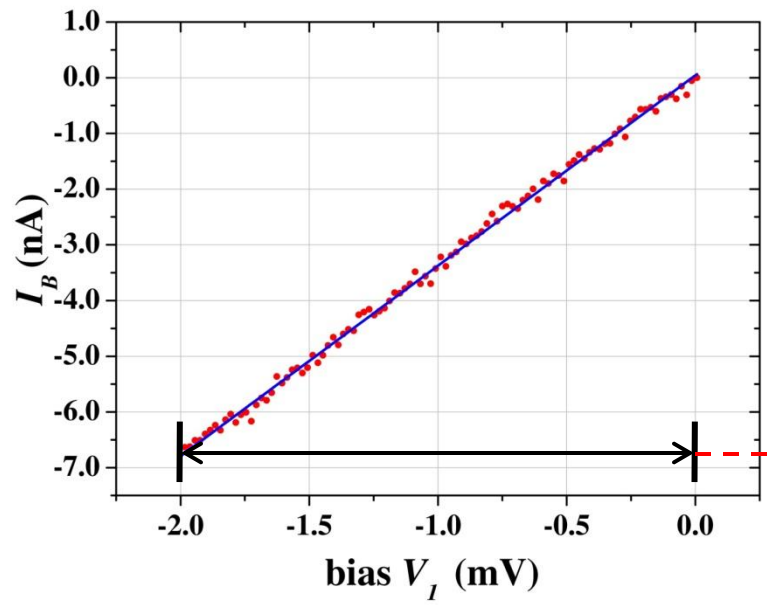
The backscattered current is a function of the local imbalance  $\Delta V(x)$  that depends on the specific scattering process.



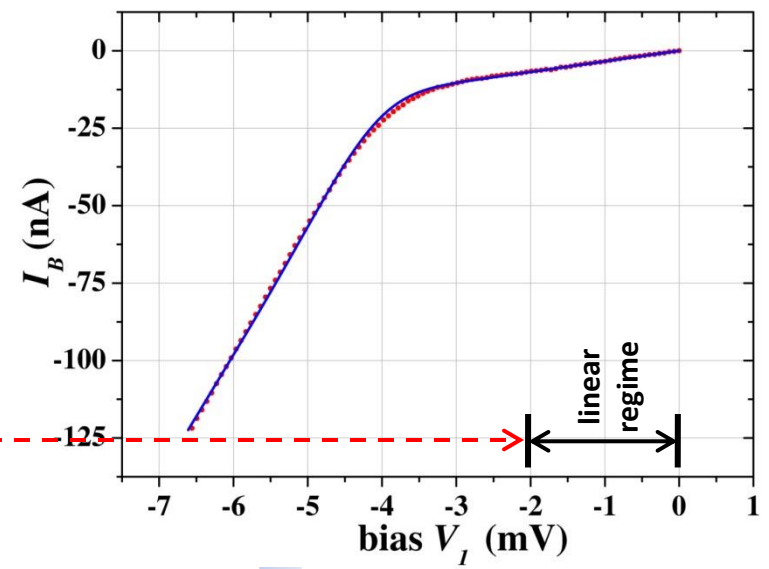
# Two mechanisms for the inter-channel scattering



$$dI = dx \frac{2eT_0}{hv_d} \int_{-\infty}^{\infty} (f_{\mu_i, T}(\epsilon) - f_{\mu_o, T}(\epsilon)) d\epsilon = dx \frac{2e^2 T_0}{hv_d} \Delta V(x)$$



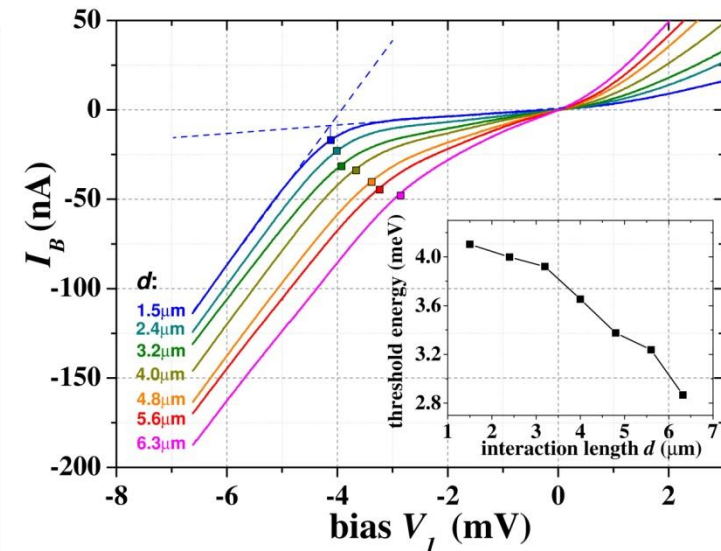
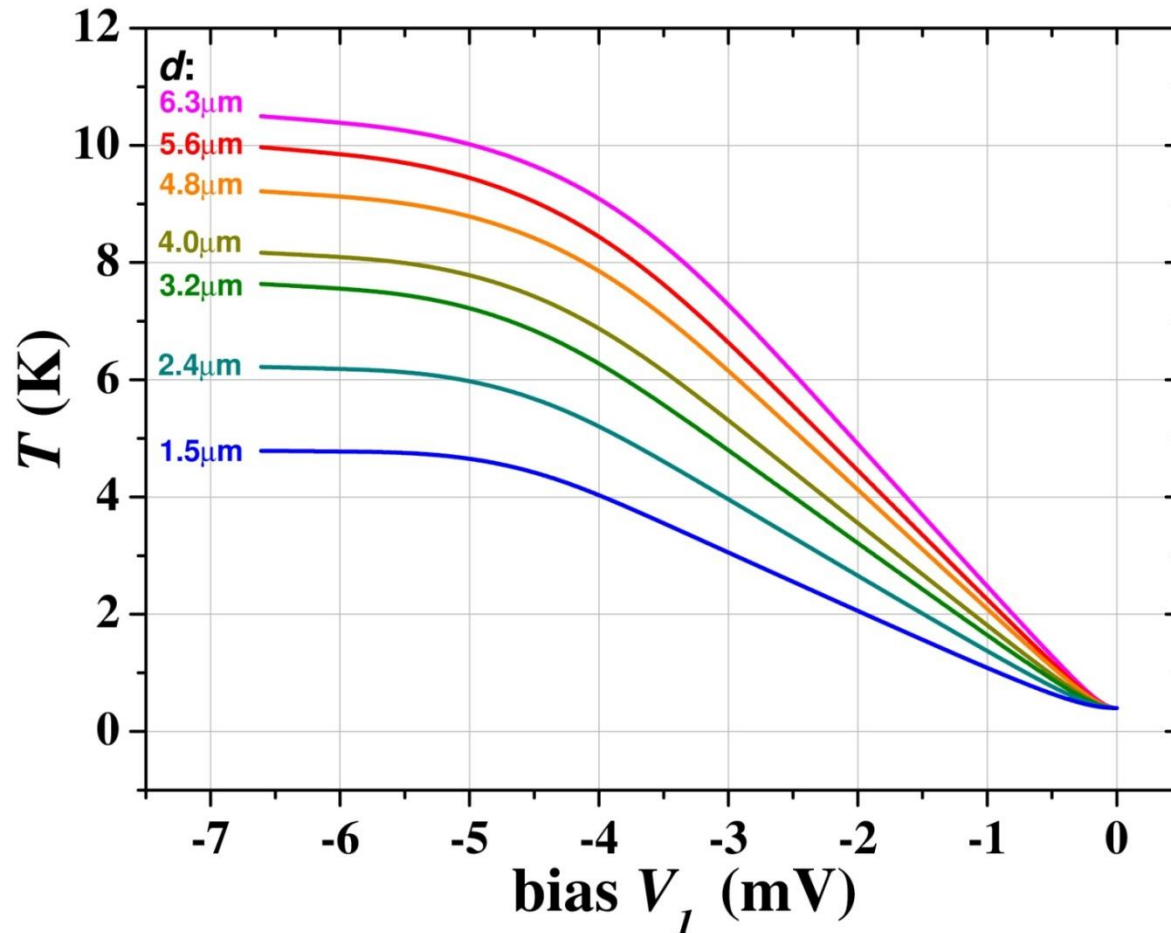
$$dI = dx \frac{2eT_1}{hv_d} \int_{-\infty}^{\infty} [f_{\mu_i, T}(\epsilon)(1 - f_{\mu_o, T}(\epsilon - \hbar\omega_c))] d\epsilon = dx \frac{2eT_1}{hv_d} \left( \frac{e\Delta V(x) - \hbar\omega_c}{1 - e^{-\frac{\hbar\omega_c - e\Delta V(x)}{k_B T}}} \right)$$



# Impact of the electron heating

Electron heating due to injection of hot carriers:

$$\frac{d}{dx}T(x) = \frac{3e^2}{4\pi^2 k_B^2 \ell_{eq}} \frac{\Delta V^2(x)}{T(x)}$$

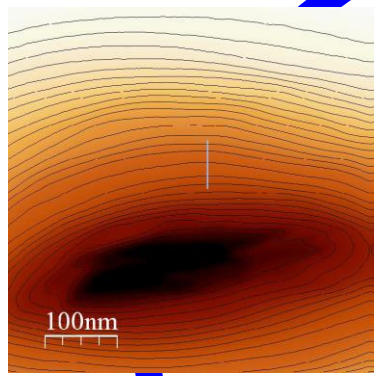
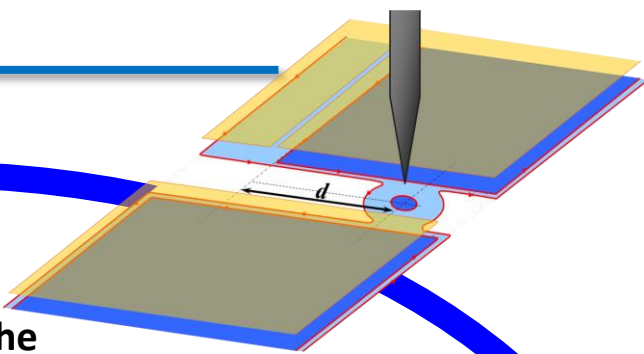


The relaxation of hot carriers induces a dramatic temperature increase. This is why the transition is smoothed and the threshold voltage reduced for high  $d$

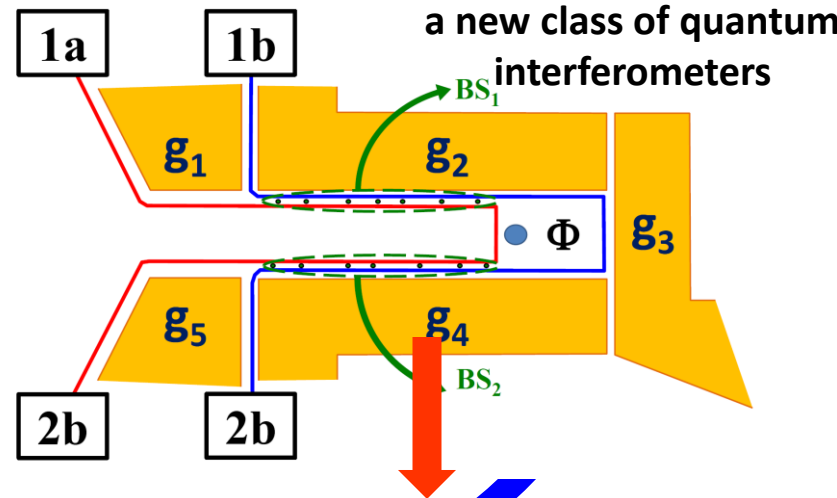
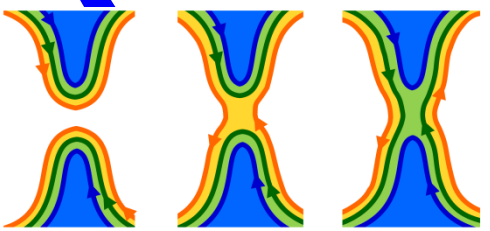
# Summary and outlook

**SGM technique in the QH regime**

size-tunable QH circuits: study of the channel mixing



Edge channel tomography: imaging of fractional stripes



**Future directions: Interference of fractional quasi-particles ?**

# Thank you for your attention!

**Nicola Paradiso**

Scuola Normale Superiore, Pisa

[nicola.paradiso@sns.it](mailto:nicola.paradiso@sns.it)

*How everything began...  
[LT-AFM installation, Pisa, October 2007]*



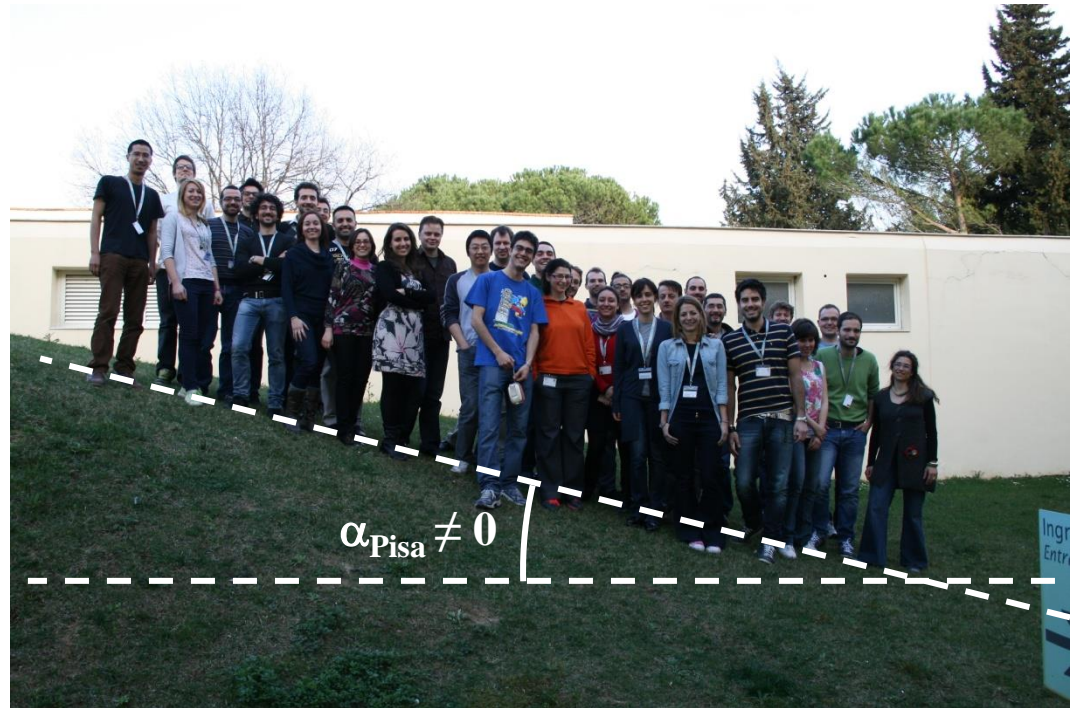
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