A programmable metrological standard based on the quantum Hall effect

Laurea Magistrale in Fisica della Materia

Luigi Caputo

Appello di maggio 2021

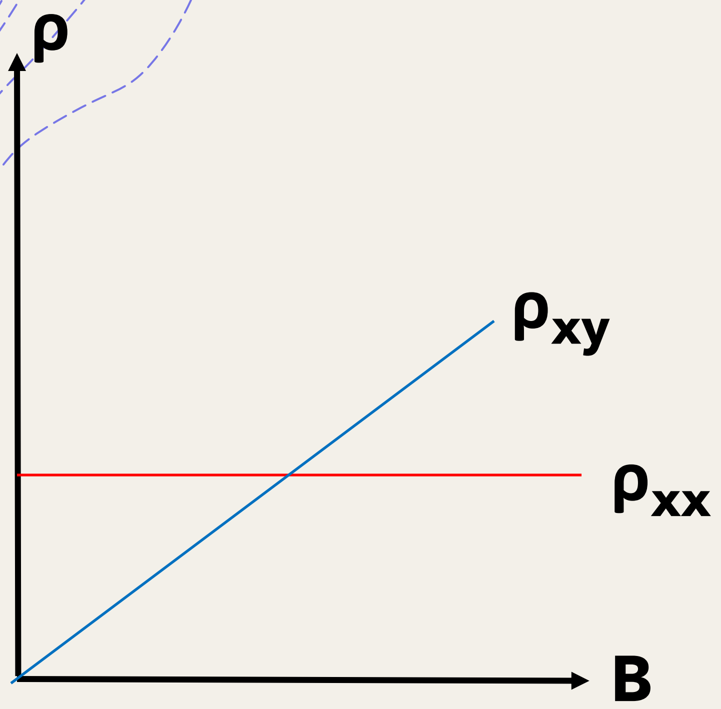
IN SUPREMÆ DIGNITATIS 1343

NEST National Enterprise for nanoScience and nanoTechnology

Presentation outline

1) **The quantum Hall effect & quantum Hall Metrology**
2) Novel device
3) Experimental results
4) Numerical results
5) Conclusions and perspectives

(Quantum) Hall effect phenomenology

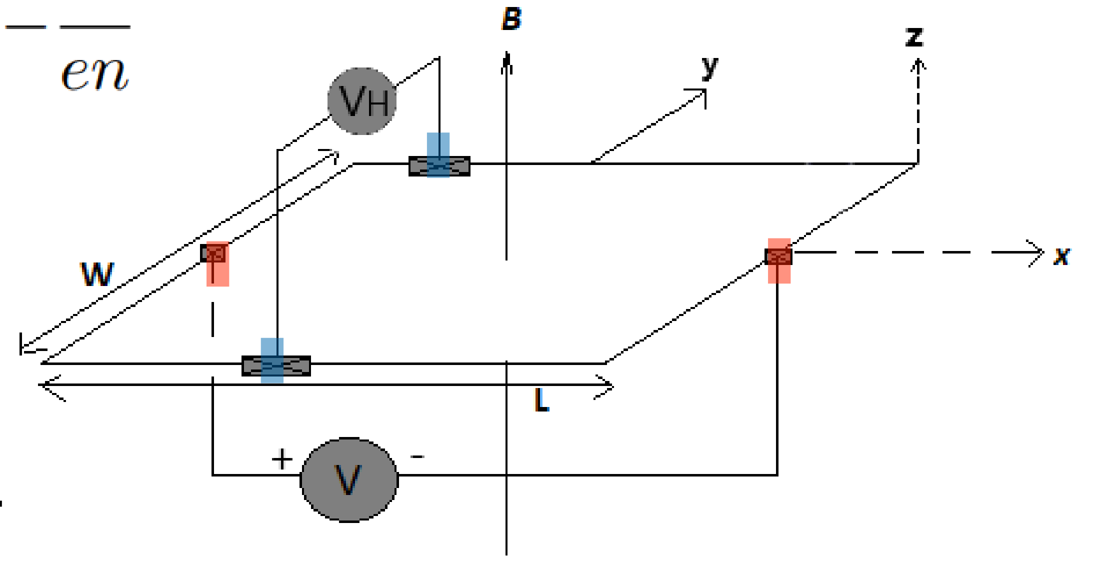


$$\rho_{xy} = \frac{B}{en}$$

$$\rho_{xx} = \frac{m^*}{\tau e^2 n}$$

$$R_H \equiv \frac{\rho_{yx}}{B} = -\frac{1}{en}$$

→ Drude theory



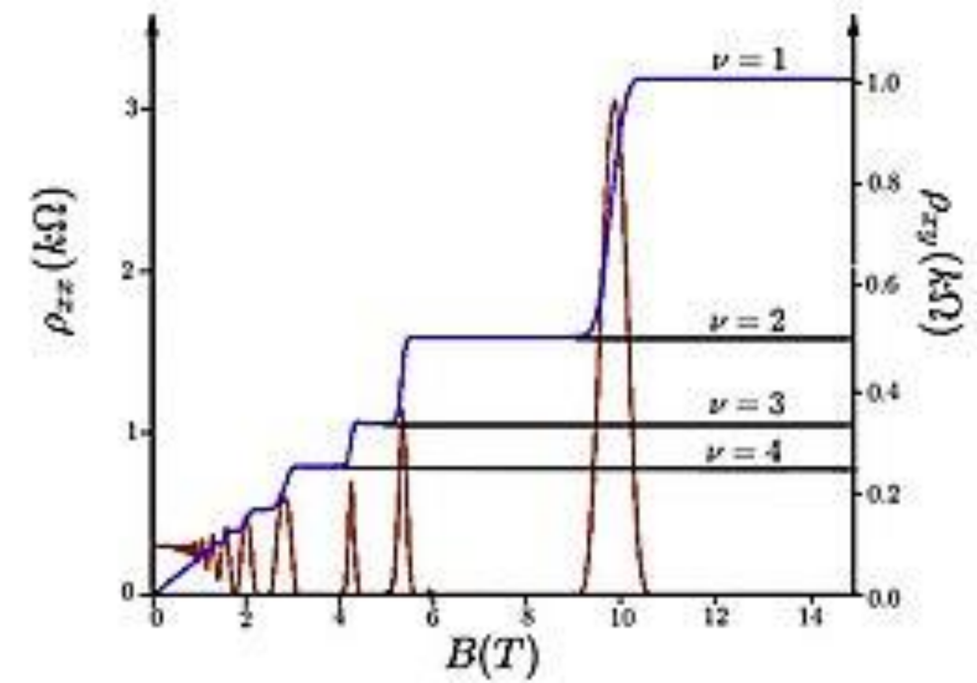
2D conductor ←

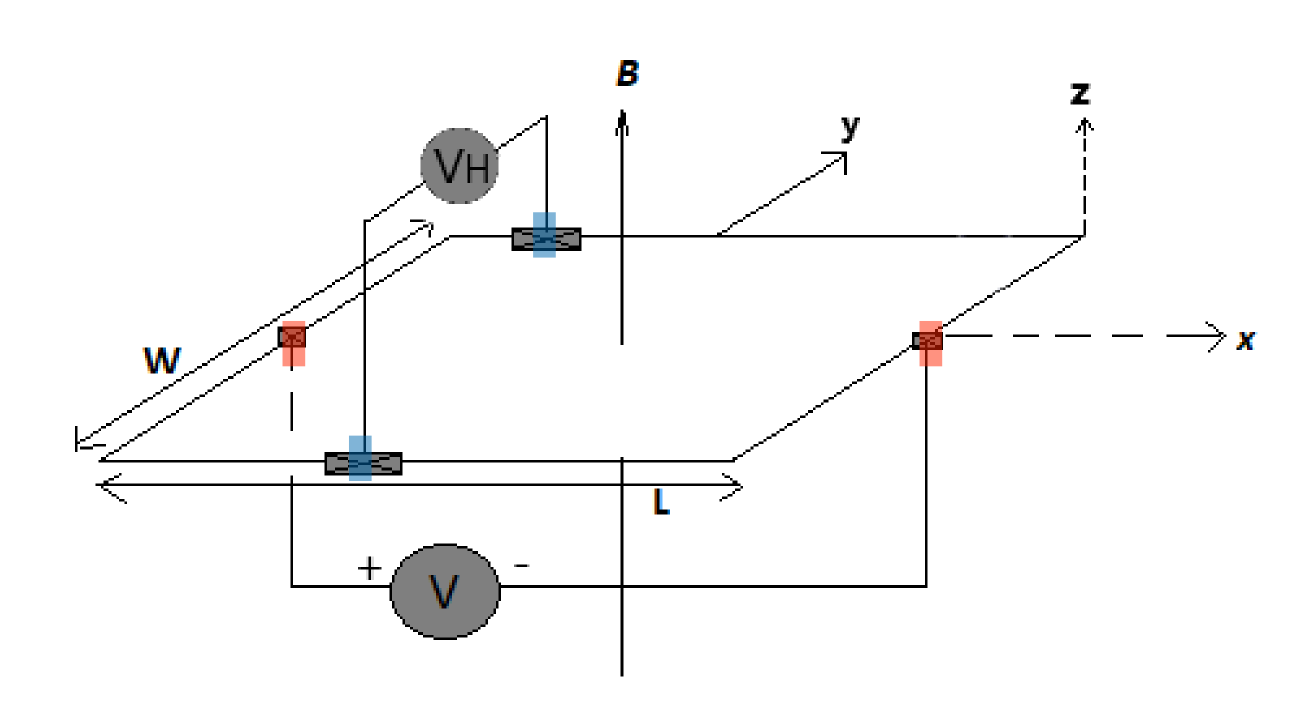
$$\nu = \frac{n\hbar}{eB}$$

Fundamental constants dependence

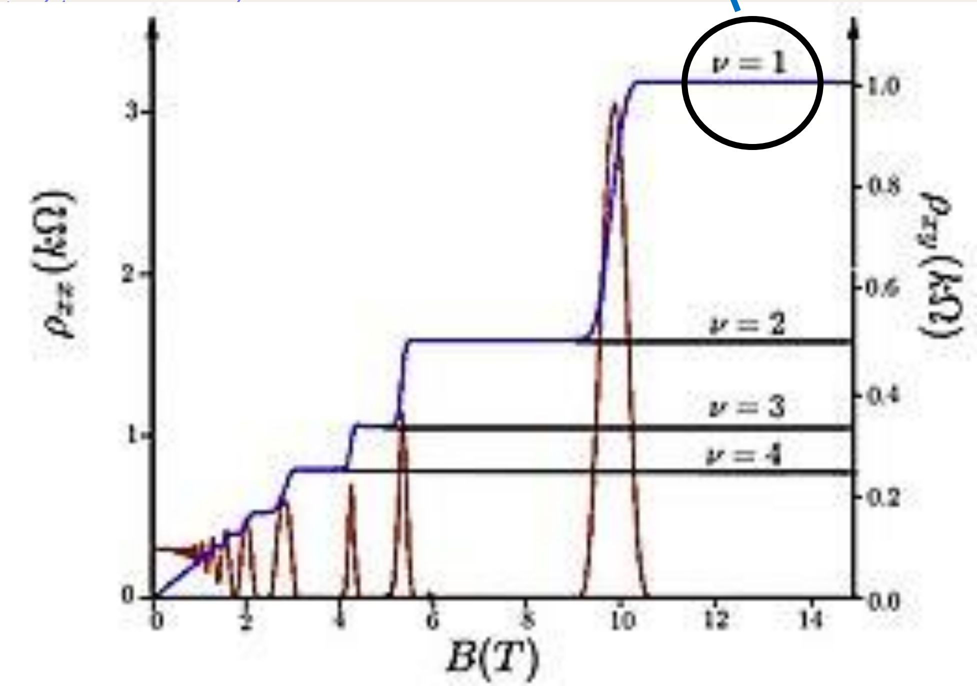
$$\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$$

$R_H = R_K/\nu$





R_K = 25.81280755710 kΩ
Verified up to 11 decimal figures

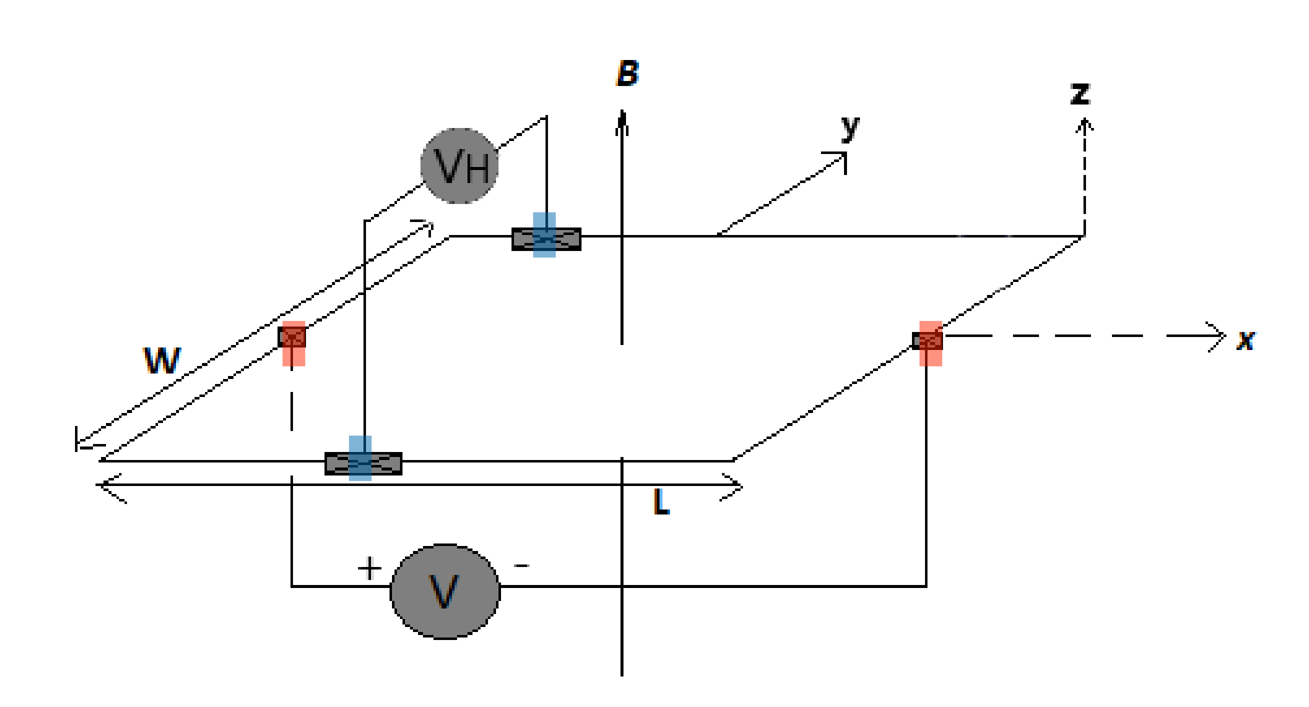


$$\nu = \frac{n\hbar}{eB}$$

$$\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$$

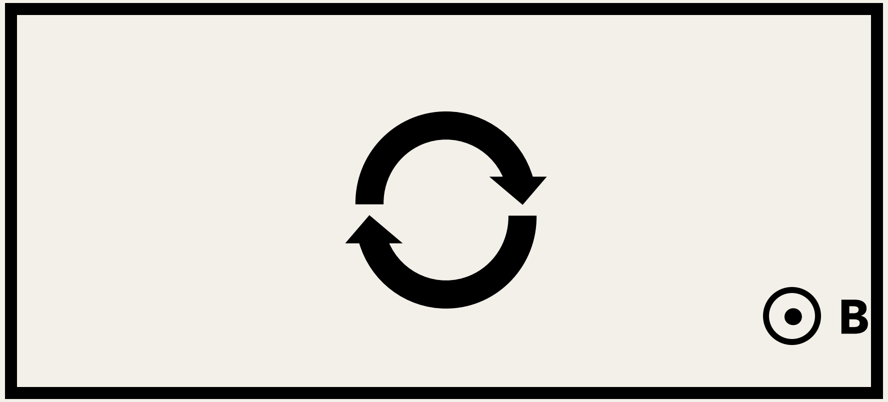
Fundamental constants dependence

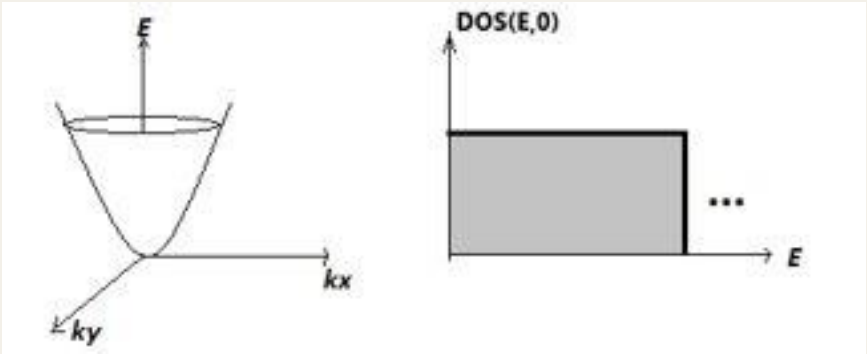
$R_H = R_K/\nu$



Quantum Hall effect physics

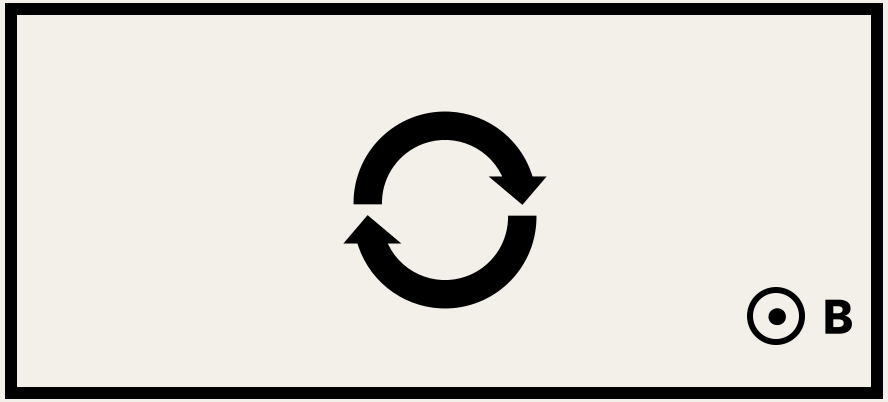
Integer quantum Hall ← **Cyclotron orbits quantization (Landau levels)**

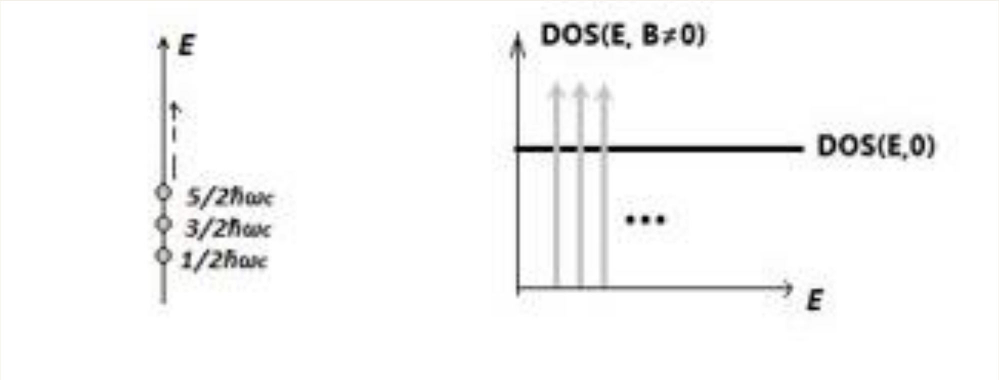




Quantum Hall effect physics

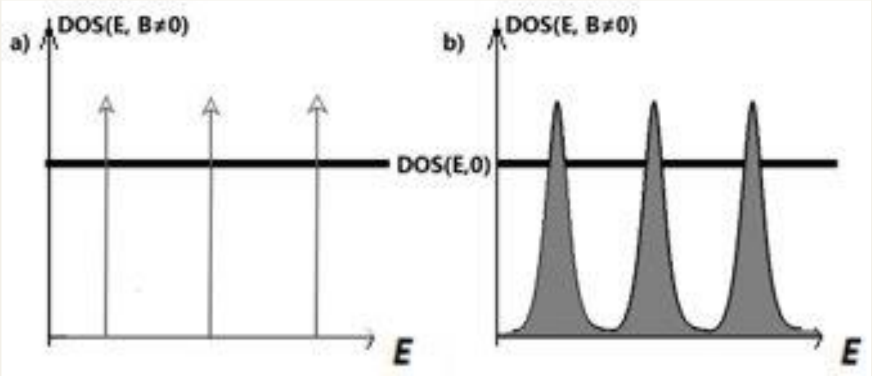
Integer quantum Hall ← **Cyclotron orbits quantization (Landau levels)**



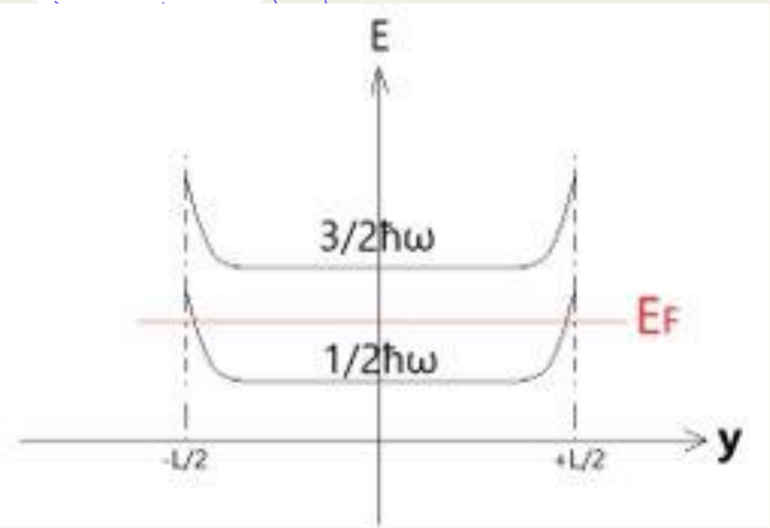


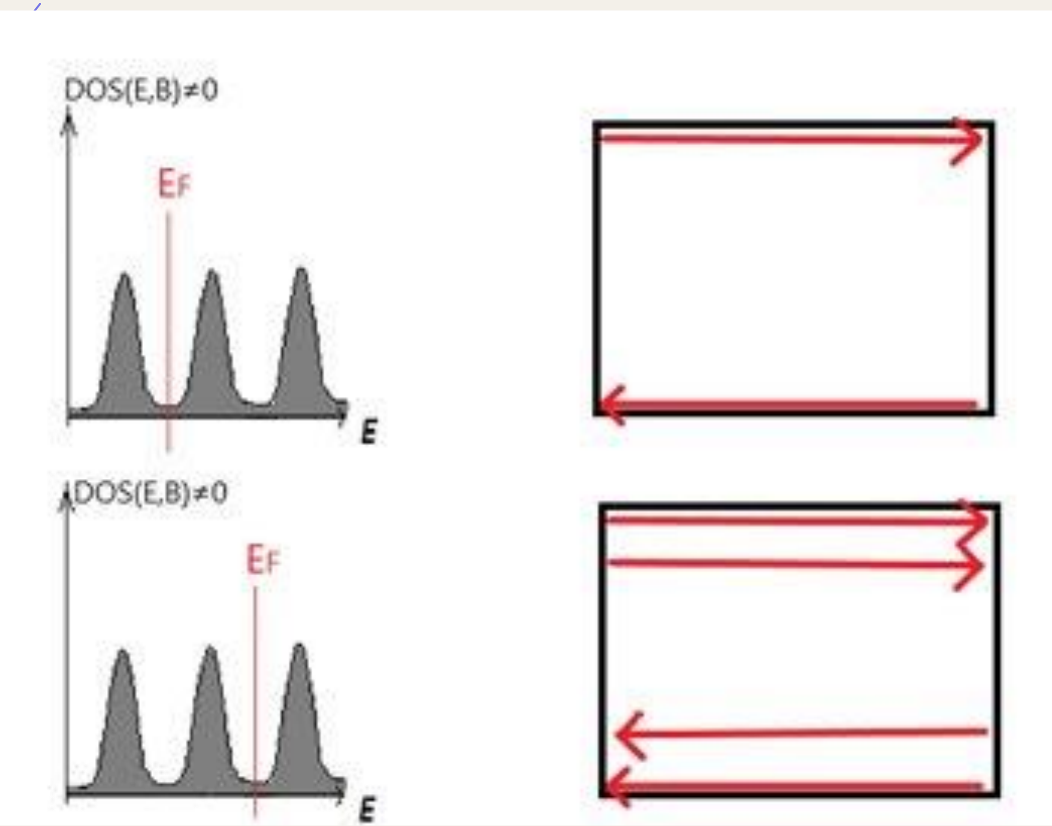
$$\Delta E = \hbar\omega_c \propto B$$

With disorder



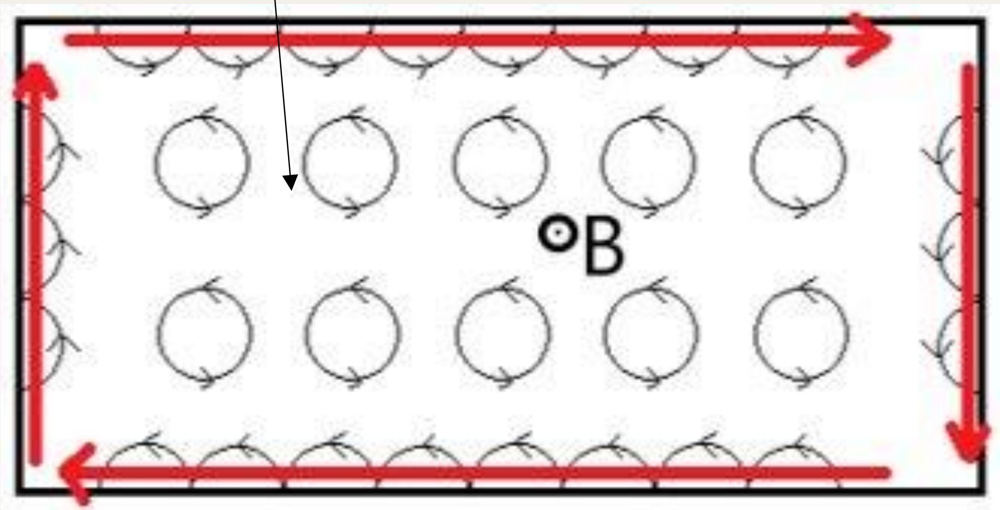
Quantum Hall effect physics



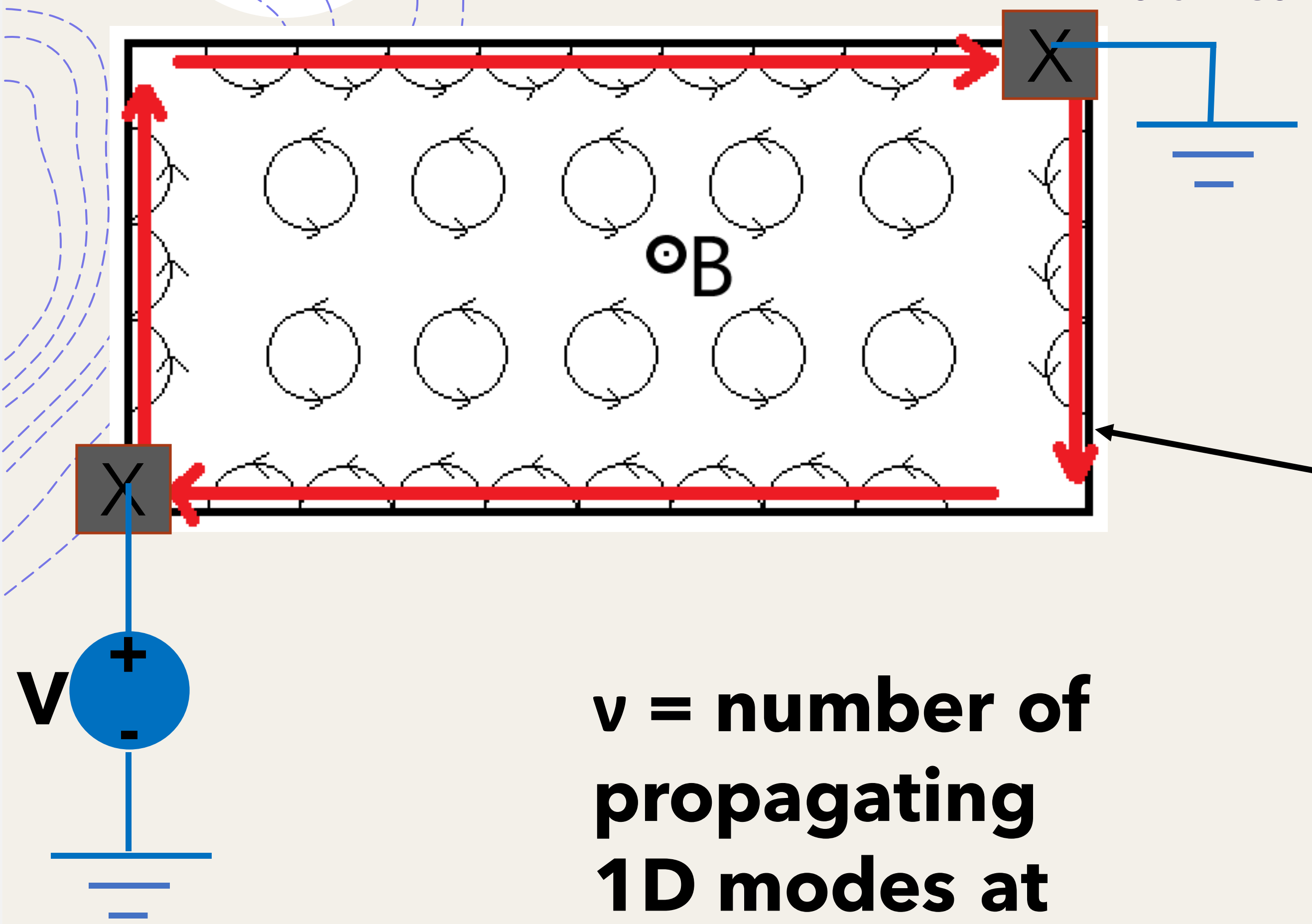


Insulator in the bulk

Conductor at the edges



Quantum Hall effect physics

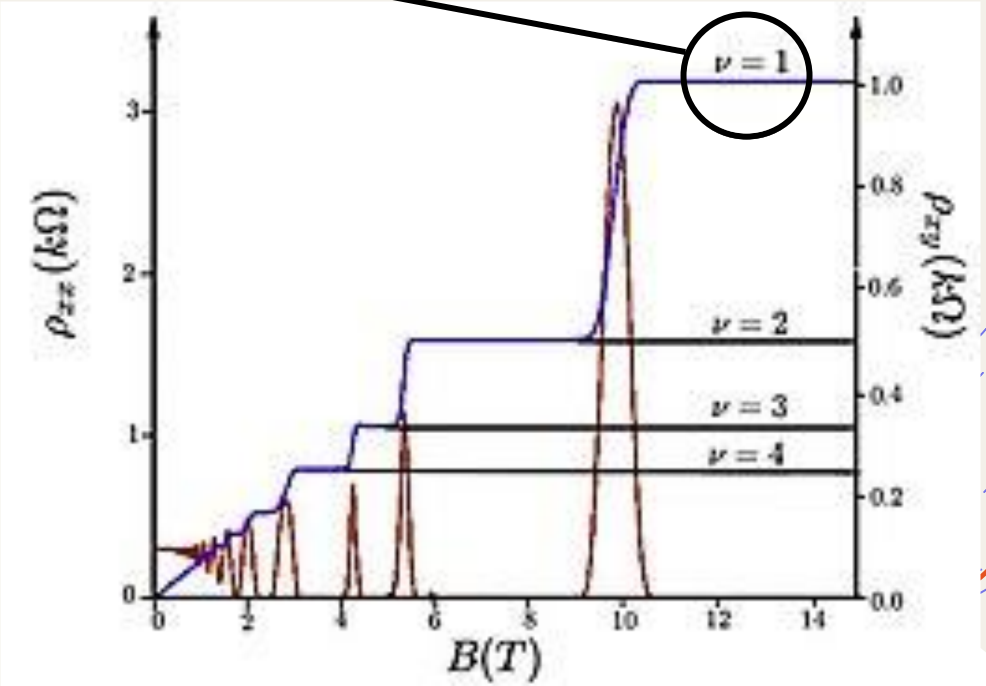


$$\mu_R - \mu_L = (-e)V$$

$$I = (-e)\int_{k_L}^{k_R} v_x(k)\frac{dk}{2\pi} = \frac{e^2}{2\pi\hbar}\int_{\mu_L}^{\mu_R} dE(k) = \frac{e^2}{h}V$$

$$R_{1edge} = \frac{V}{I} = \frac{h}{e^2} = R_K$$

ν = number of propagating 1D modes at the edges and Landau levels occupied.

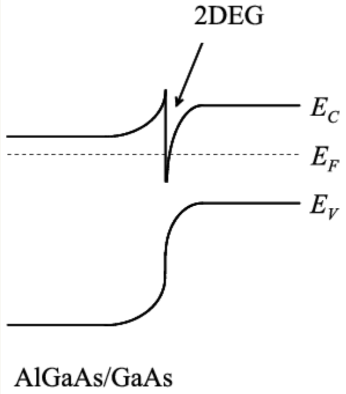


Quantum Hall metrology

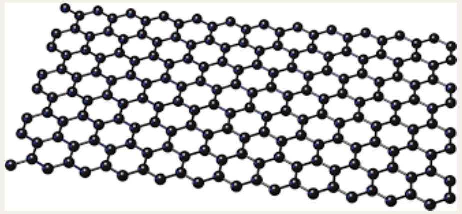
Electrical resistance metrology

QH universality tests

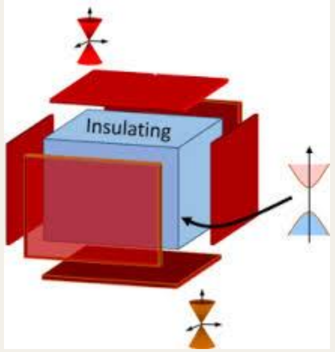
Resistance calibration



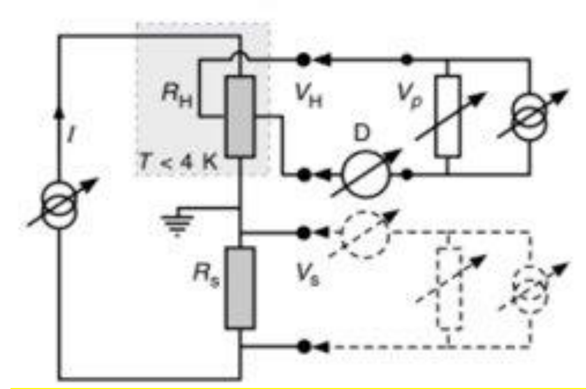
2D electron gas in heterostructures



2D materials like graphene

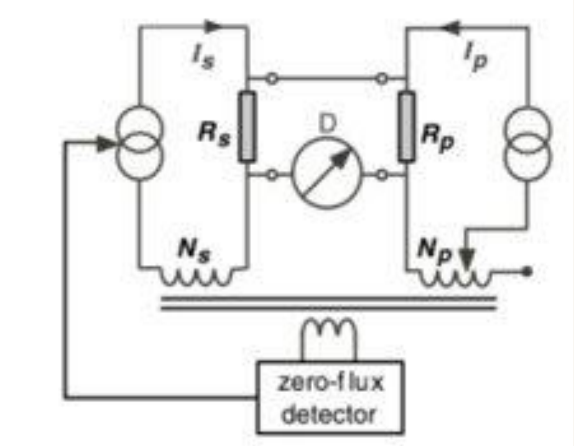


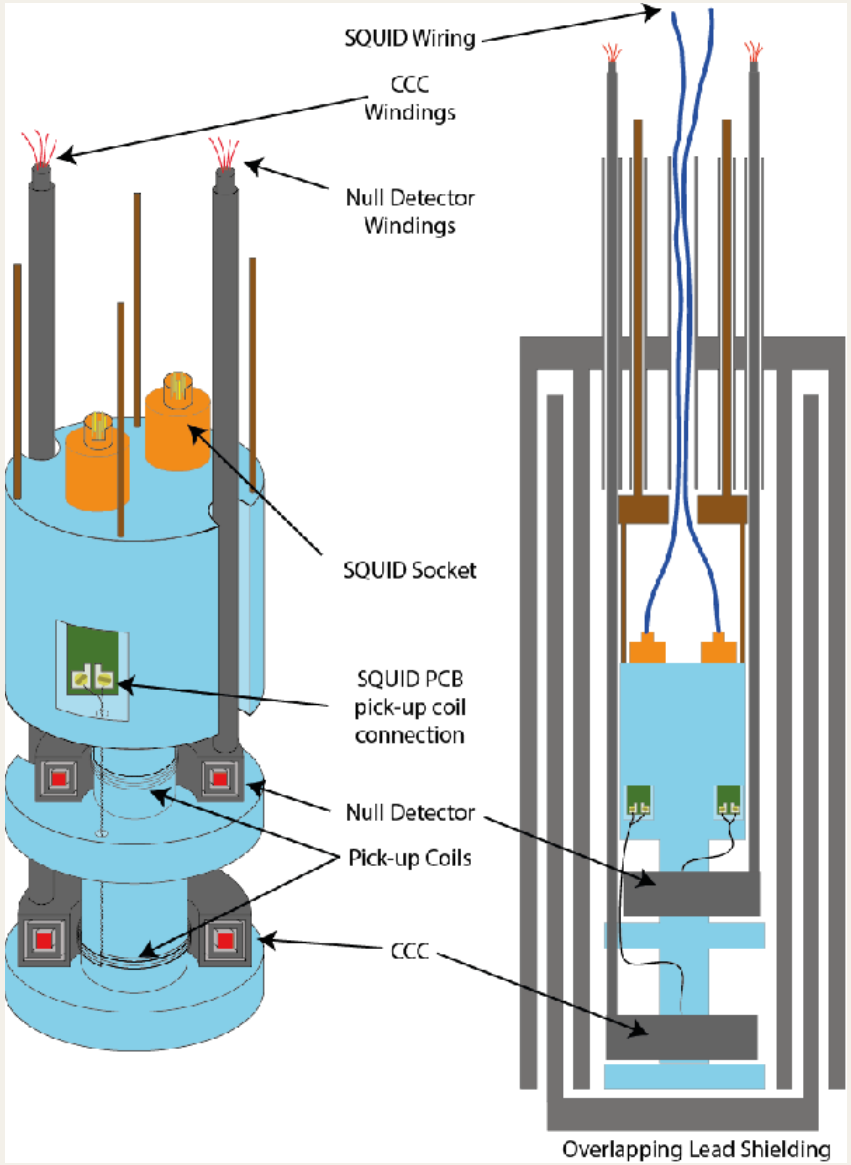
Topological insulators
Spin QH effect at B=0



How to compare resistances different from R_K without losing precision ? ...

… Comparing currents with high precision



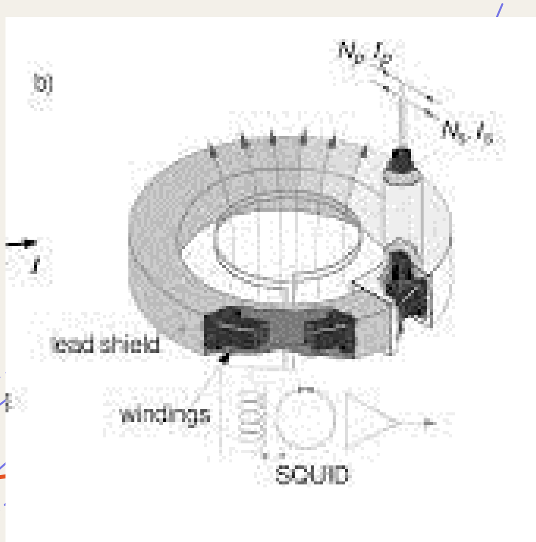


CCCs
Cryogenic Current Comparators

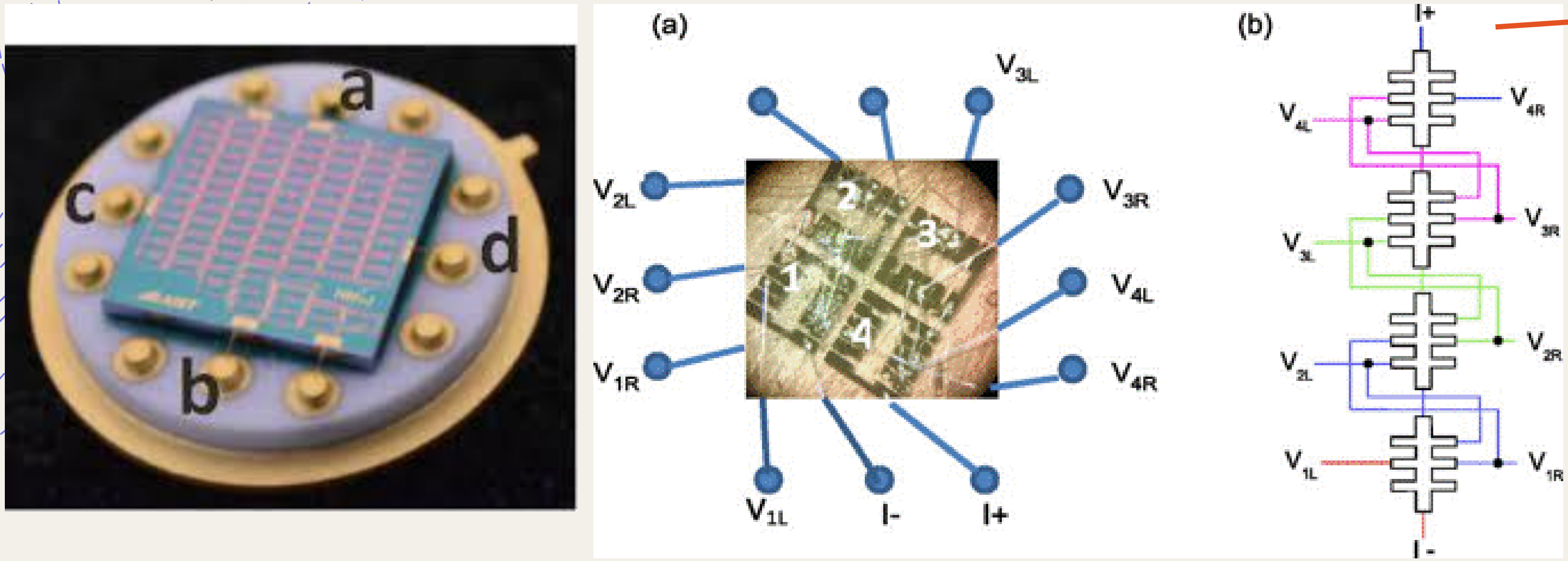
Advantages:
- Very high precision due to SQUID magnetometers

Problems:
- **Intricate setup**
- **Incompatibility of measuring apparatus with the presence of a magnetic field**

Separate insulated cryogenic setup for the SQUID



… Generating new standards by scaling R_K value



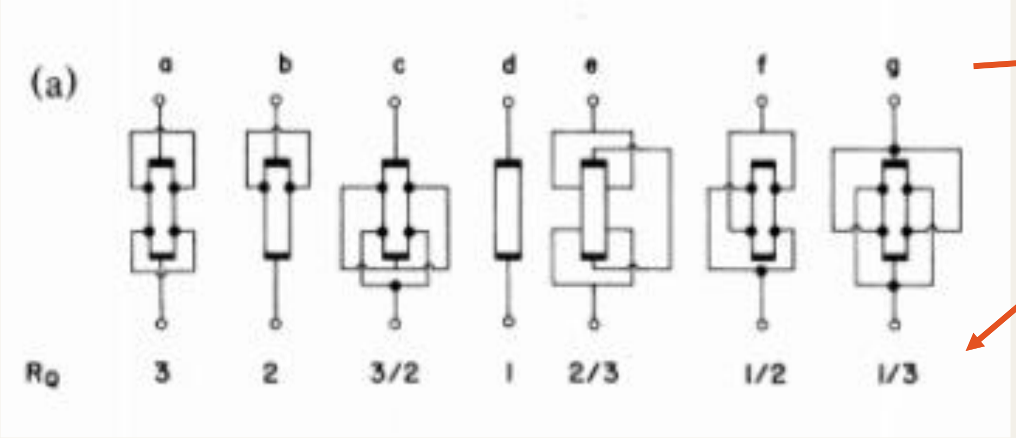
QHARS:
Quantum Hall Array Resistance Standard

Advantages:

- Conventional cryogenic setup

Problems:

- **Presence of Ohmic resistances**
- **One standard per device**
- **Number of standards scales with the number of elements**



Hall bar with multiple lateral shorts:

- **Few fractions of R_K obtainable**

… Are there other ways ?

Presentation outline

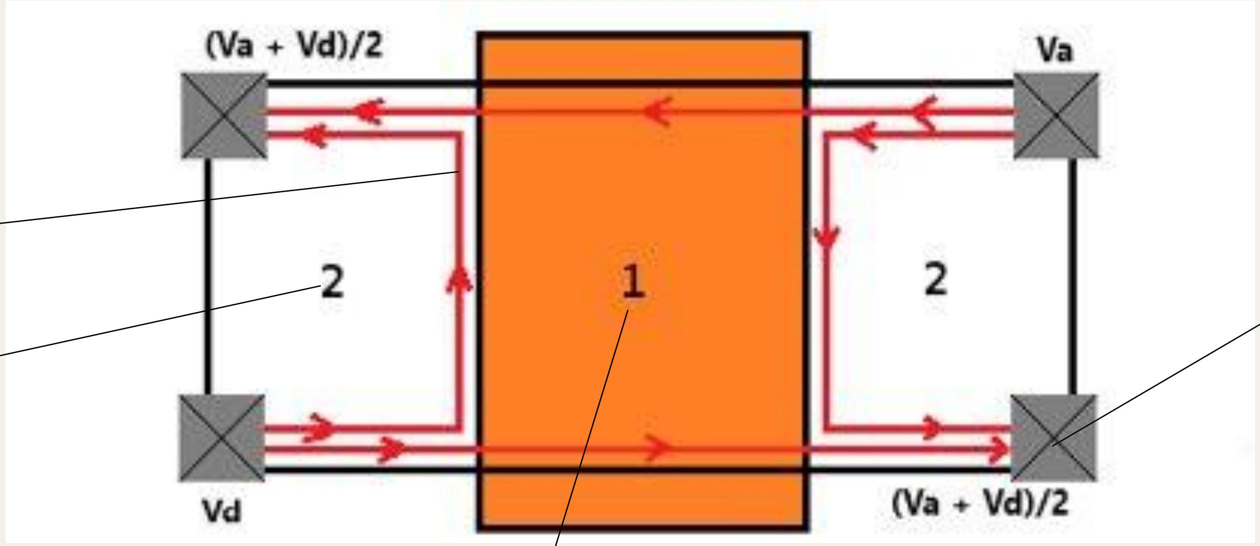
1) The quantum Hall effect & quantum Hall Metrology
2) **Novel approach**
3) Experimental results
4) Numerical results
5) Conclusions and perspectives

Building block

New way to obtain more R_K fraction with less elements without Ohmic connections

Cascaded Quantum Hall Bisection and Applications to Quantum Metrology
Zahra Sadre Momtaz, Stefan Heun, Giorgio Biasiol, and Stefano Roddaro
Phys. Rev. Applied **14**, 024059 – Published 20 August 2020

Edge mixer



Edge reflection

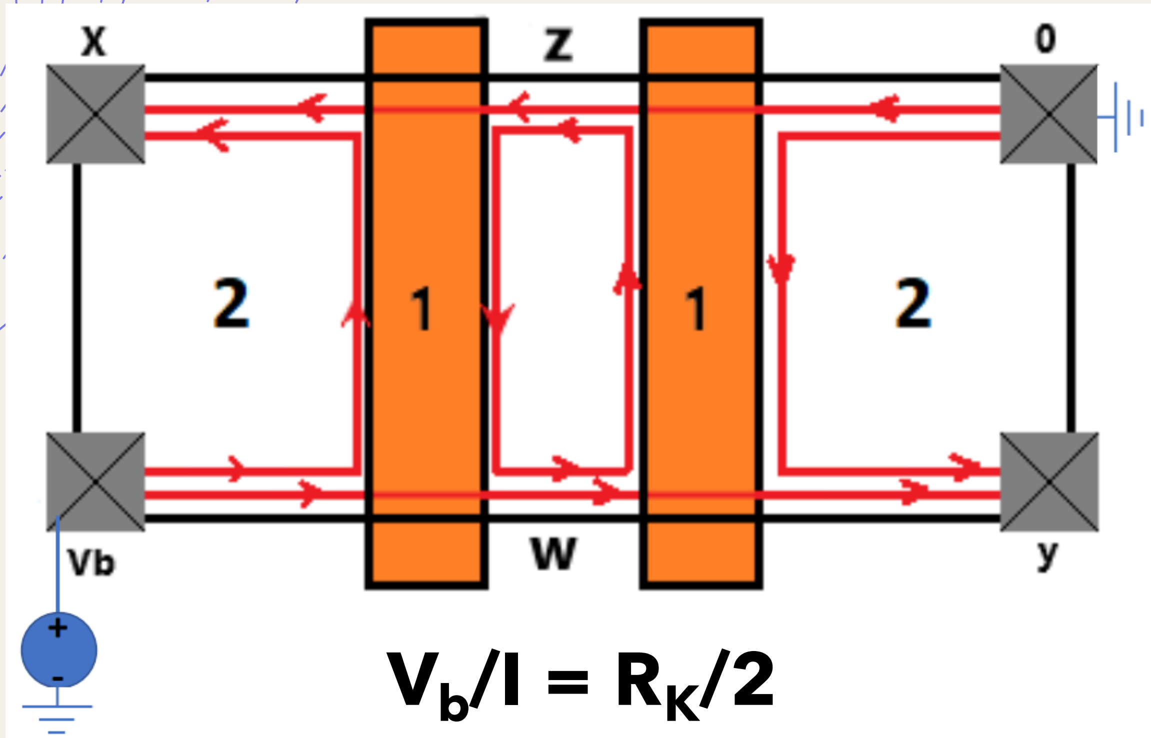
ν = 2

ν-Reconfigurable region

Average of input voltages Vd and Va

Equilibrating edges coming from different regions

Two barriers in series



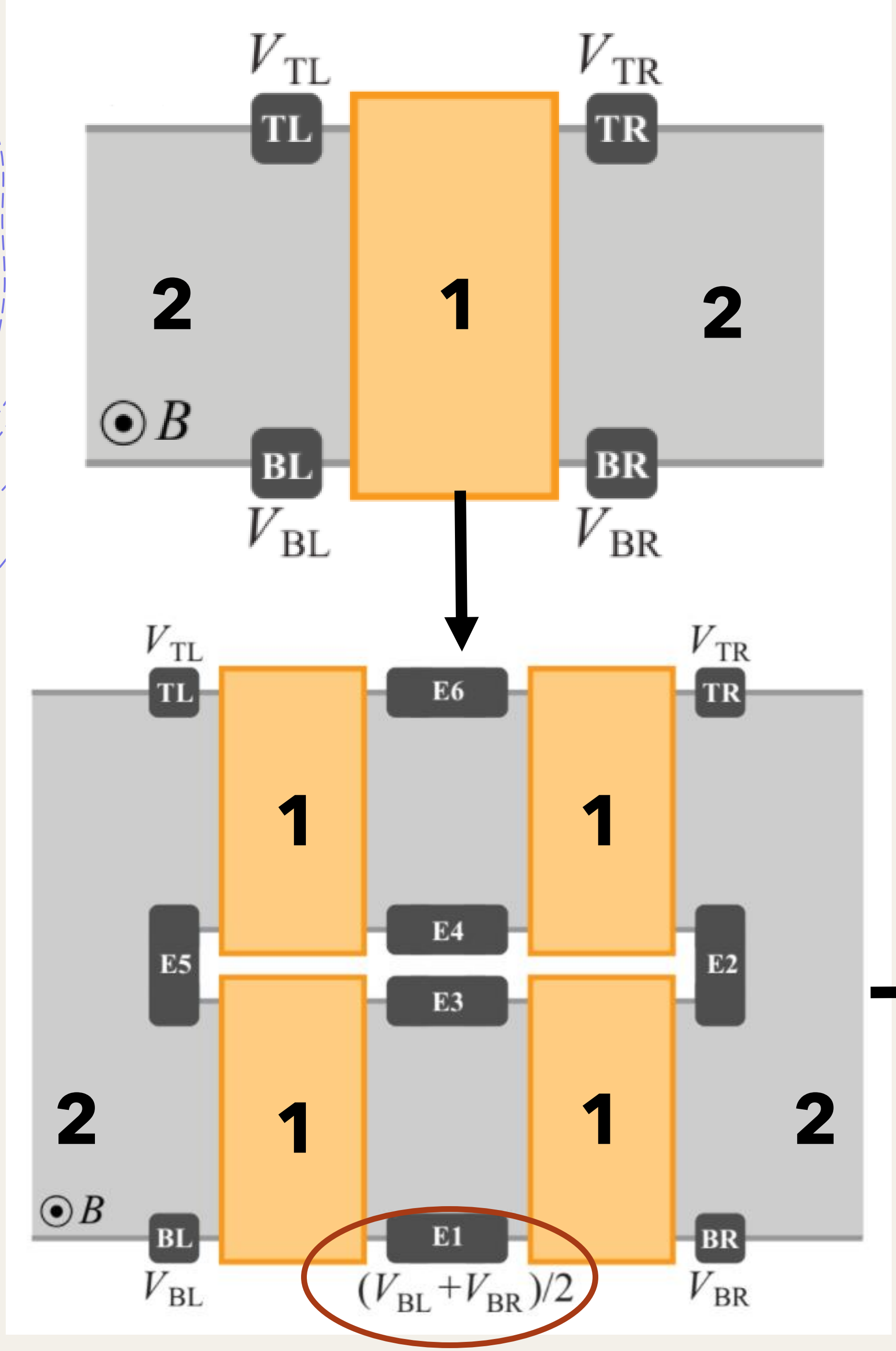
$$V_b/I = R_K/2$$

$$\begin{cases} x = \frac{V_b}{2} + \frac{z}{2} \\ y = z = \frac{w}{2} \end{cases}$$

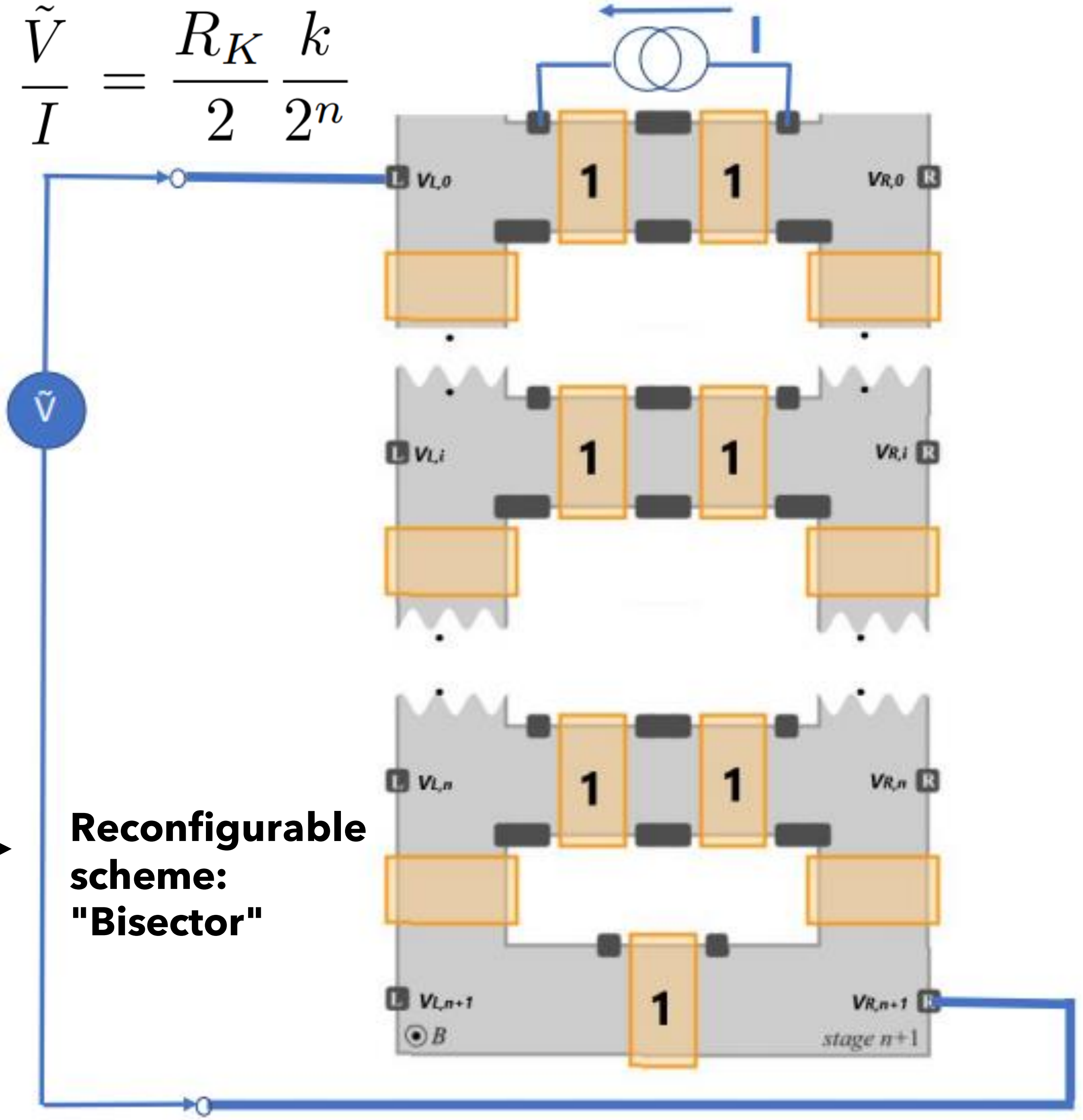
$$y - x = -\frac{V_b}{3}$$

$$V_b - y = \frac{2V_b}{3}$$

Various fractions generated without interconnected elements.



$$\frac{\tilde{V}}{I} = \frac{R_K}{2}\frac{k}{2^n}$$



Reconfigurable scheme: "Bisector"

Recap and thesis targets

Open problems in resistive metrology

Solved with the scheme in the paper

- Generating ad hoc standards on the same device
- Stray resistances due to interconnected elements
- Reduce the number of elements and increase number of standards
- Double cryogenic setup for CCC

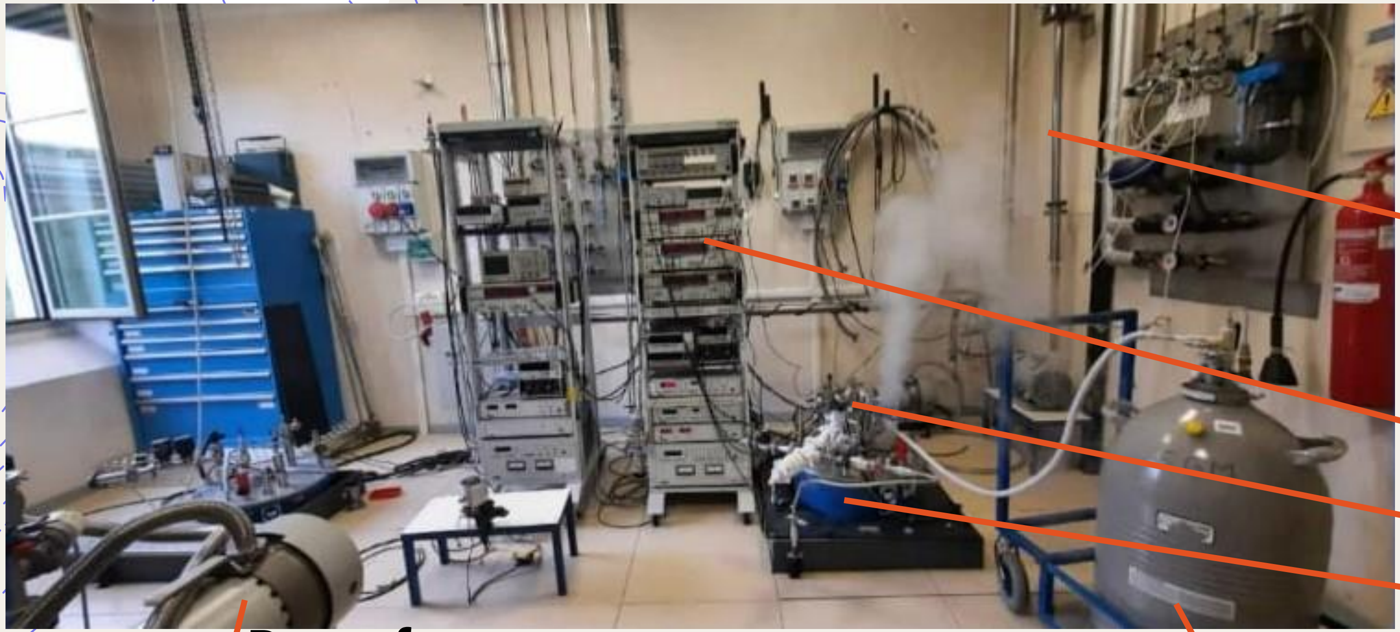
Open questions in the article studied

- **How precise is the edge mixing ?**
- **Large voltage bias or current is need to achieve high precision: What is the maximum voltage applicable to the new device ? (breakdown voltage)**

Presentation outline

1) The quantum Hall effect & quantum Hall Metrology
2) Novel approach
3) **Experimental results**
4) Numerical results
5) Conclusions and perspectives

Lab and cryogenic setup



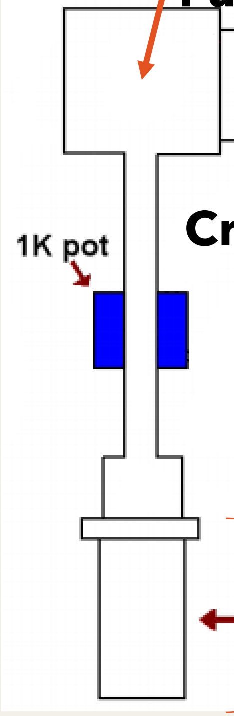
Other cryostat insert

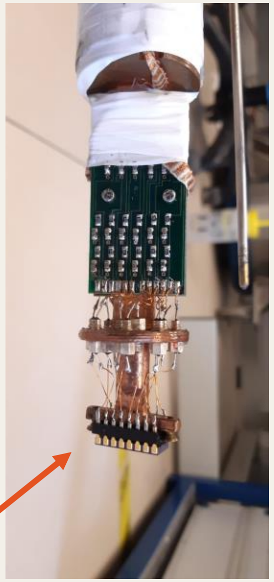
Lock-ins

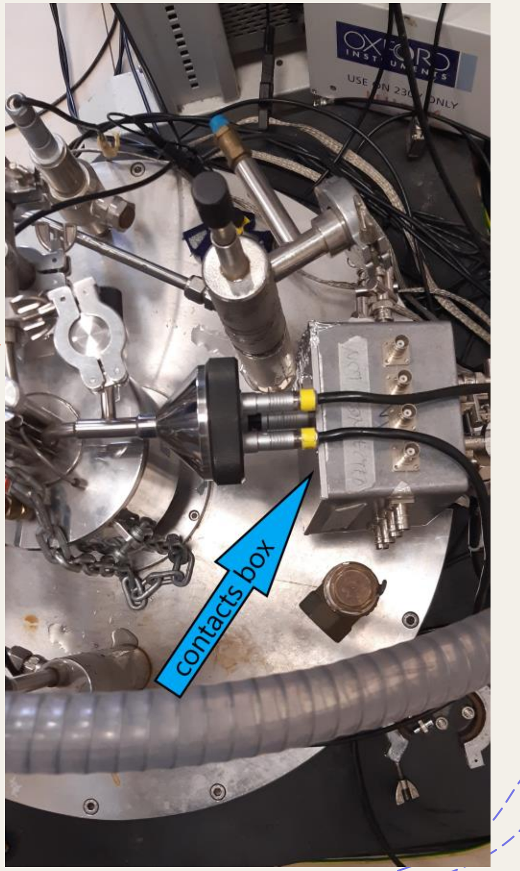
He-3 Cryostat

N2 refilling tank

Pump for vacuum



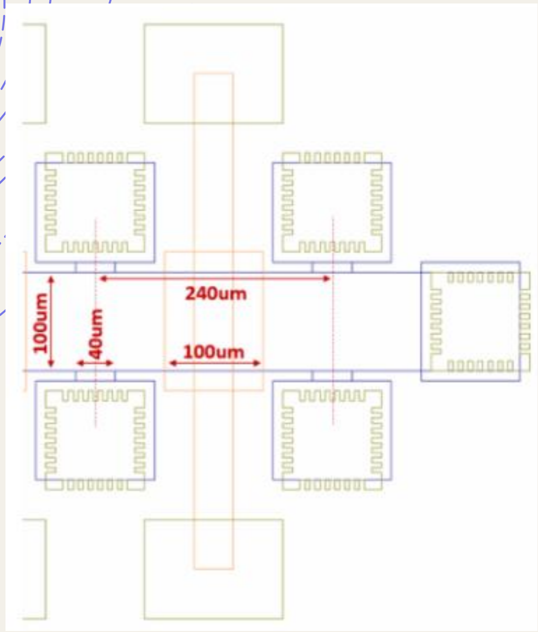


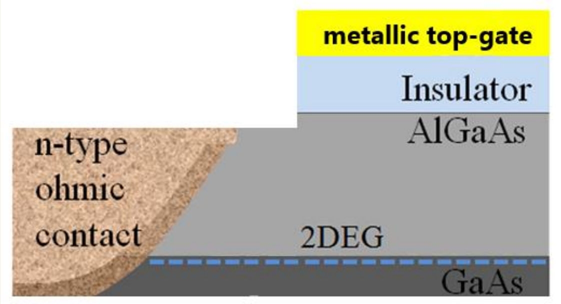


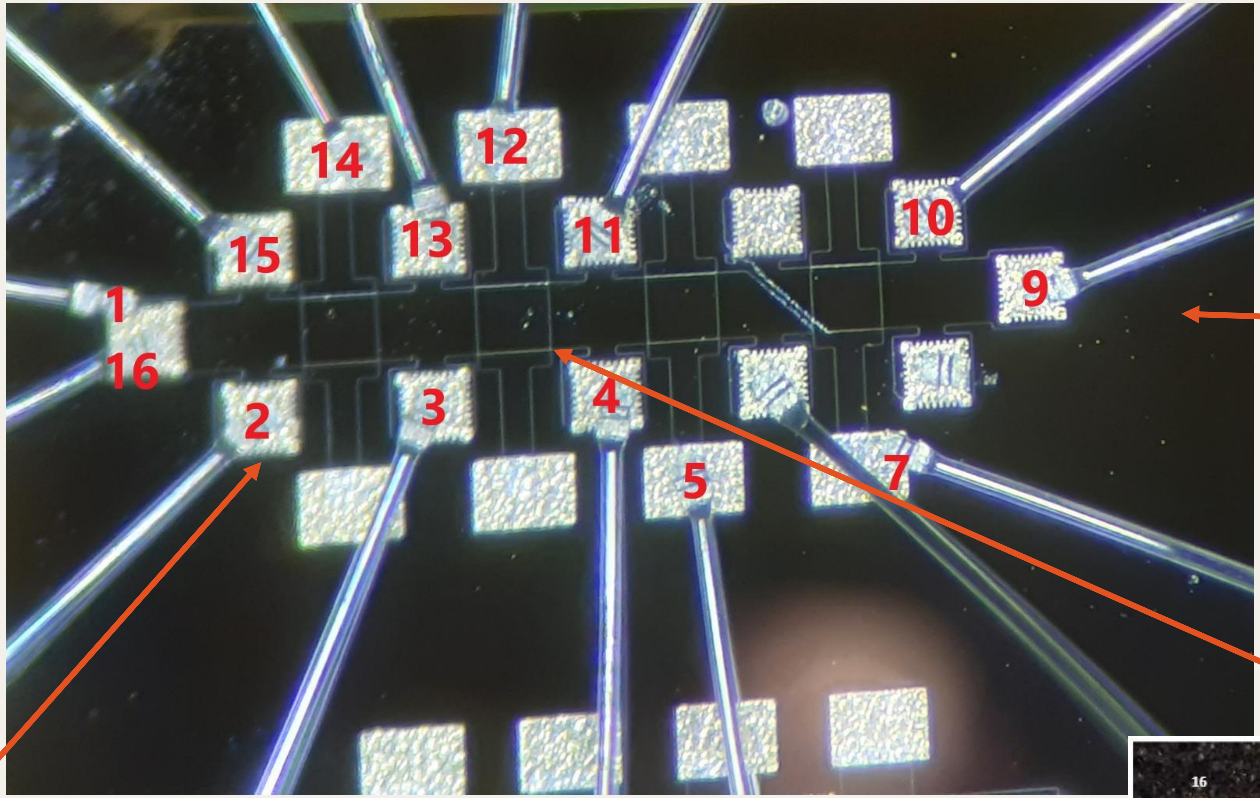
Minimum temperature Around 300 mK

Device architecture

Doping	% Al	Spacer	Depth		300 K		4K DARK		4K LIGHT	
					n	μ	n	μ	n	μ
Uniform	25/36	370	670	Å	3.0E+11	7.2E+03	2.5E+11	3.3E+06	3.9E+11	5.0E+06
					cm^{-2}	cm^2/Vs				

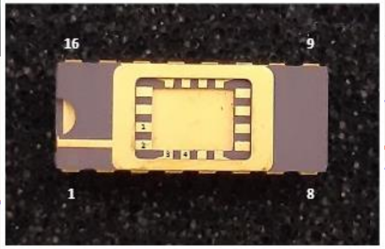






High mobility GaAs/AlGaAs wafer from Weizmann institute grown by Vladimir Y. Umansky

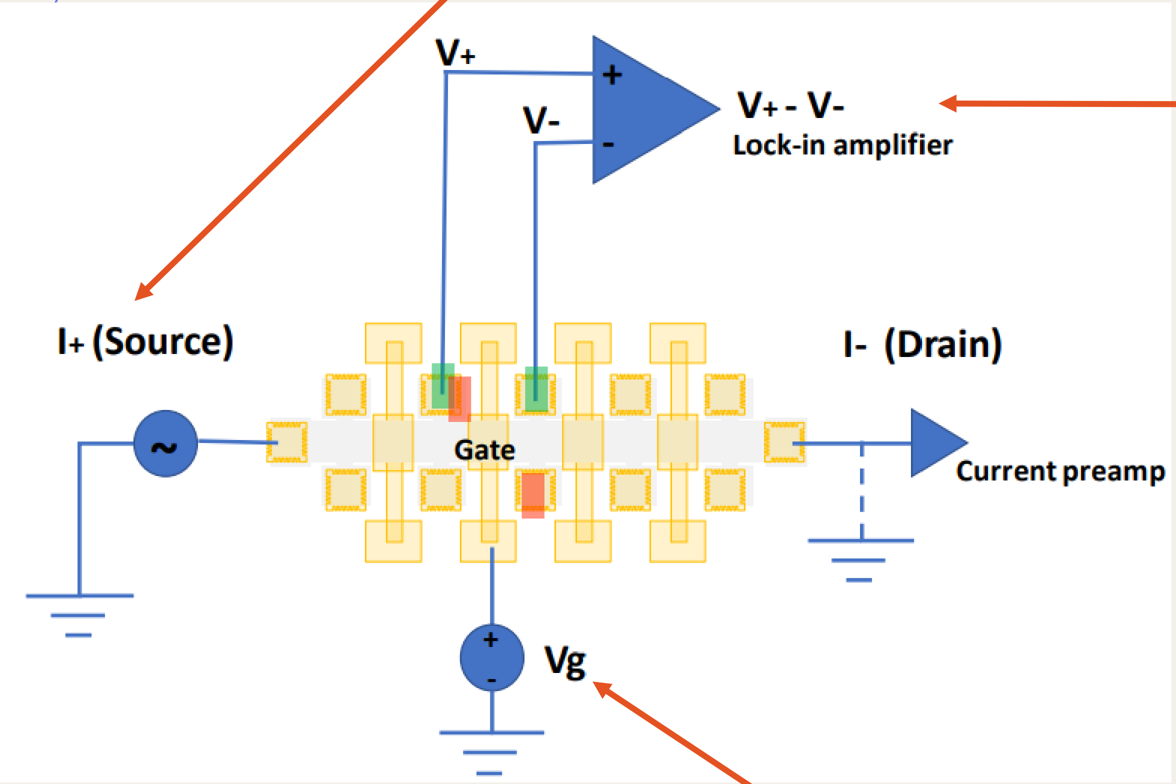
Aluminium gates (barriers)



AuGeNi Ohmic contacts

Measurement setup

AC current bias or voltage bias



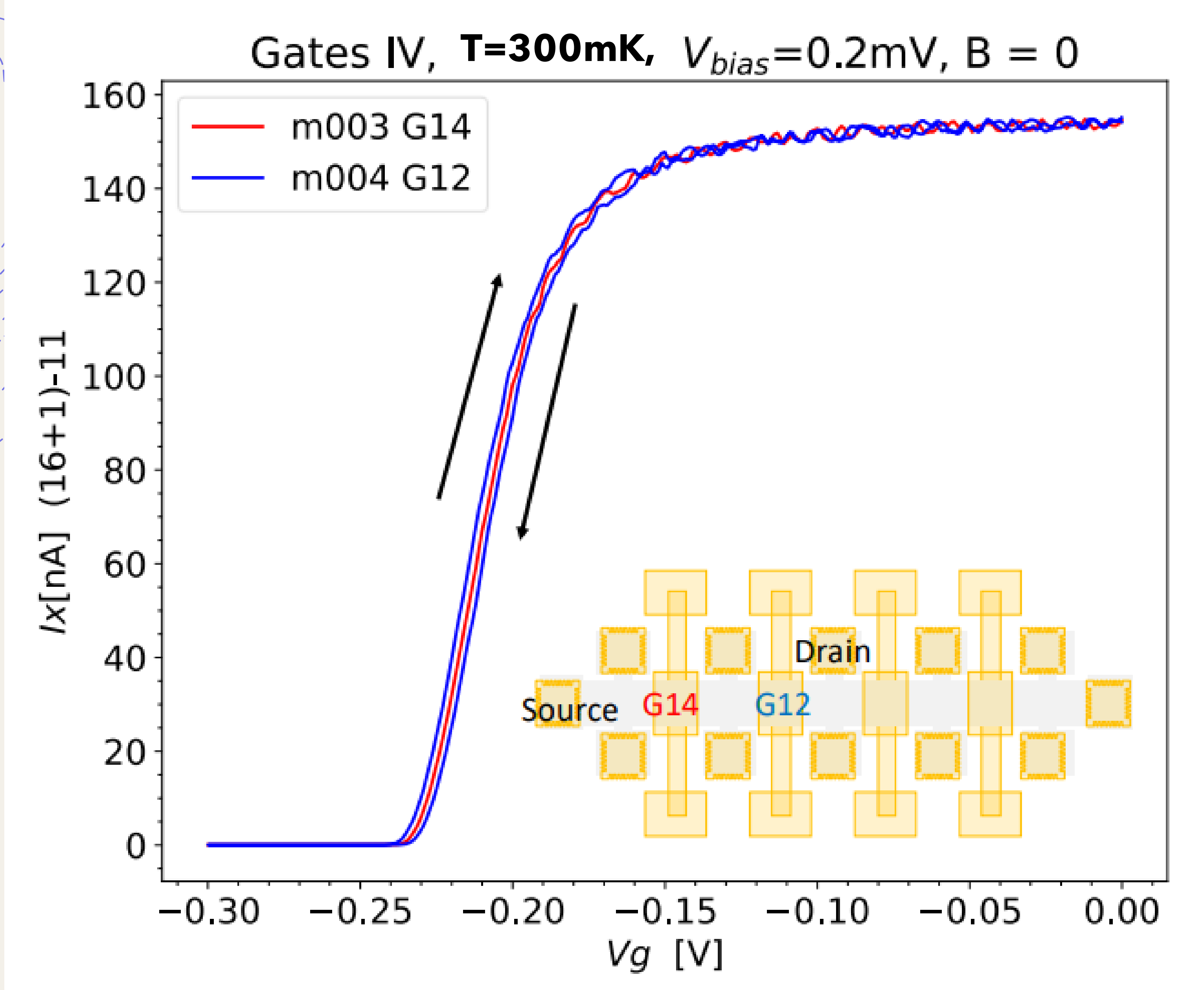
One lockin to measure longitudinal voltage drops

One lockin to measure diagonal or transversal voltage drops

- How good is the equilibration or mixing (zero voltage) ?
- Mixing breakdown voltage at finite gate bias?

DC gate voltage bias to reconfigure ν

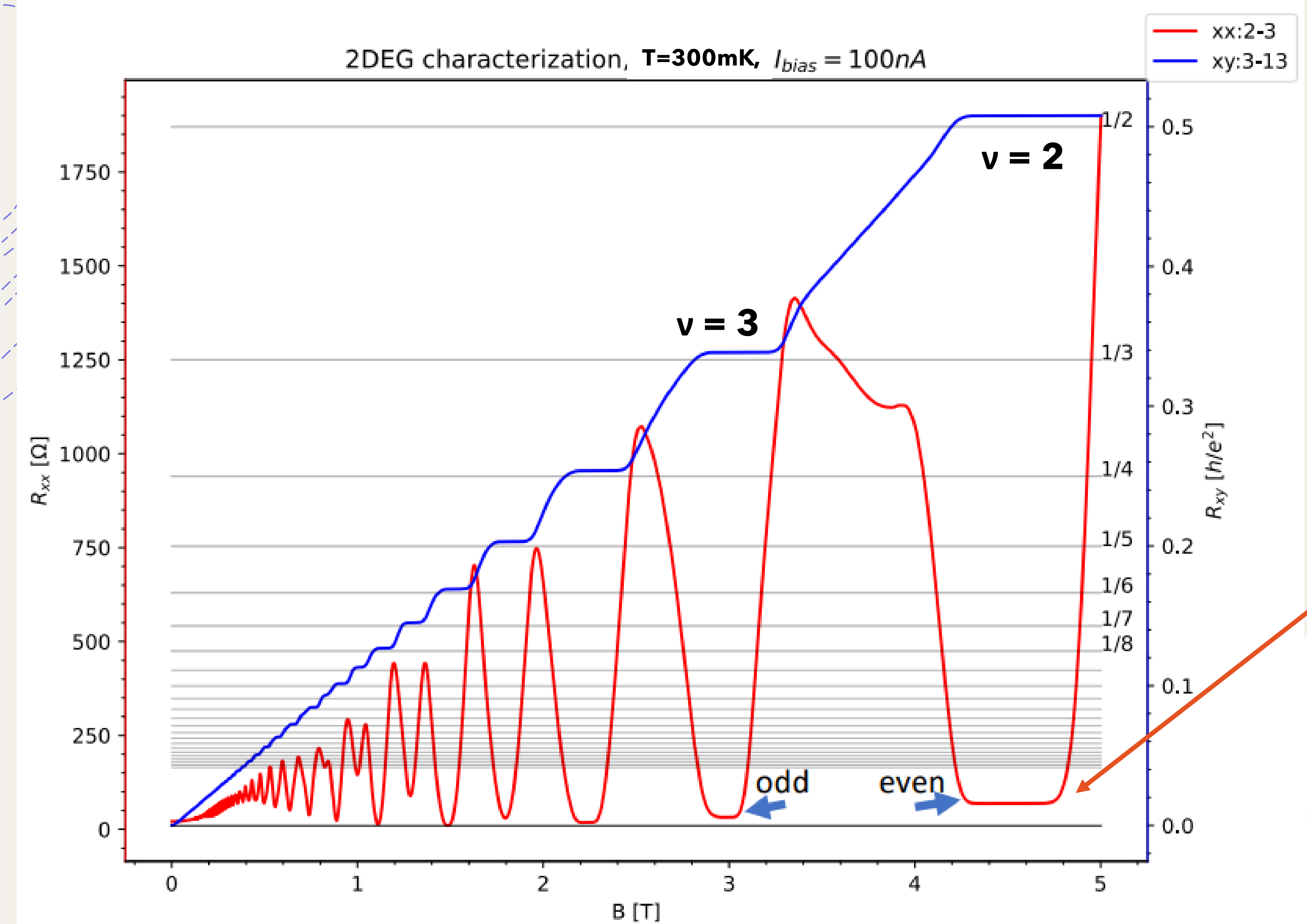
Gates characterization



Pinch-off at –0.25 V
(**barrier realization**)
Slight hysteresis

Contacts working correcly
At **B = 0**

2DEG characterization: transport parameters

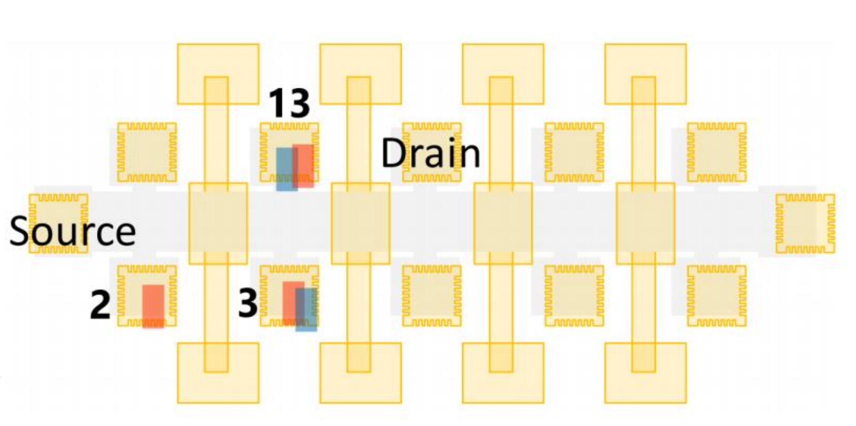


$W/L_{Hallbar} = 2.4$

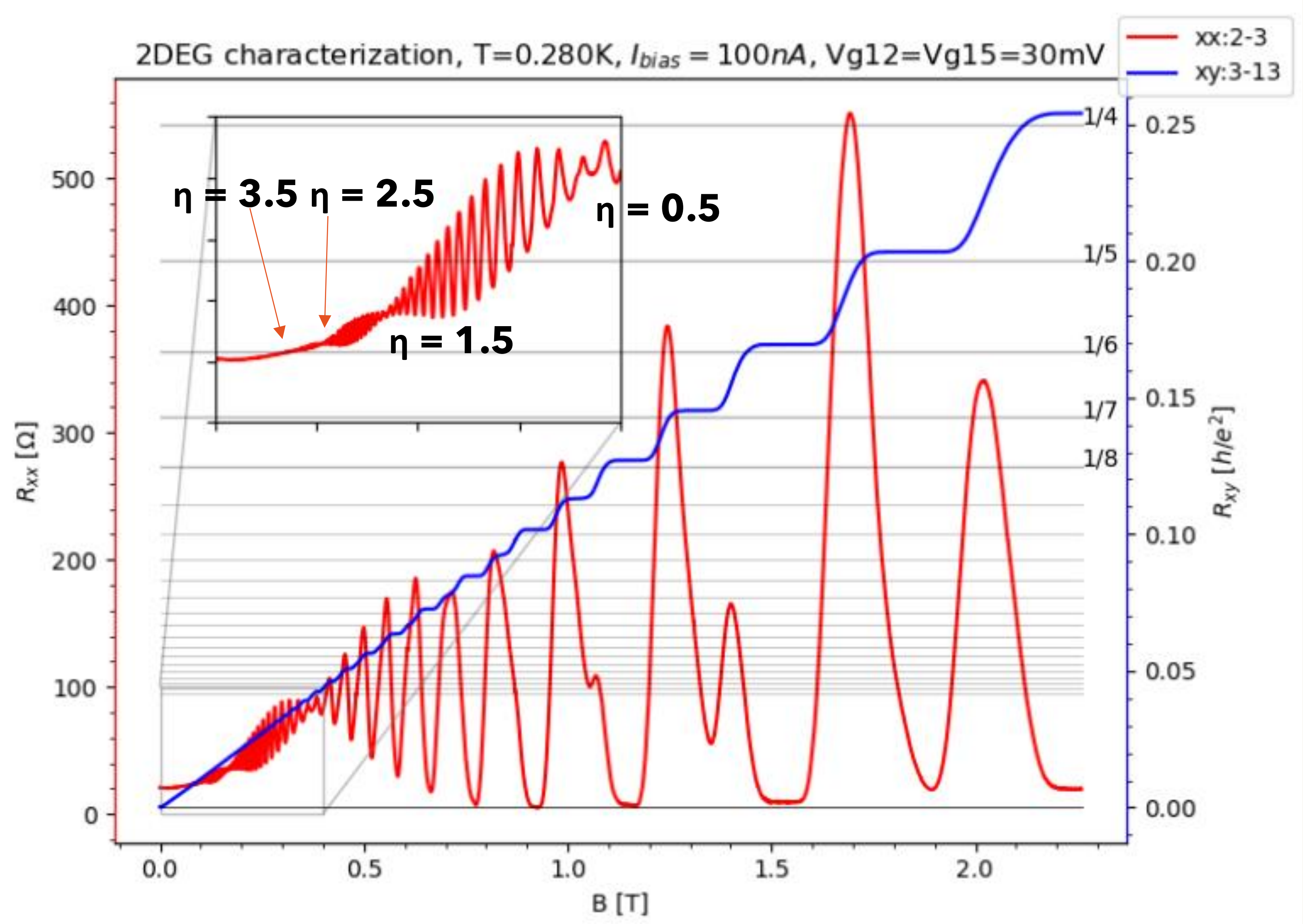
$\mu = 3.37 \times 10^6$ cm2/(Vs)
$n = 2.2 \times 10^{11}$ cm^{-2}
$\tau = 128$ ps

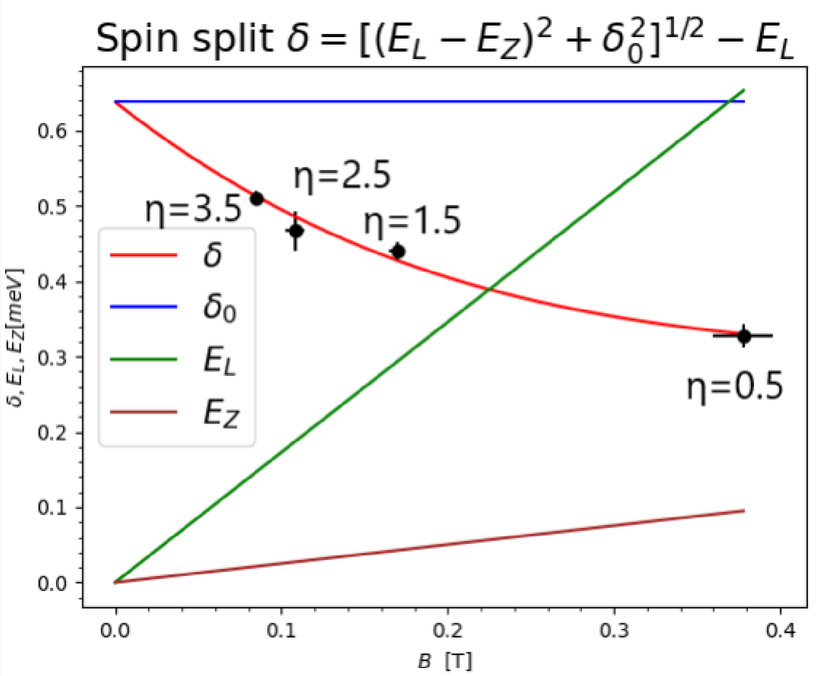
Parallel conduction ?

B > 0 anticlockwise chirality
B < 0 clockwise chirality

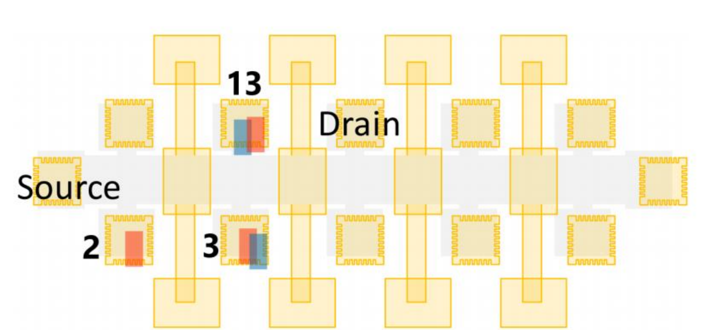


2DEG characterization: beating features





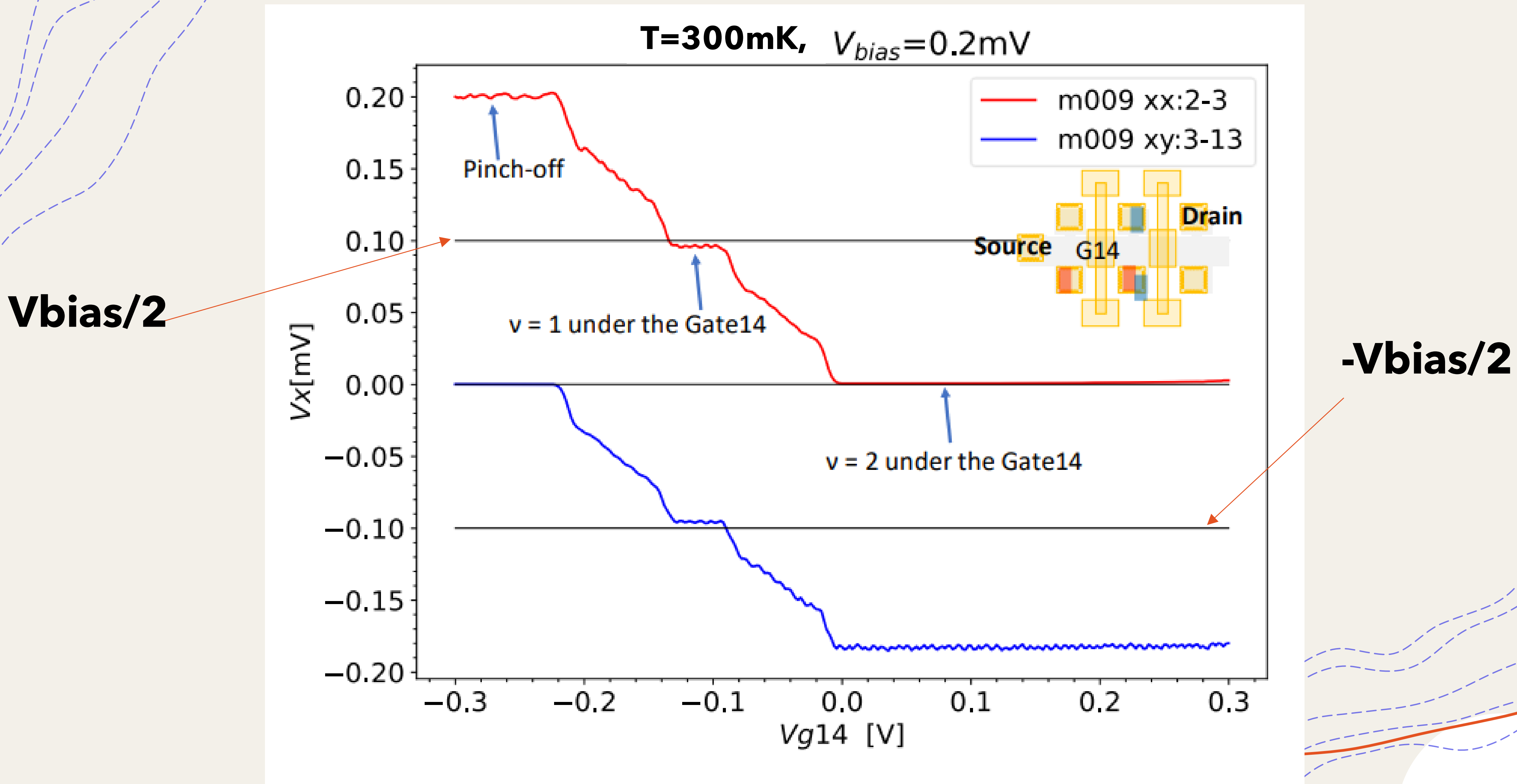
$g* = -0.29 \pm 0.06$
$\delta_0 = 0.638 \pm 0.007$ meV
EL(ν = 2) = 0.5 eV
EZ(ν = 2) = 0.1 meV



$\delta = \eta\hbar\omega c(B_{node})$

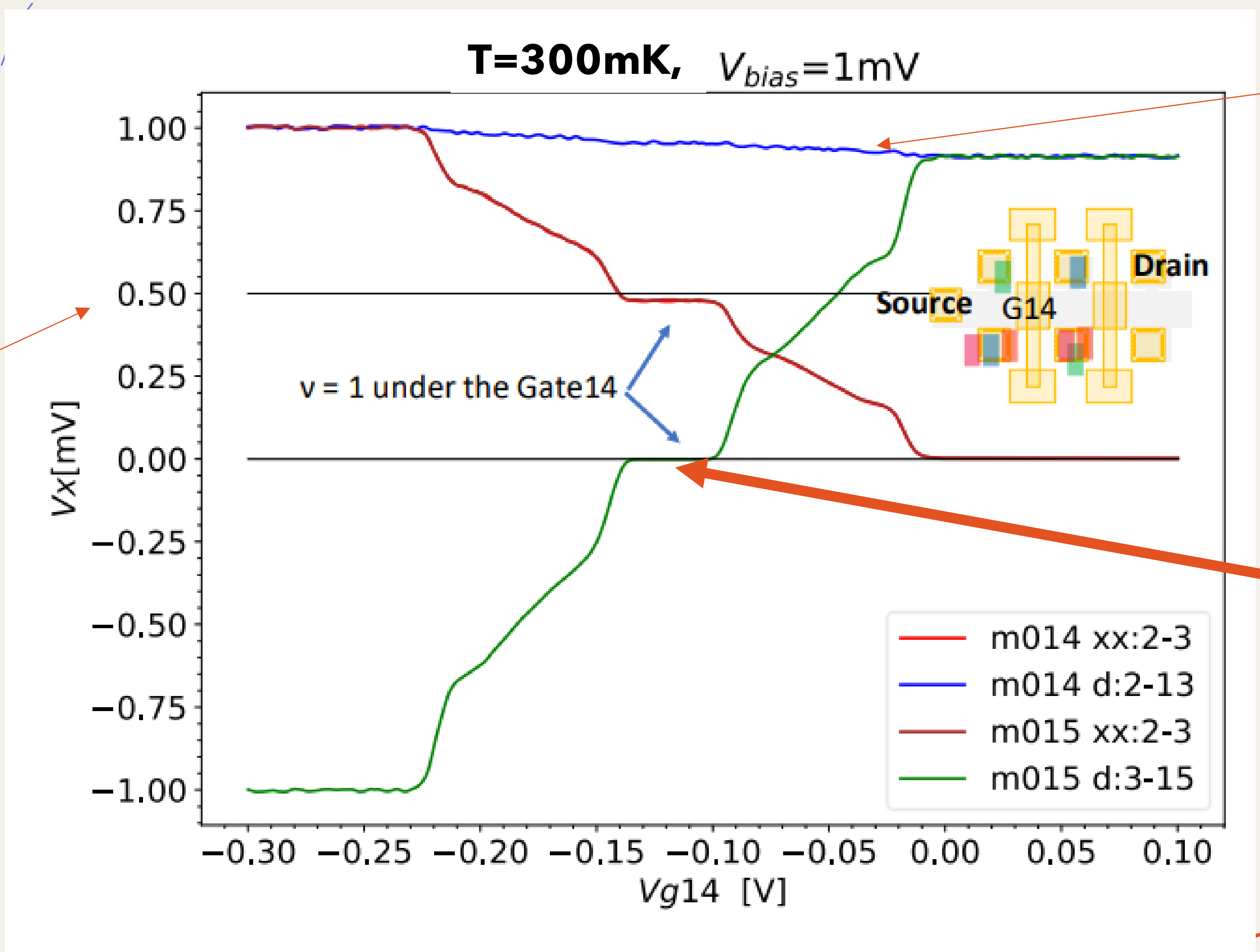
Gate characteristics at ν = 2

Gate14 mixer calibration. Correct control of ν under the gate.



Gate characteristics at ν = 2

Gate14 single mixer operation verification



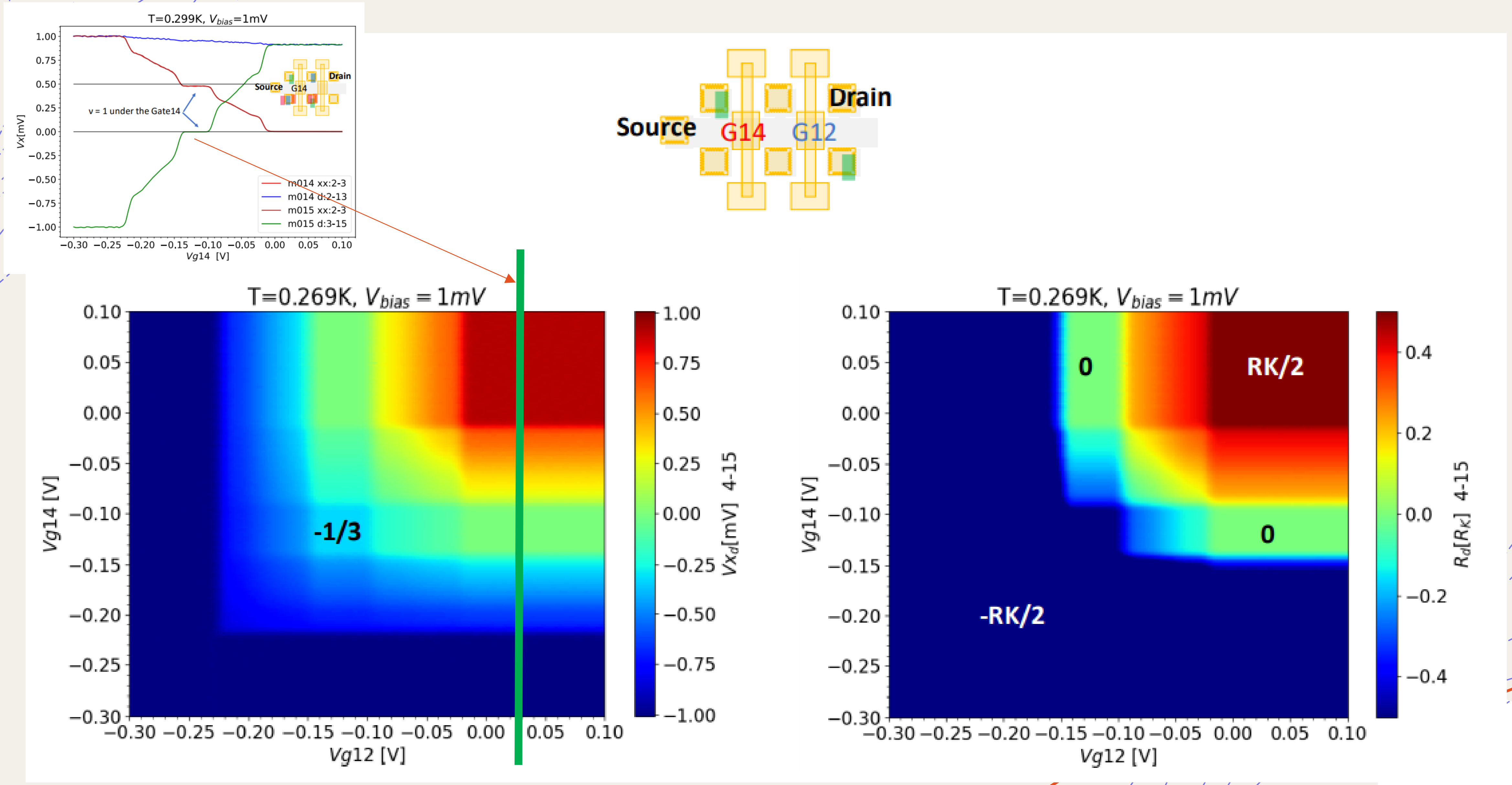
Deviation from Vbias due to Rxx ≠ 0

Vbias/2

Correct equilibration Averages on the diagonal

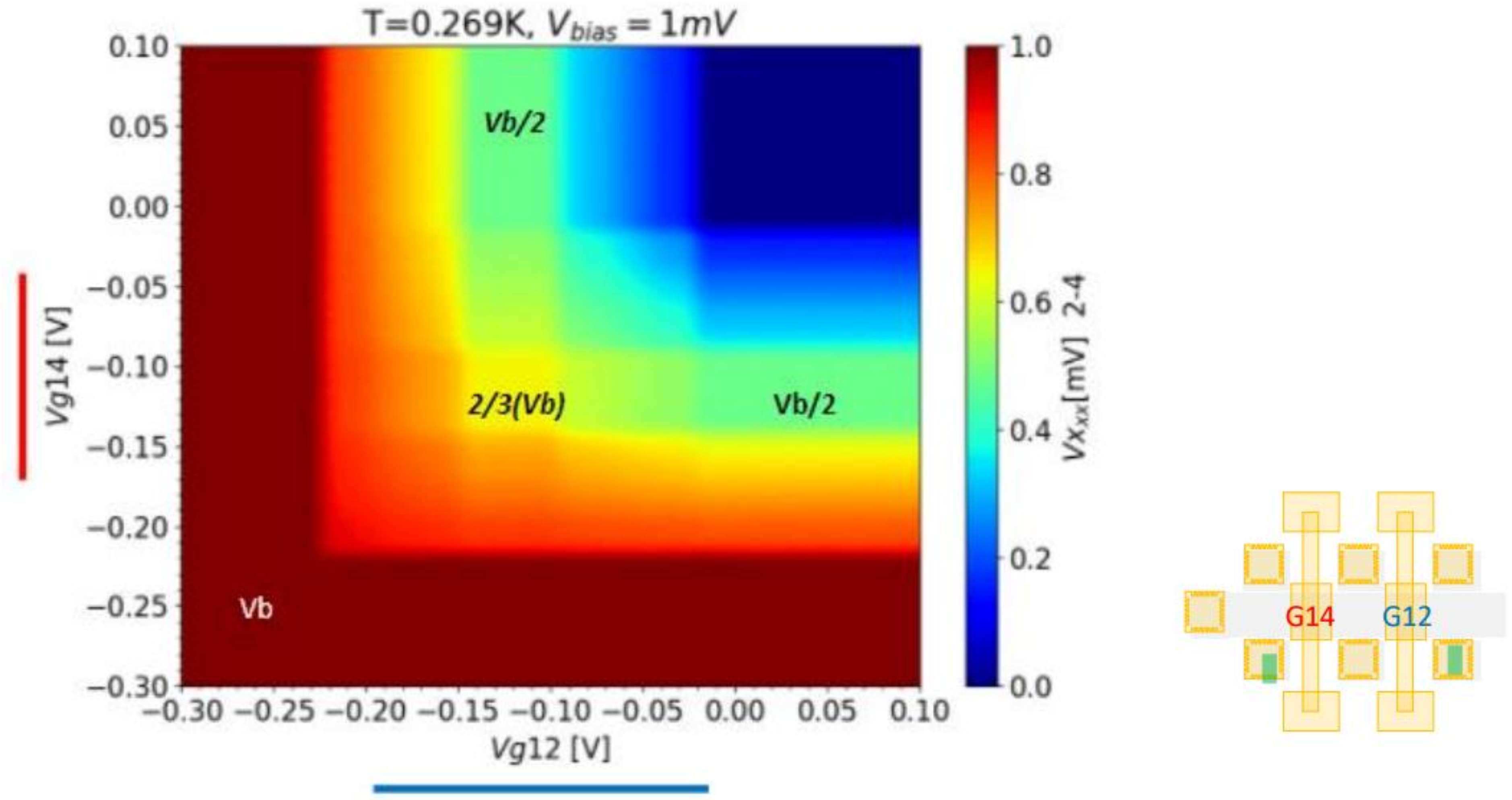
Gate characteristics at ν = 2

Gate14&12 diagonal voltage drops and diagonal 4w resistance (2D map)

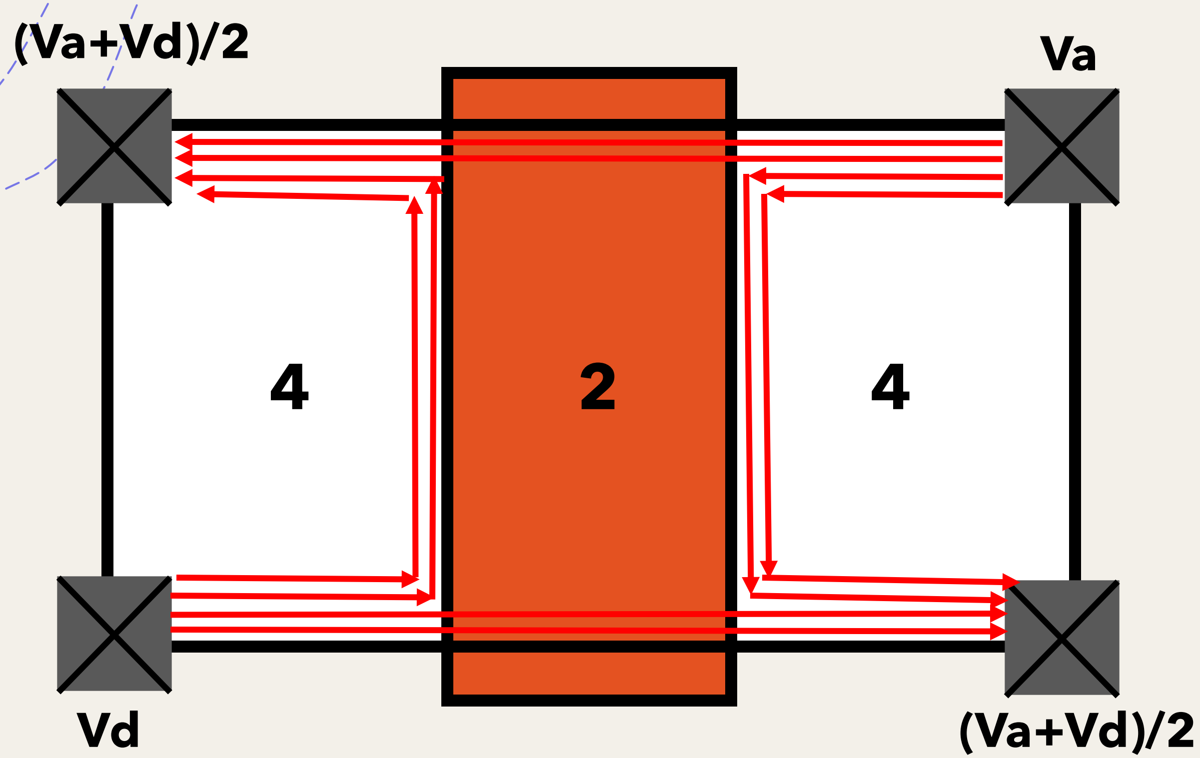


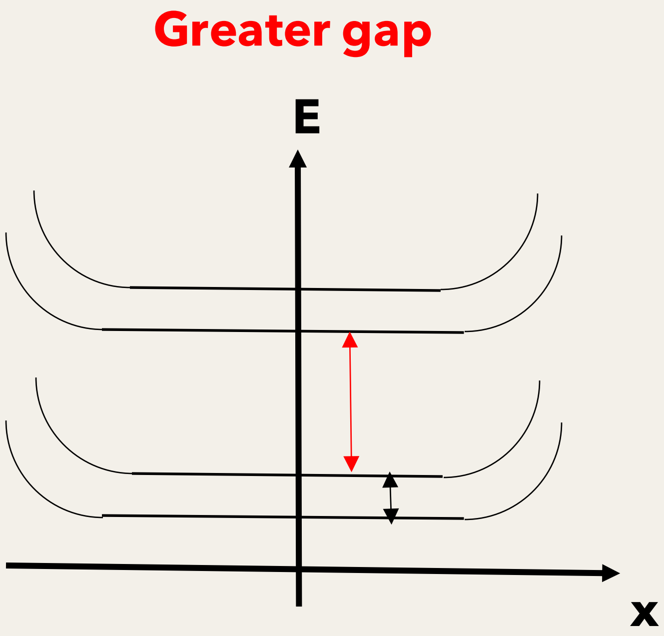
Gate characteristics at ν = 2

Gate14&12 longitudinal voltage drops (2D map)



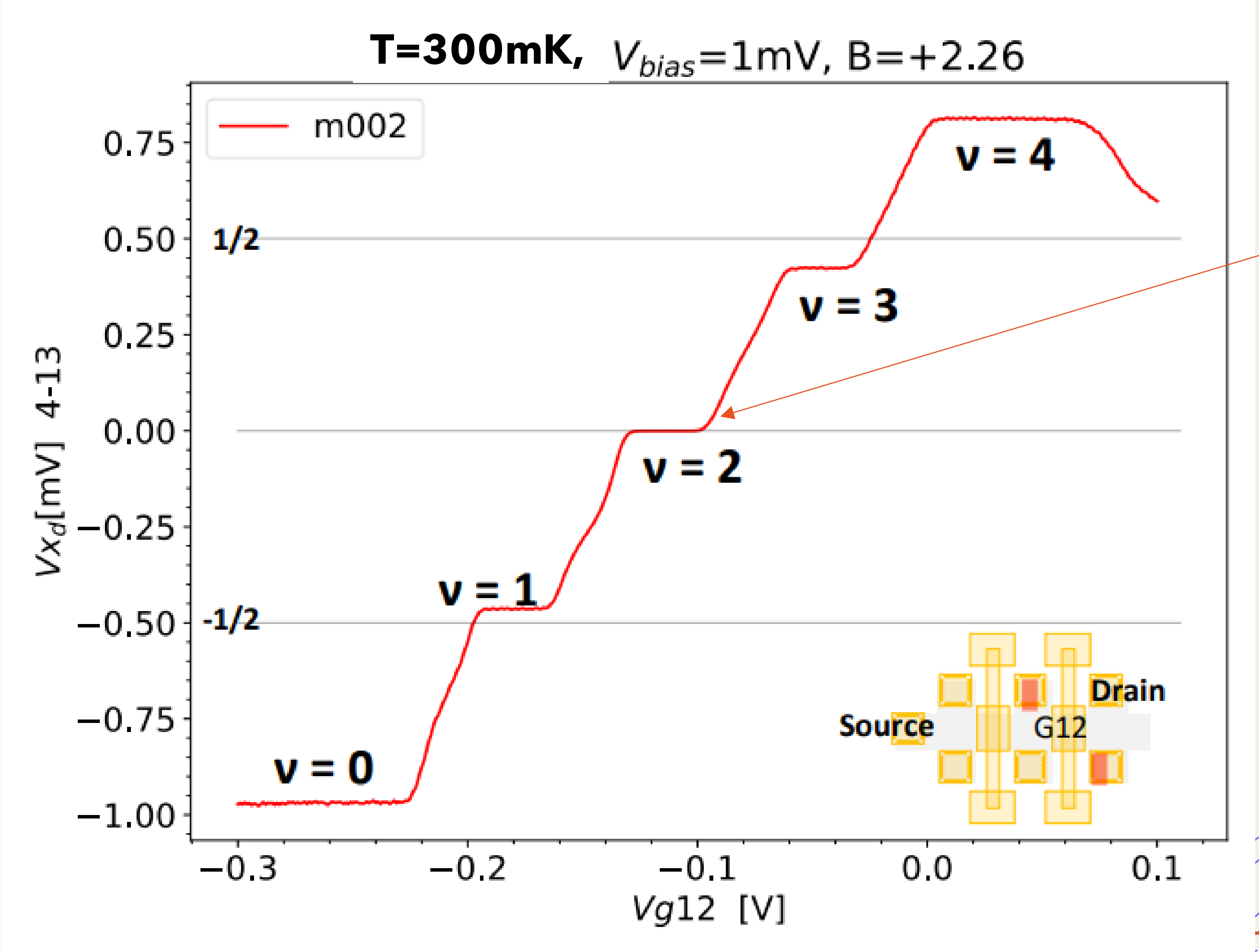
Alternative working scheme for the mixer





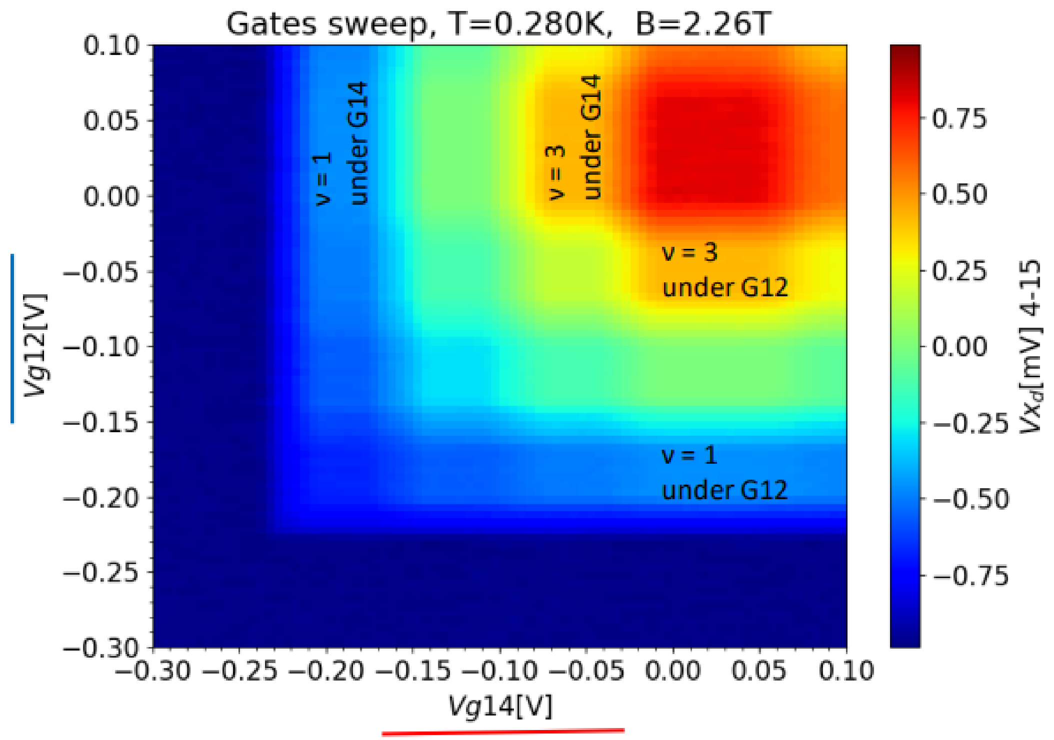
Gate characteristics at ν = 4

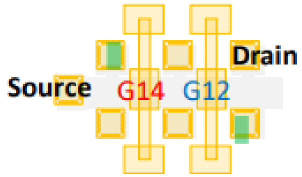
Gate14 mixer equilibration check



Correct equilibration Averages on the diagonal

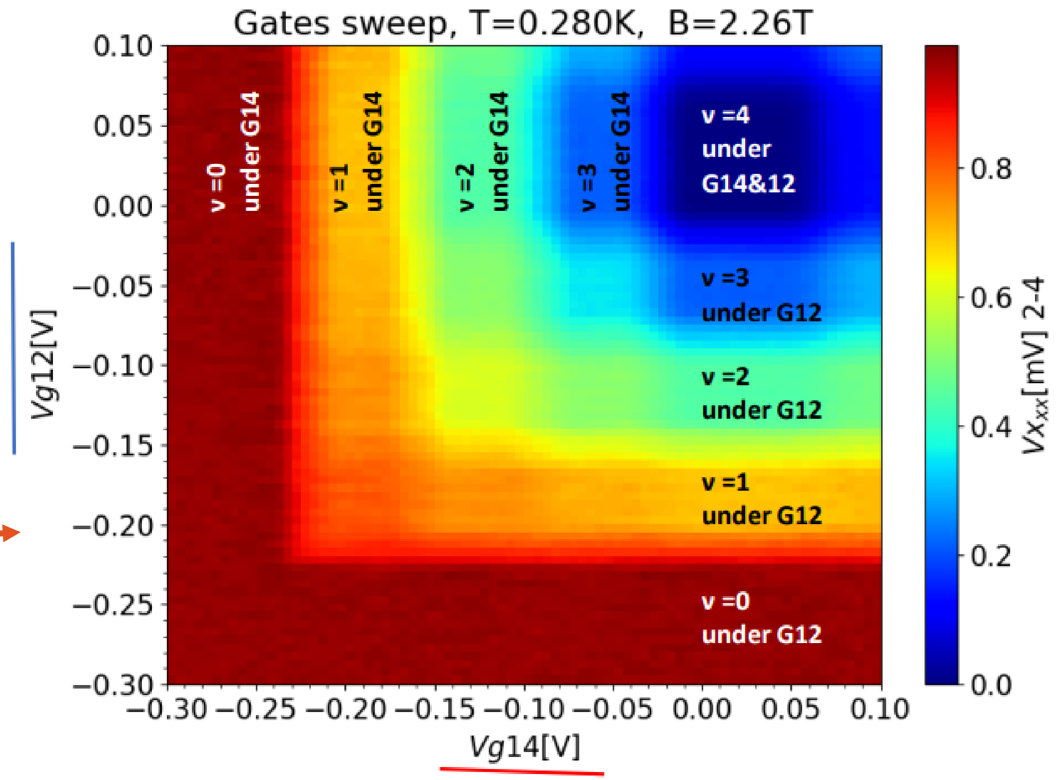
Gate characteristics at ν = 4
Gate12&14 diagonal voltage drops (2D maps)

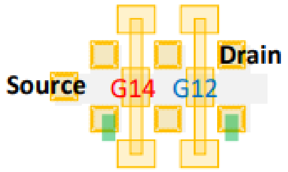




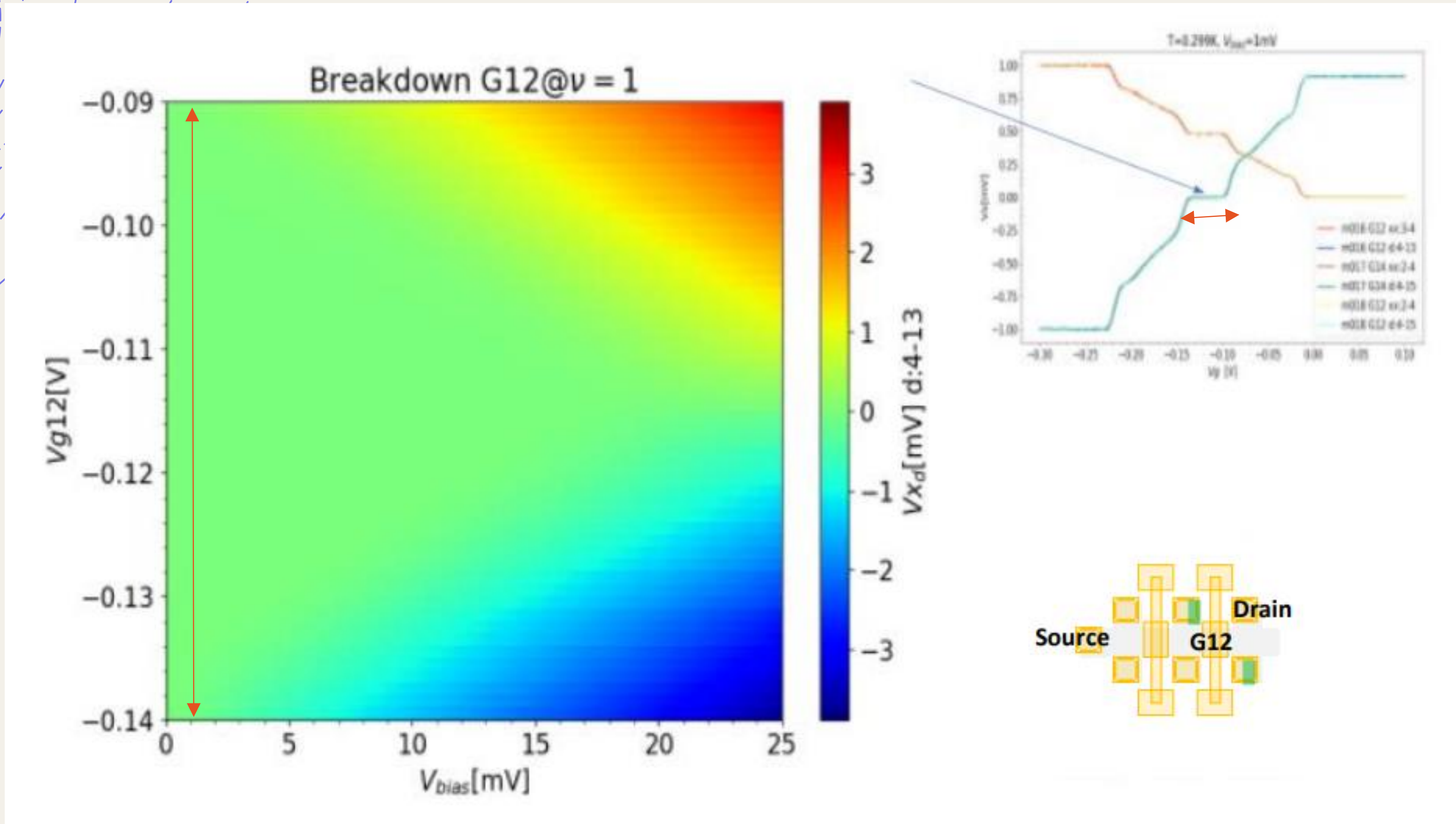
Diagonal

Longitudinal





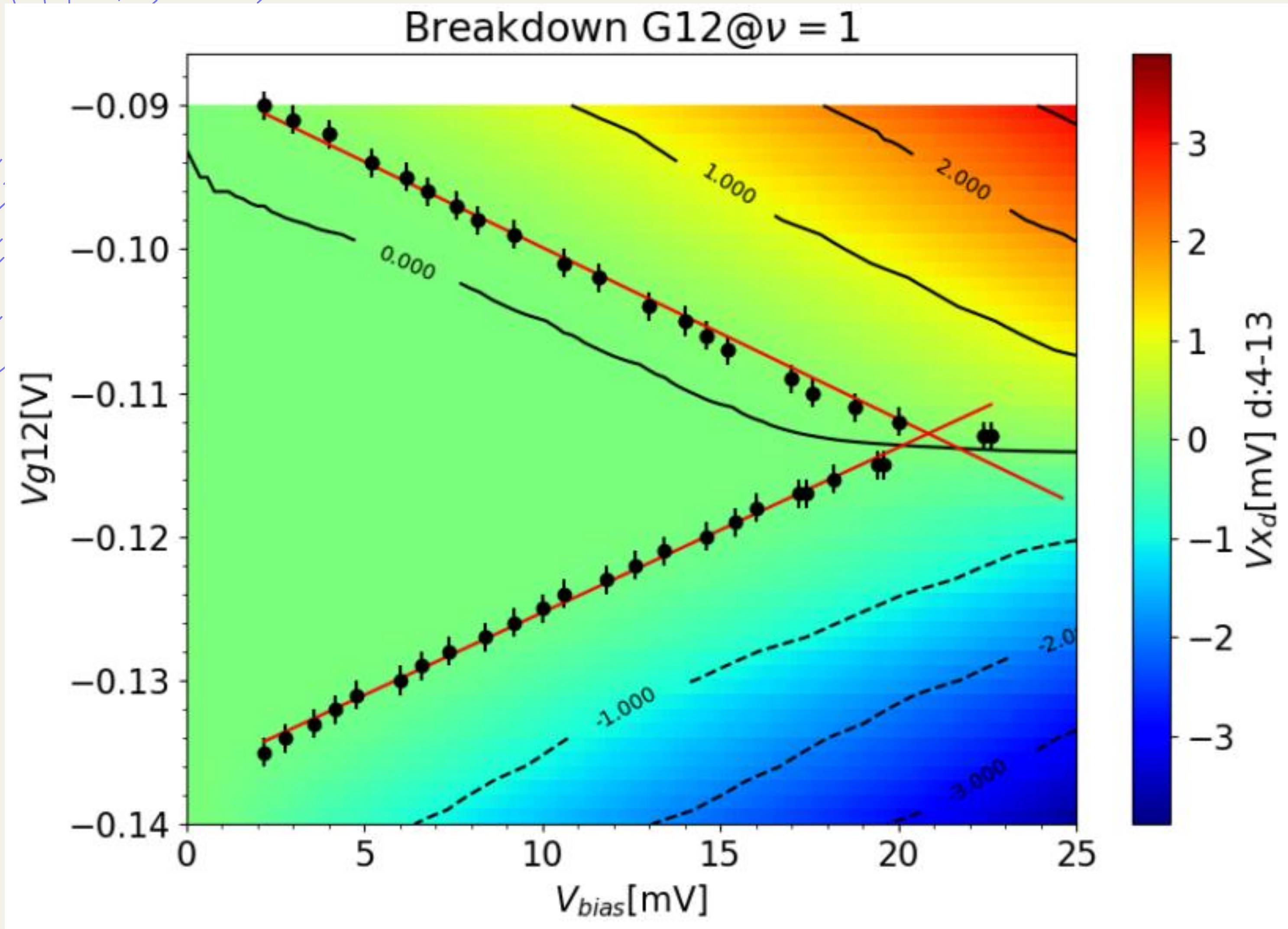
Quantum Hall breakdown
(max voltage bias applicable)



Colormap of gate12 at ν = 1 diagonal voltage

And Hall bar at ν = 2

Max voltage bias values



0.794 ± 0.008 μA

v gate v bar	1	2	3	4
2	20.8 ± 0.2 mV	-	-	-
4	16.0 ± 0.6 mV	14.4 ± 0.2 mV	15 ± 1 mV	10.3 ± 0.1 mV

Ic > 1 μA

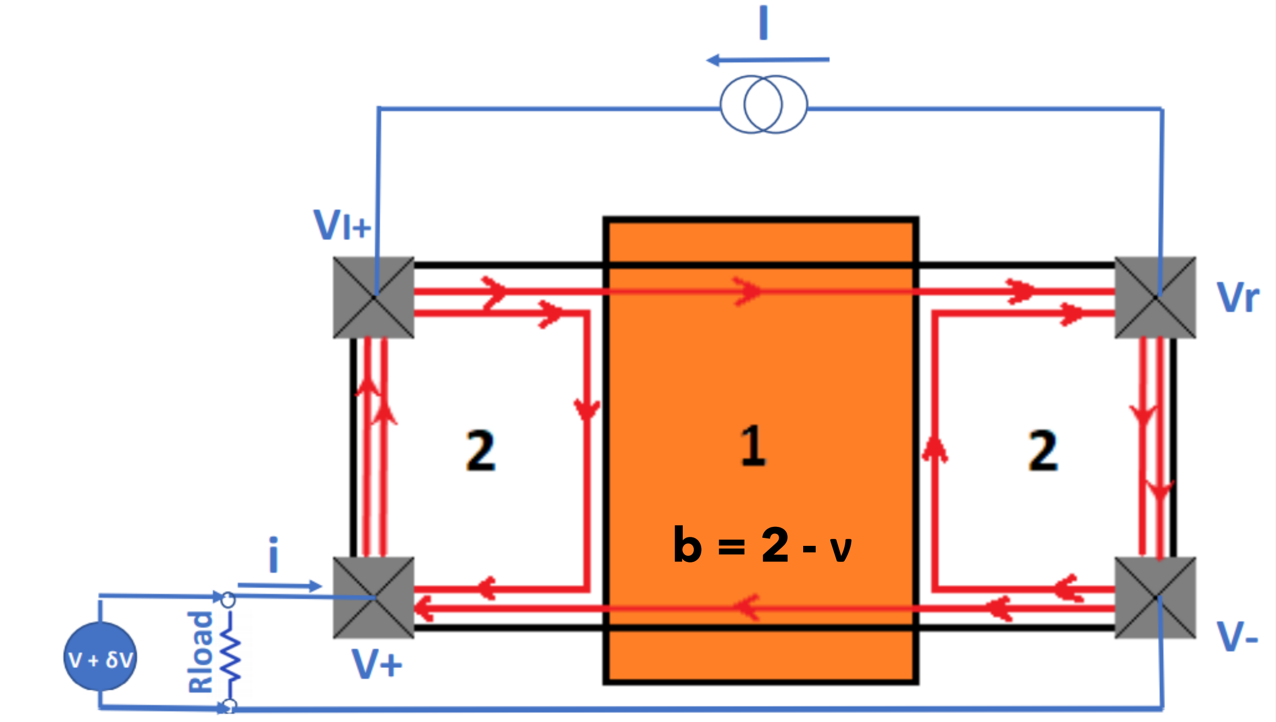
Ic >= 0.3 μA

Max voltage ~ 10 mV

Presentation outline

1) The quantum Hall effect & quantum Hall Metrology
2) Novel approach
3) Experimental results
4) **Numerical results**
5) Conclusions and perspectives

Study of external influences on the precision: R_{out}



$$\delta V = -iRout$$

$$(V + \delta V)/I = (Rload/(Rload + Rout))R$$

R_{4w}
4-w real resistance

4-w ideal resistance

$$R = \mathbf{b(2 - b)}(RK/2)$$

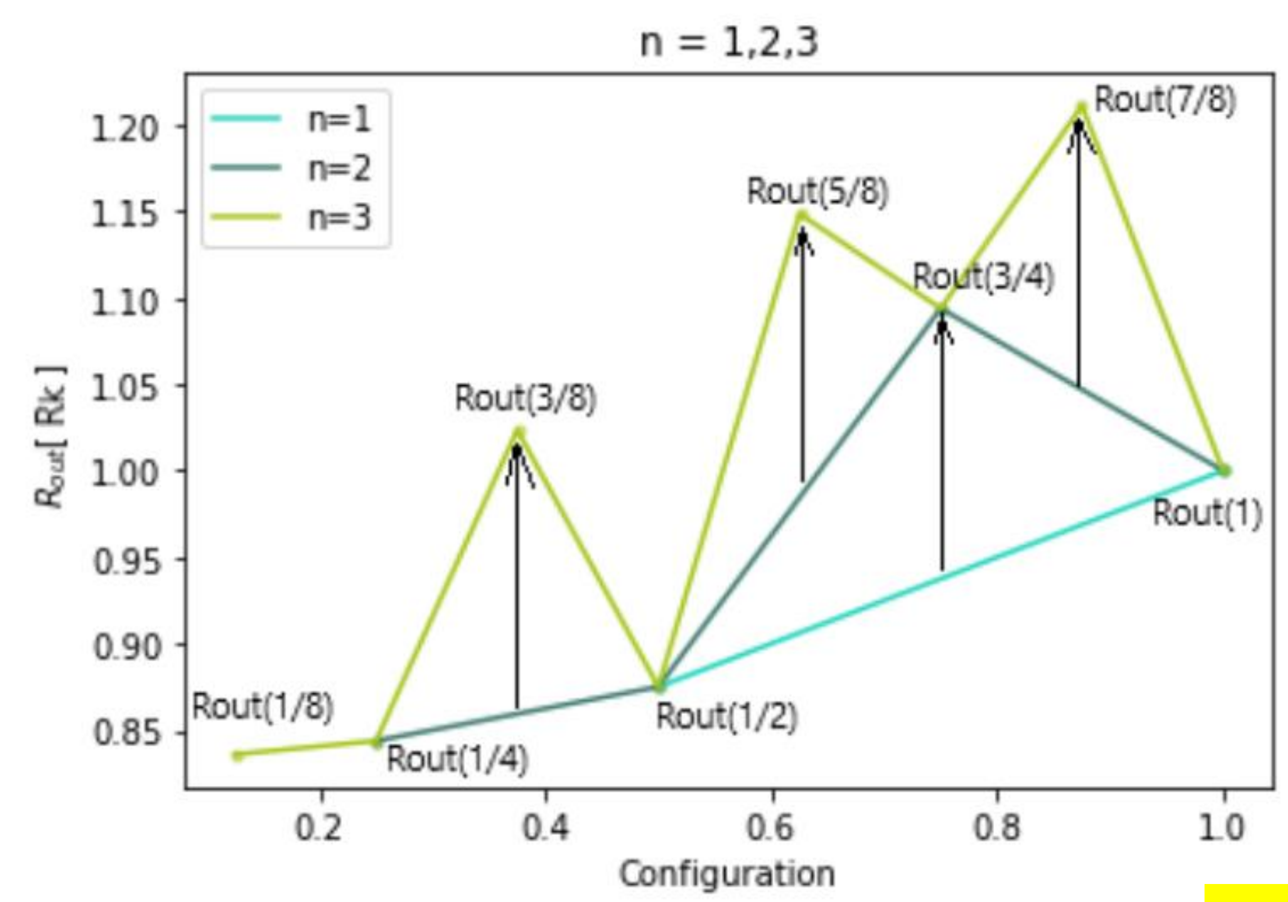
$$R_{4w} = \mathbf{(bR_L/(R_K + (2\text{-}b)R_L))}(R_K/2)$$

$$R_{out} = R_K/(2\text{-}b)$$

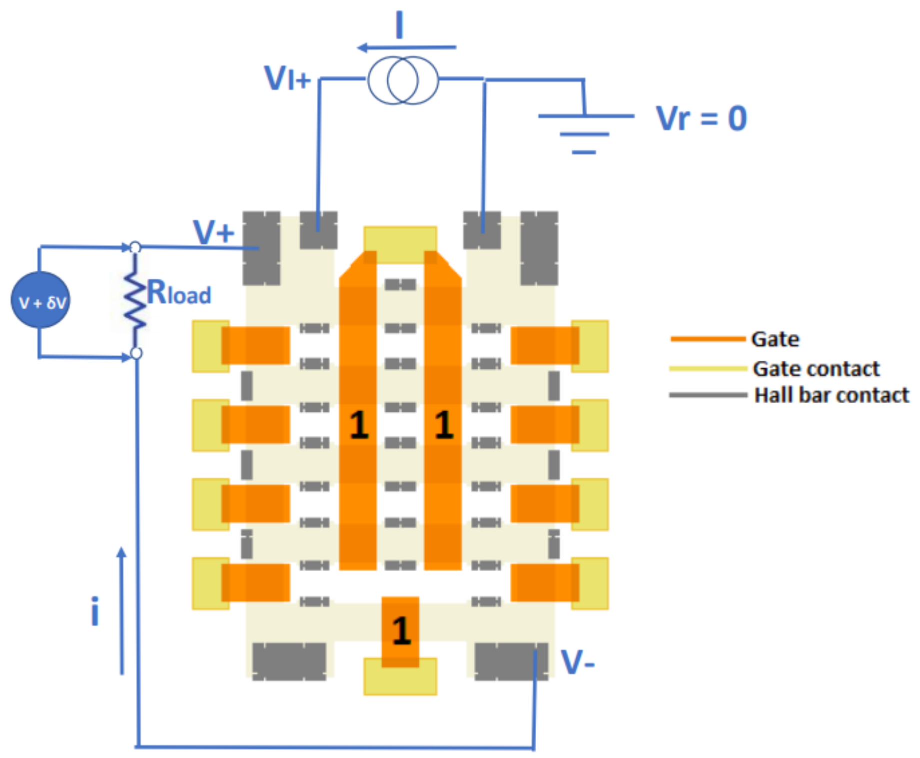
Configuration dependent !

Numerical R_{out} results for the generic bisector

$$R_{out}((f_1 + f_2)/2) = \frac{R_{out}(f_1) + R_{out}(f_2)}{2} + h(n)$$



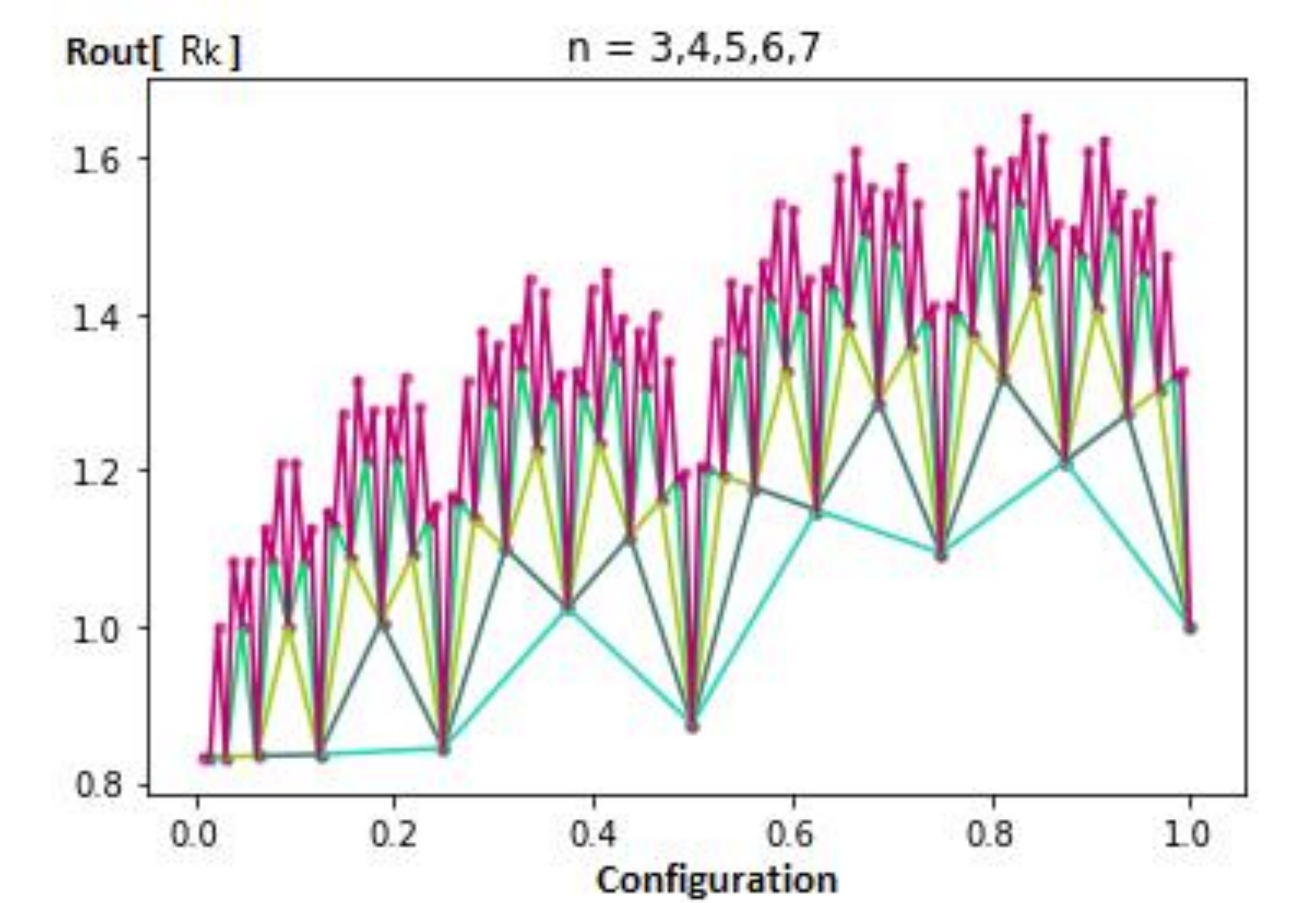
k/2ⁿ



- $n = 1 \quad 2^n = 2 \quad k = 1, 2 \quad f = 0.5, 1$
- $n = 2 \quad 2^n = 4 \quad k = 1, 2, 3, 4 \quad f = 0.25, 0.5, 0.75, 1$
- …

Fractal behaviour

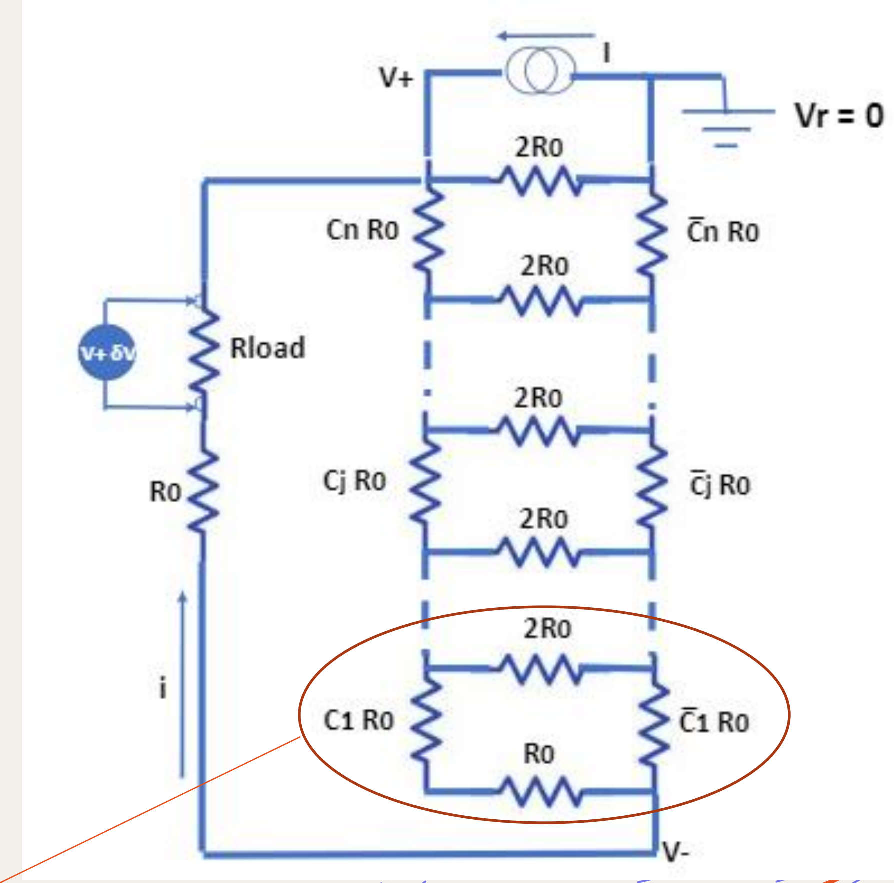
Modeling QH potential drops as a resistive network



k/2^n

$$h(n) = \frac{1}{8}\sum_{i=0}^{n-1} 2^{-2i}$$

Rout is limited for n->∞



Simplifying each stage and finding Rout

Conclusions

- Correct equilibration of the edge channels for the bisection scheme
- Breakdown voltage was found to be ~ 10 mV
- Limited and predictable Rout

What application ?
Other 2D materials ?

Grazie per l'attenzione