

# **Cavity optomechanics: – interactions between light and nanomechanical motion**

**Florian Marquardt**

**University of Erlangen-Nuremberg, Germany, and  
Max-Planck Institute for the Physics of Light (Erlangen)**



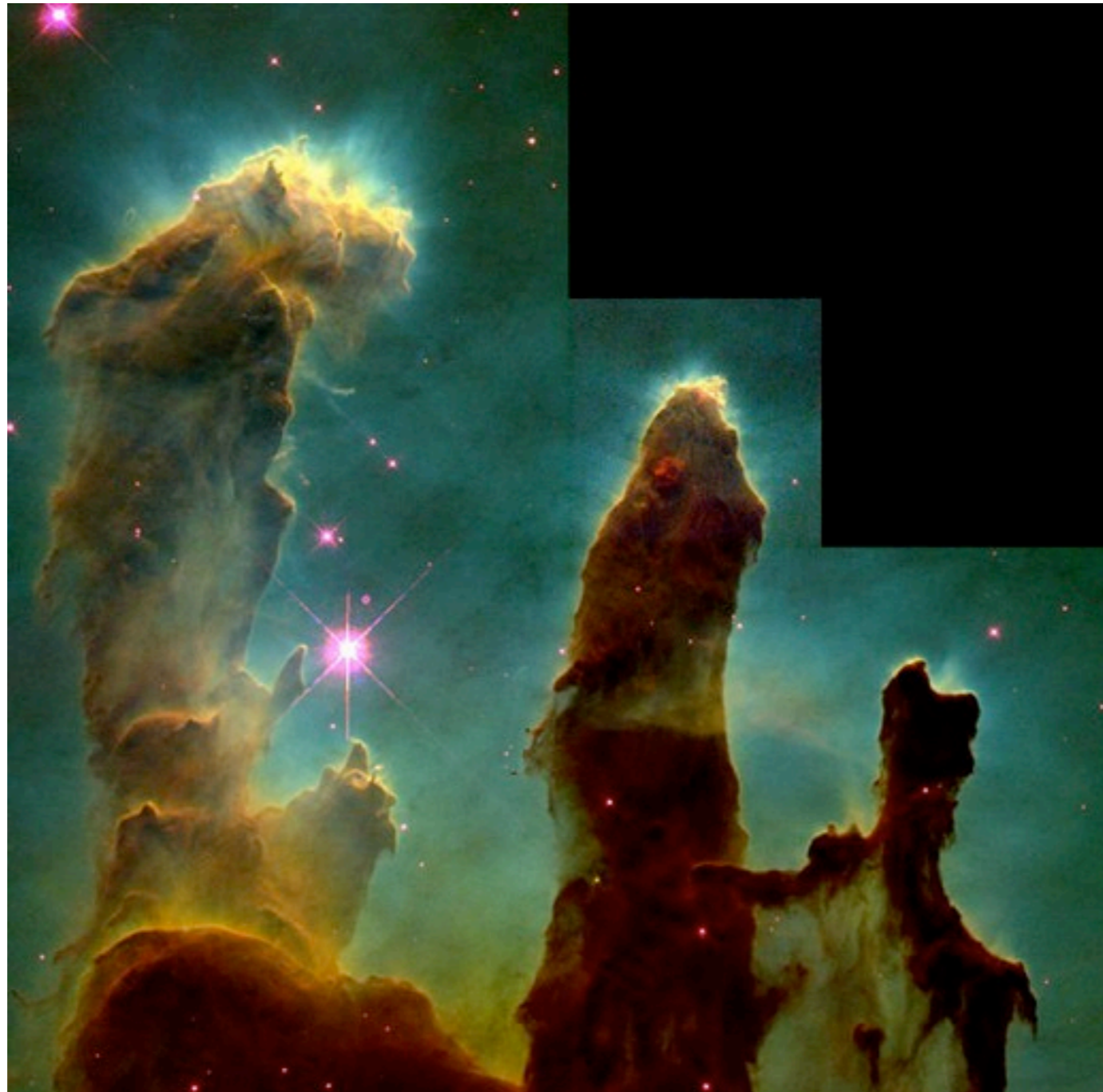
# Radiation pressure



**Johannes Kepler**  
De Cometis, 1619

(Comet Hale-Bopp; by Robert Allevo)

# Radiation pressure



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De Cometis, 1619

# Radiation pressure

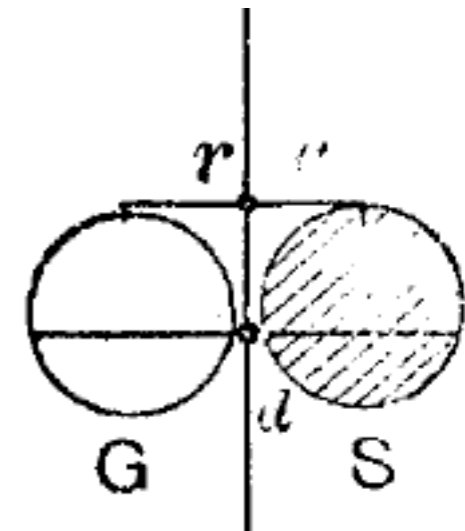
**Nichols and Hull, 1901**

**Lebedev, 1901**

A PRELIMINARY COMMUNICATION ON THE  
PRESSURE OF HEAT AND LIGHT  
RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

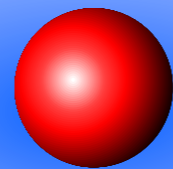
**M**AXWELL,<sup>1</sup> dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."



$$F = \frac{2I}{c}$$

Nichols and Hull, Physical Review **13**, 307 (1901)

# Radiation forces



Trapping and cooling

- Optical tweezers
- Optical lattices


...but usually no back-action from motion onto light!

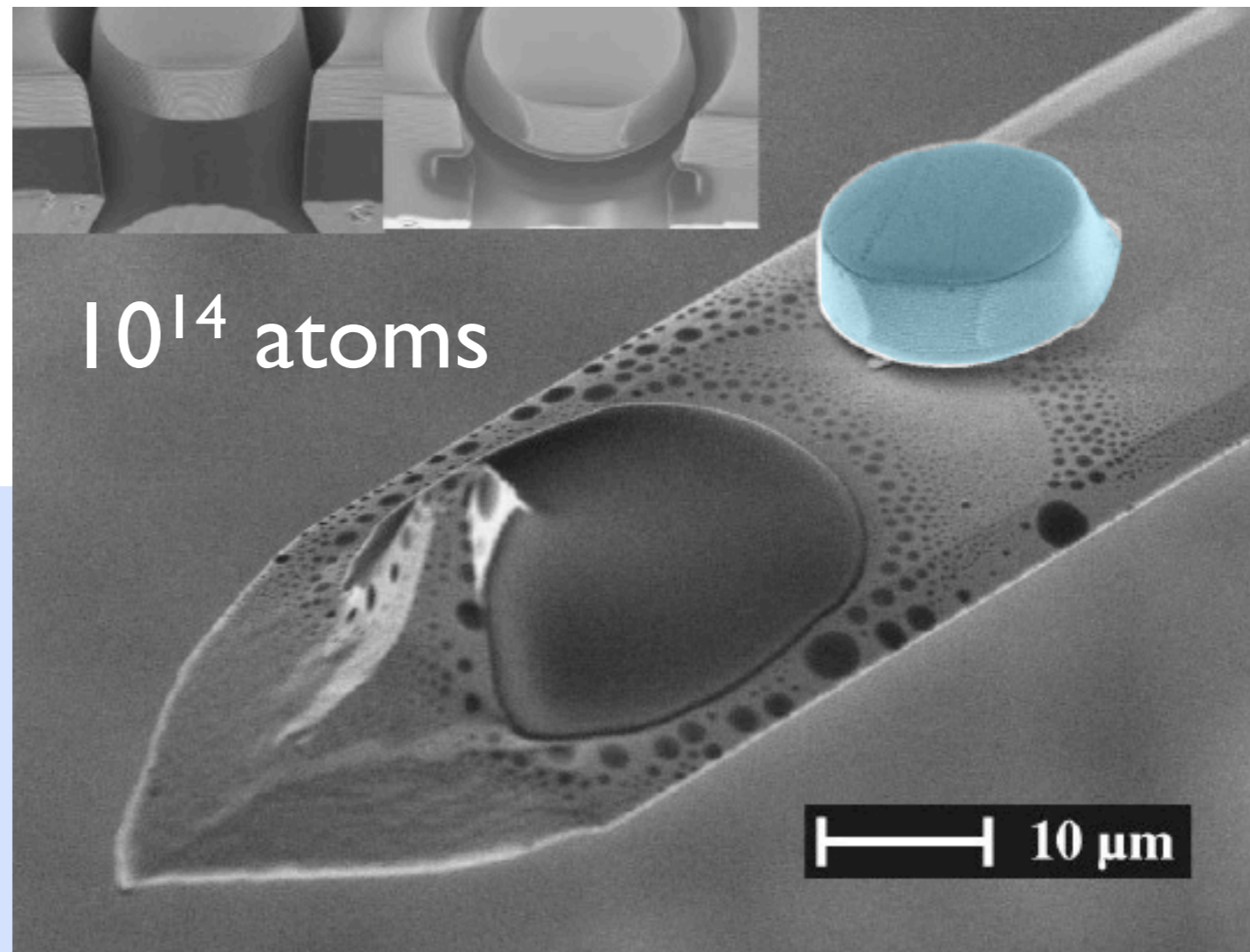
# Optomechanics on different length scales



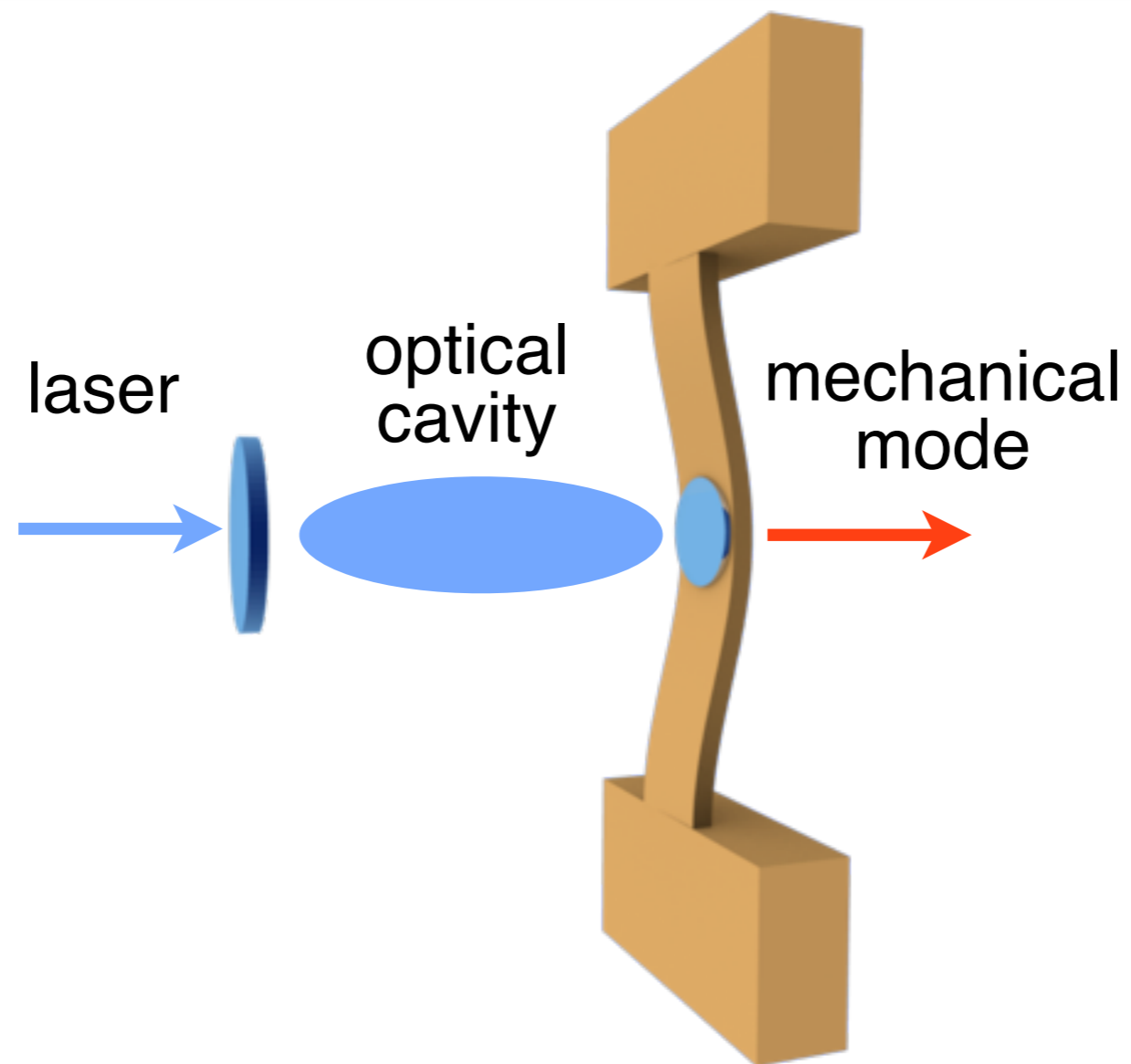
**LIGO – Laser Interferometer  
Gravitational  
Wave Observatory**

**Mirror on cantilever –  
Bouwmeester lab, Santa Barbara  
(2006)**

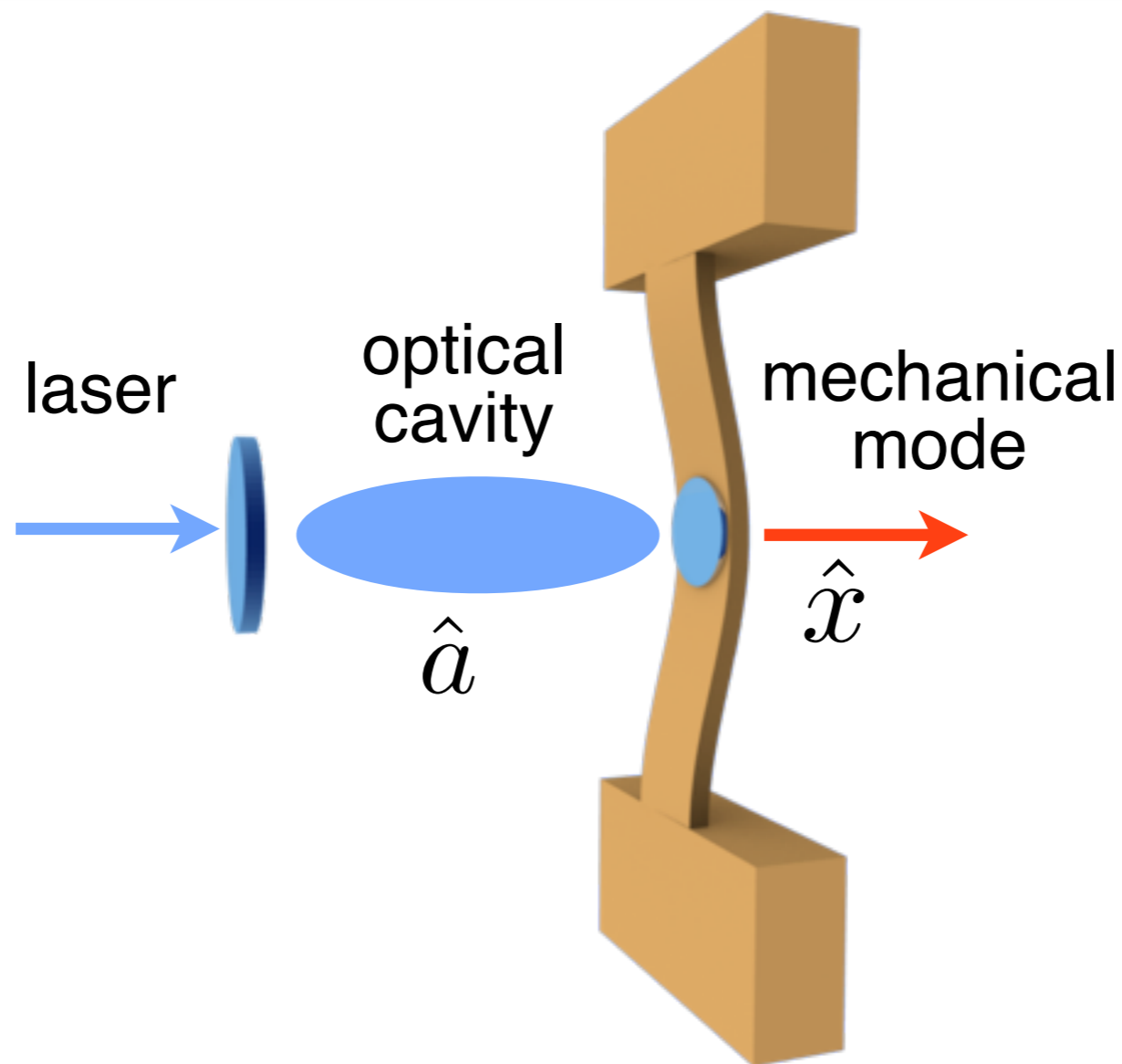
$$\omega_M \sim 1\text{kHz} - 1\text{GHz}$$
$$m \sim 10^{-12} - 10^{-10}\text{kg}$$
$$x_{\text{ZPF}} \sim 10^{-16} - 10^{-14}\text{m}$$
$$x_{\text{ZPF}} = \sqrt{\hbar/(2m\omega_M)}$$




# Optomechanical Hamiltonian



# Optomechanical Hamiltonian



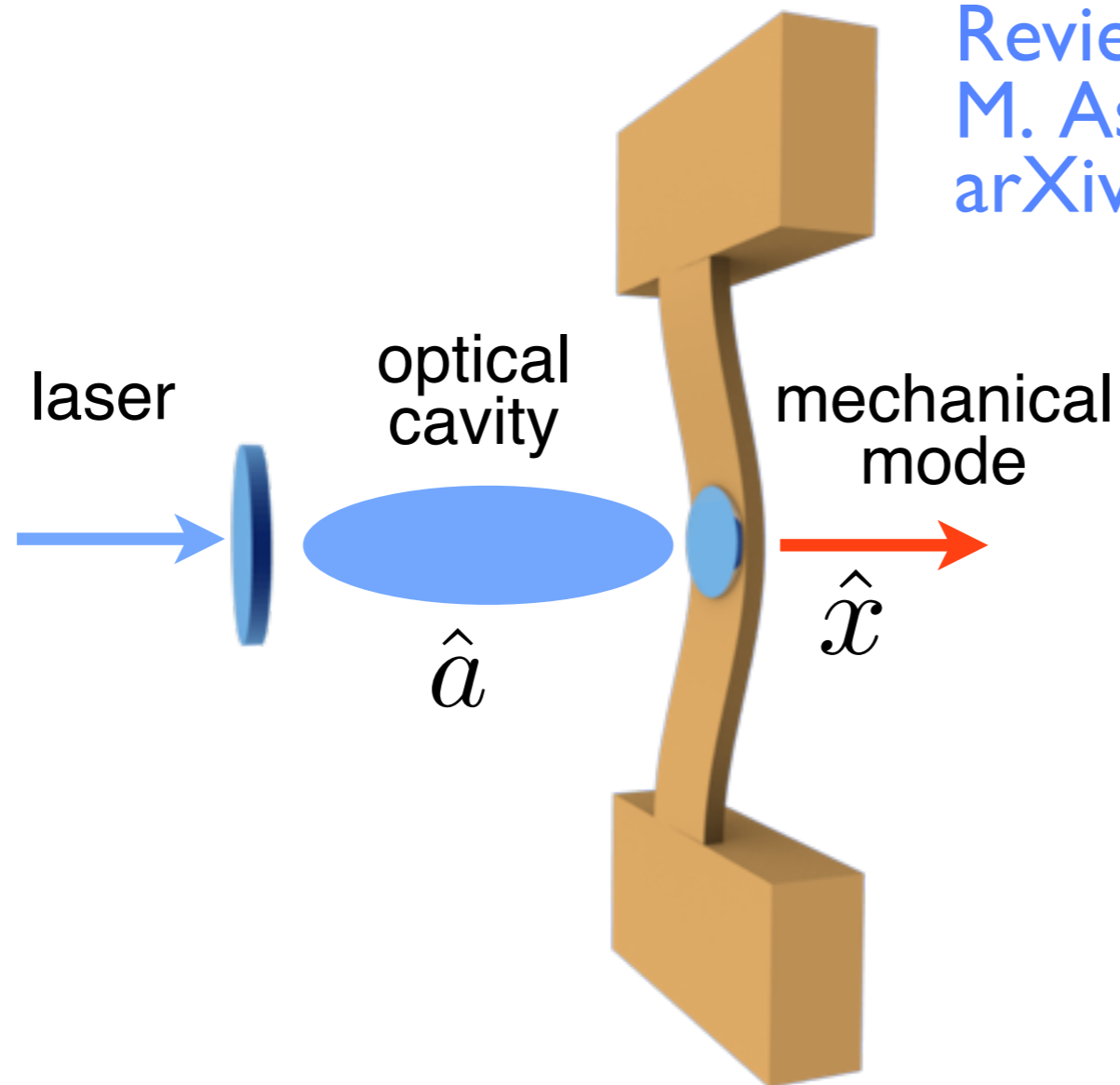
$$\hat{H} = \hbar\omega_{\text{cav}} \cdot (1 - \hat{x}/L)\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \dots$$

$$\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger) \quad x_{\text{ZPF}} = \sqrt{\frac{\hbar}{2m\Omega}}$$



# Optomechanical Hamiltonian

Review “Cavity Optomechanics”:  
M. Aspelmeyer, T. Kippenberg, FM  
arXiv:1303.0733



$$g_0 \sim \text{Hz} - \text{MHz}$$

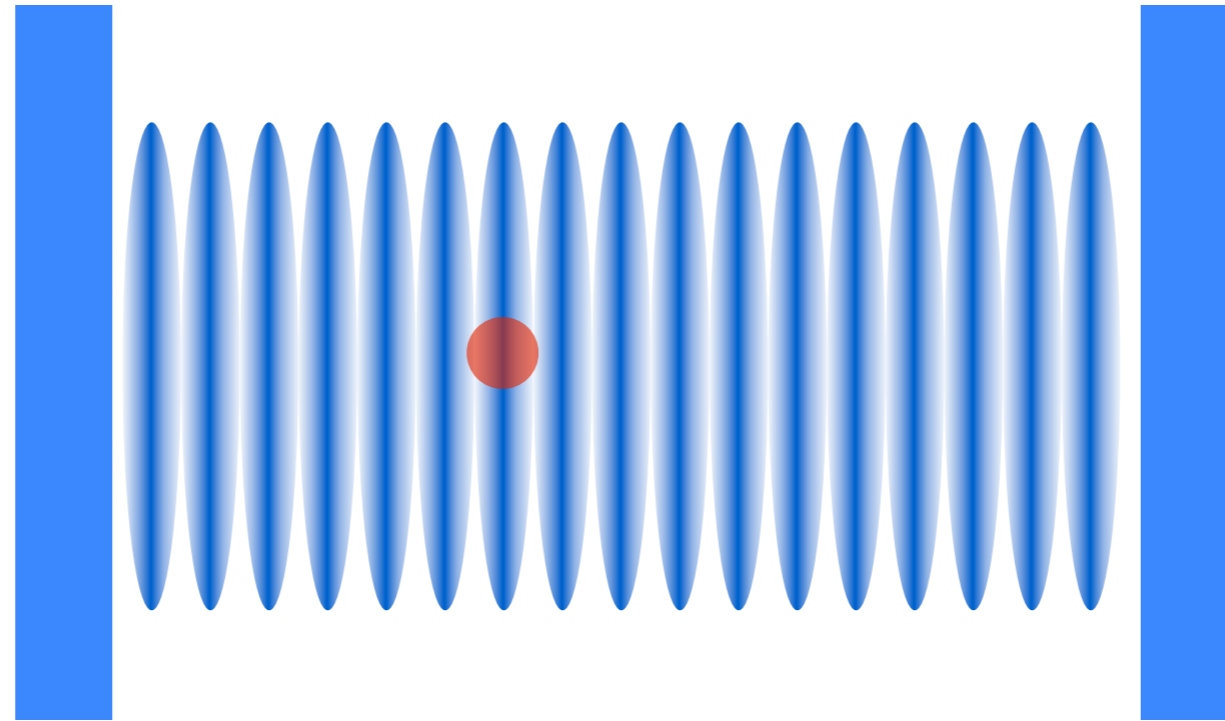
$$\hat{H} = -(\Delta + g_0(\hat{b} + \hat{b}^\dagger))\hat{a}^\dagger\hat{a} + \Omega\hat{b}^\dagger\hat{b} + \dots$$

laser detuning  
 $\Delta = \omega_L - \omega_{\text{cav}}$

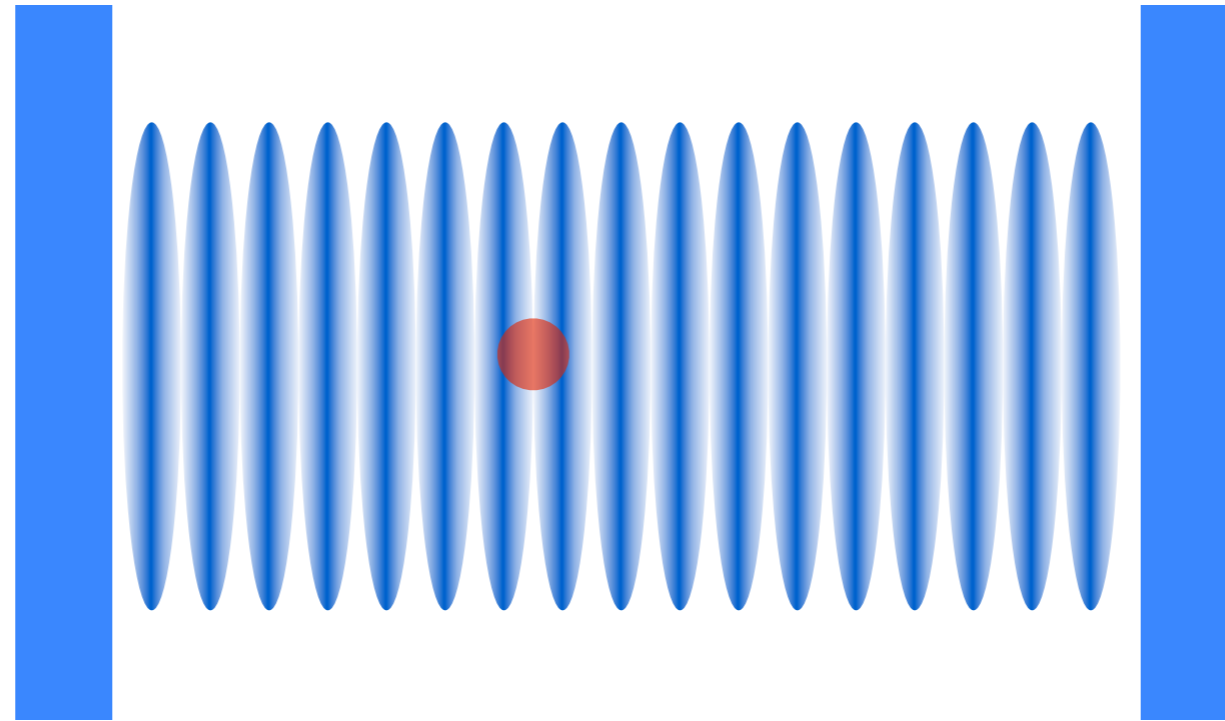
optomech.  
coupling

$$\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger)$$
$$x_{\text{ZPF}} = \sqrt{\hbar/2m\Omega}$$

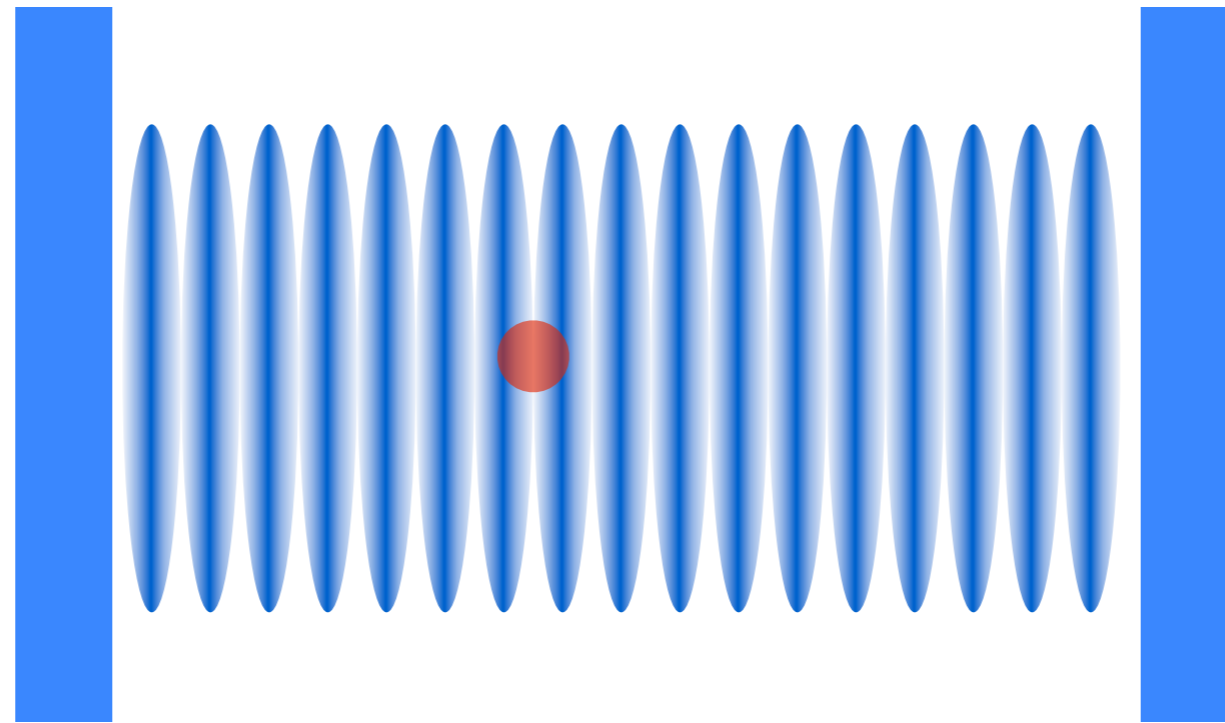
# Optomechanical Hamiltonian



# Optomechanical Hamiltonian



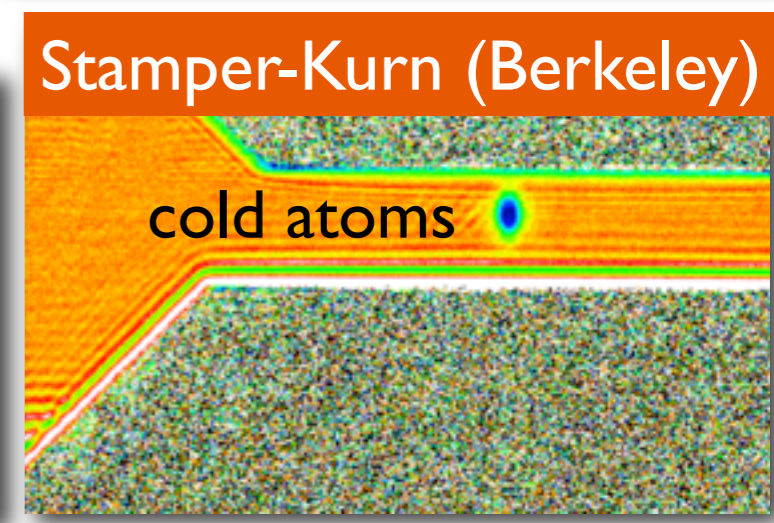
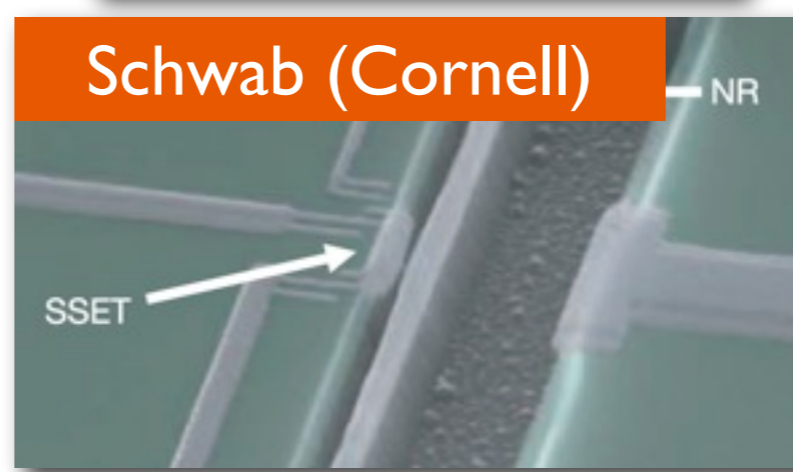
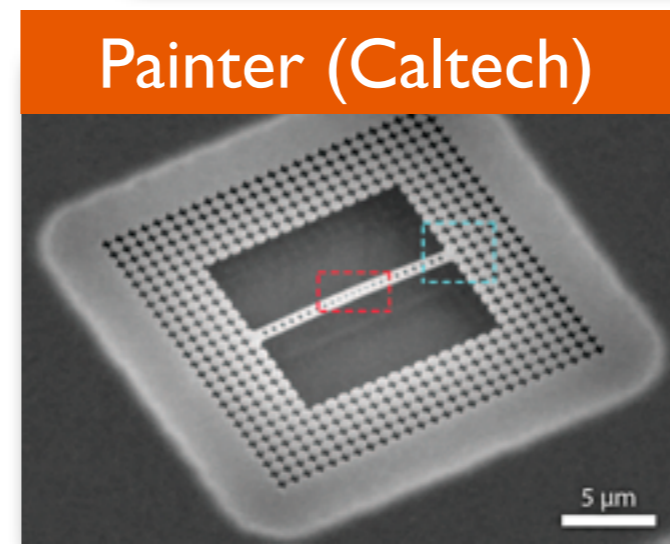
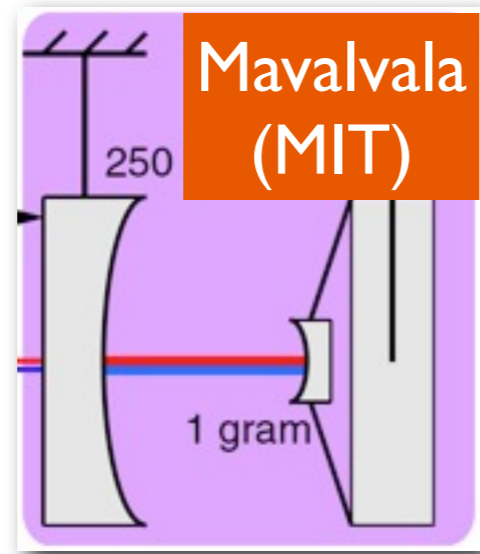
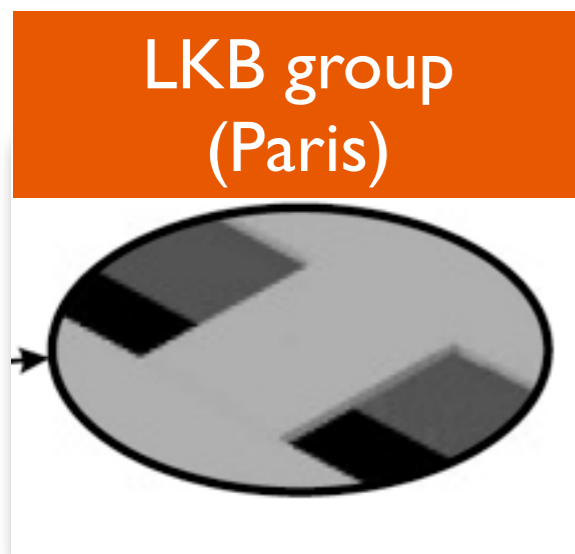
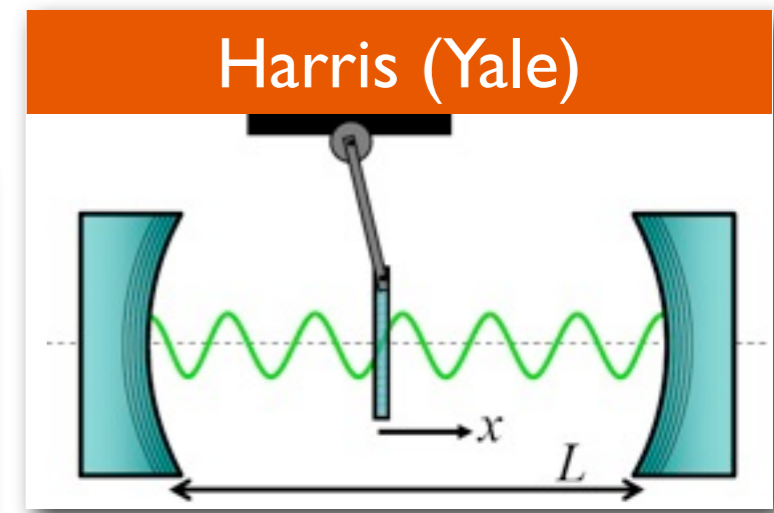
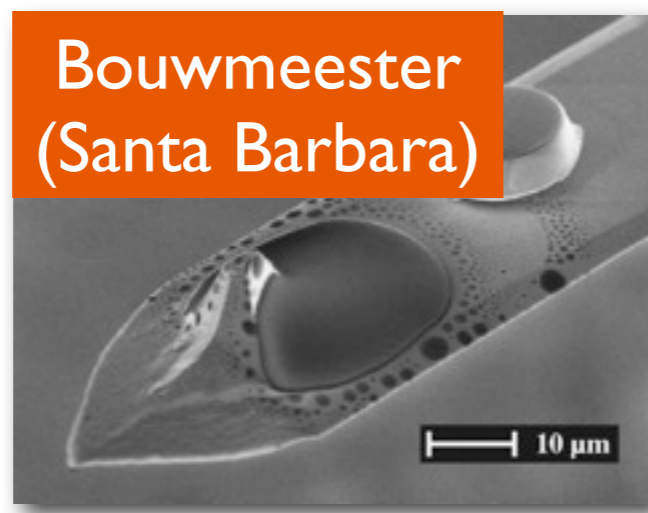
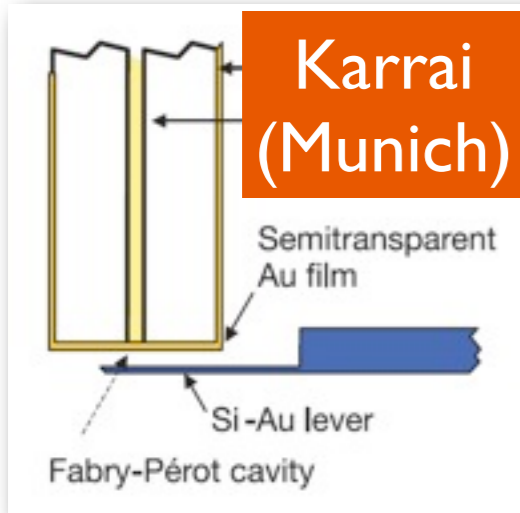
# Optomechanical Hamiltonian



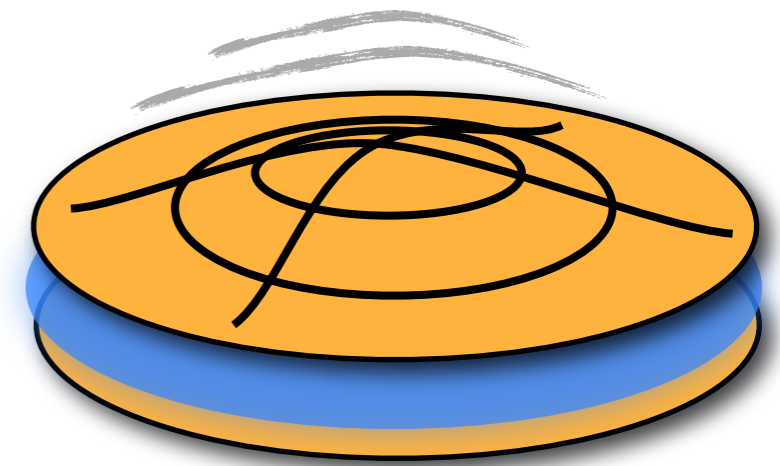
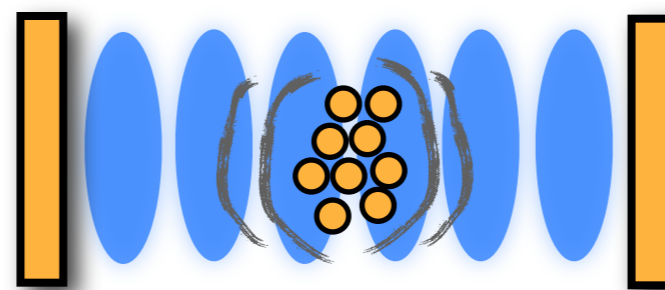
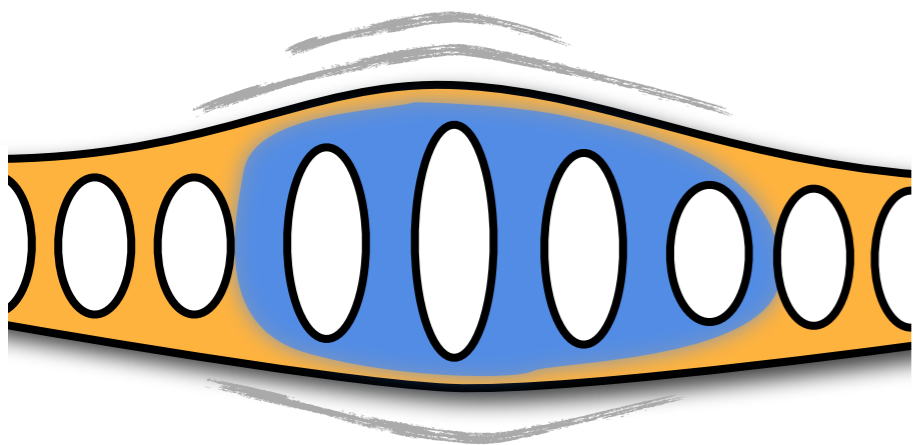
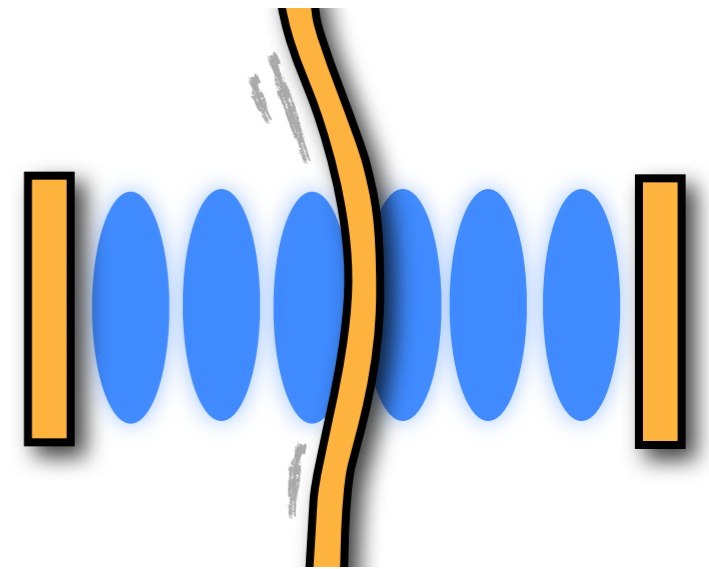
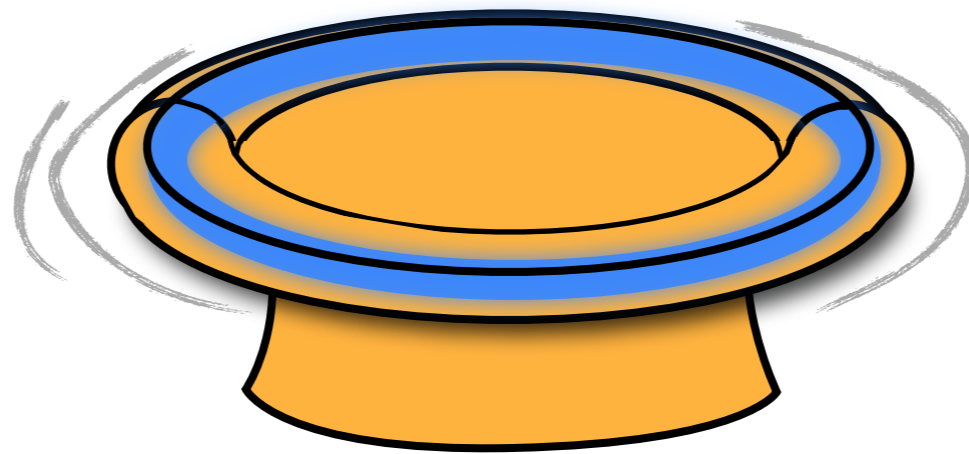
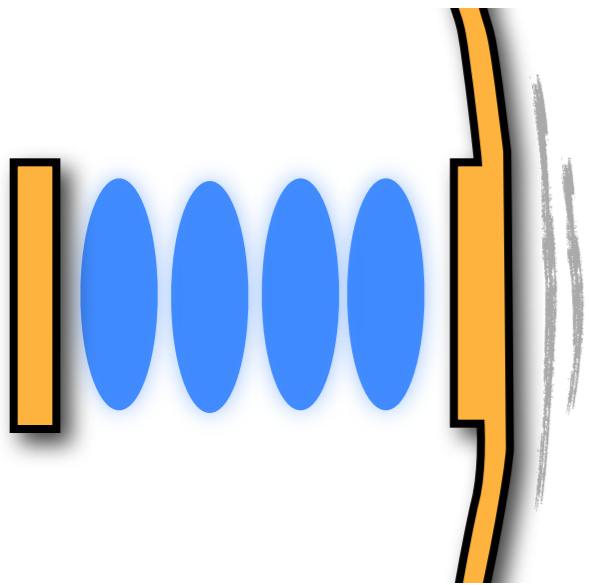
$$\hat{H} = \hbar\omega_{\text{cav}}(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \dots$$

...any dielectric moving inside a cavity  
generates an optomechanical interaction!

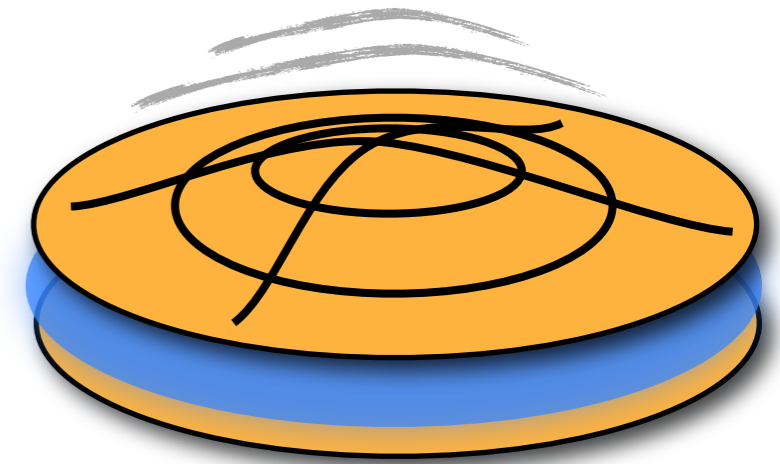
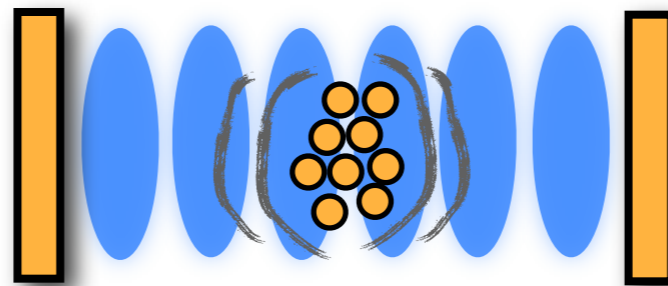
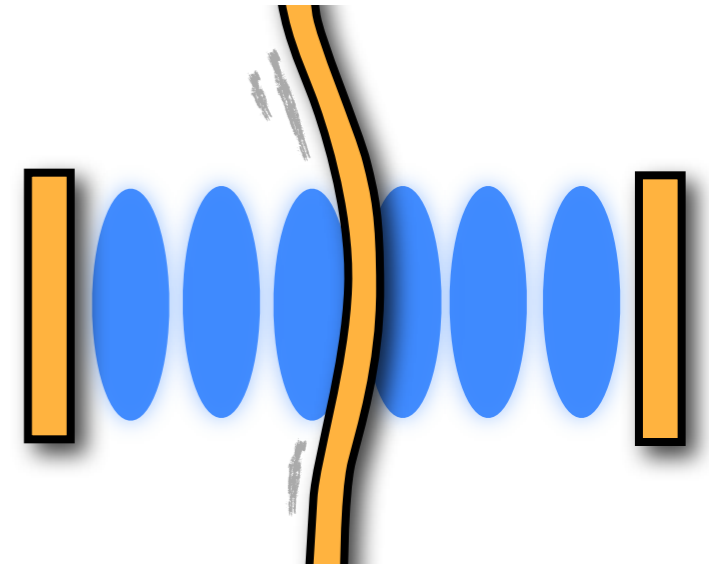
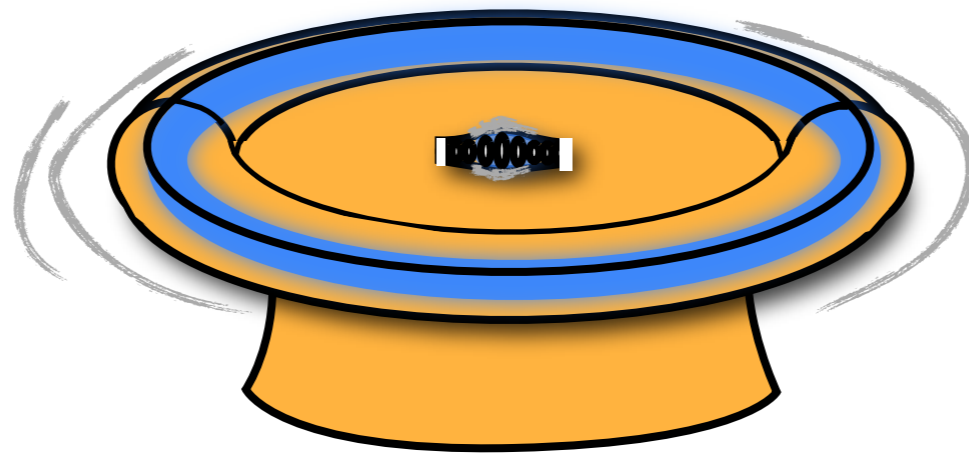
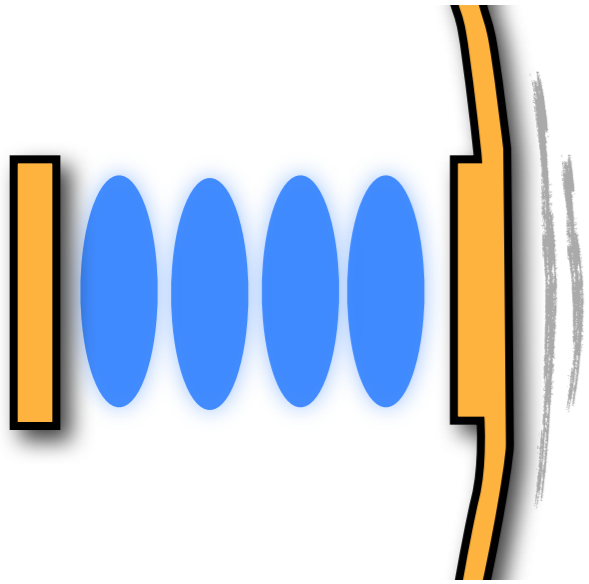
# The zoo of optomechanical (and analogous) systems



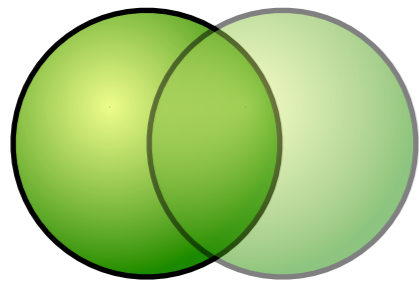
# The zoo of optomechanical (and analogous) systems



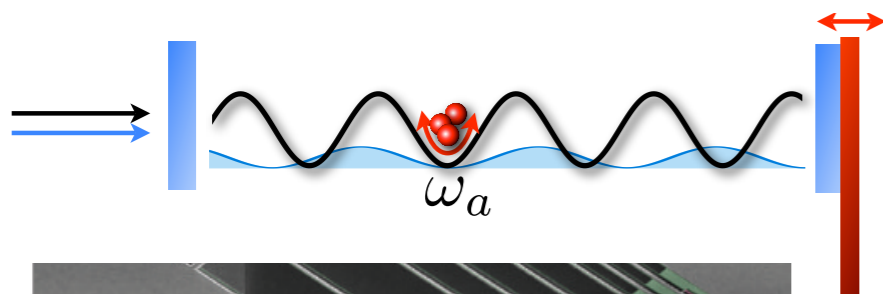
# The zoo of optomechanical (and analogous) systems



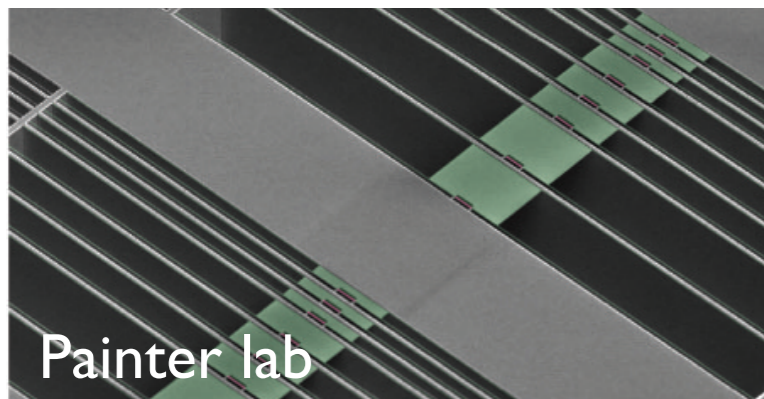
# Optomechanics: general outlook



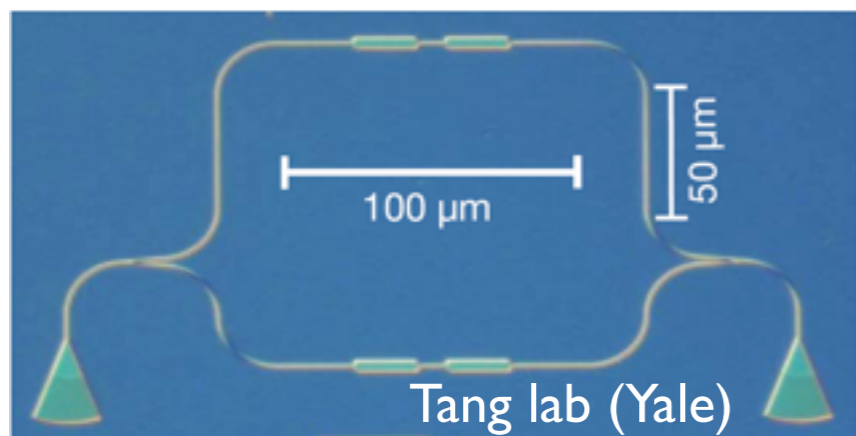
**Fundamental tests of quantum mechanics in a new regime:** entanglement with ‘macroscopic’ objects, unconventional decoherence? [e.g.: gravitationally induced?]



**Mechanics as a ‘bus’ for connecting hybrid components:** superconducting qubits, spins, photons, cold atoms, ....



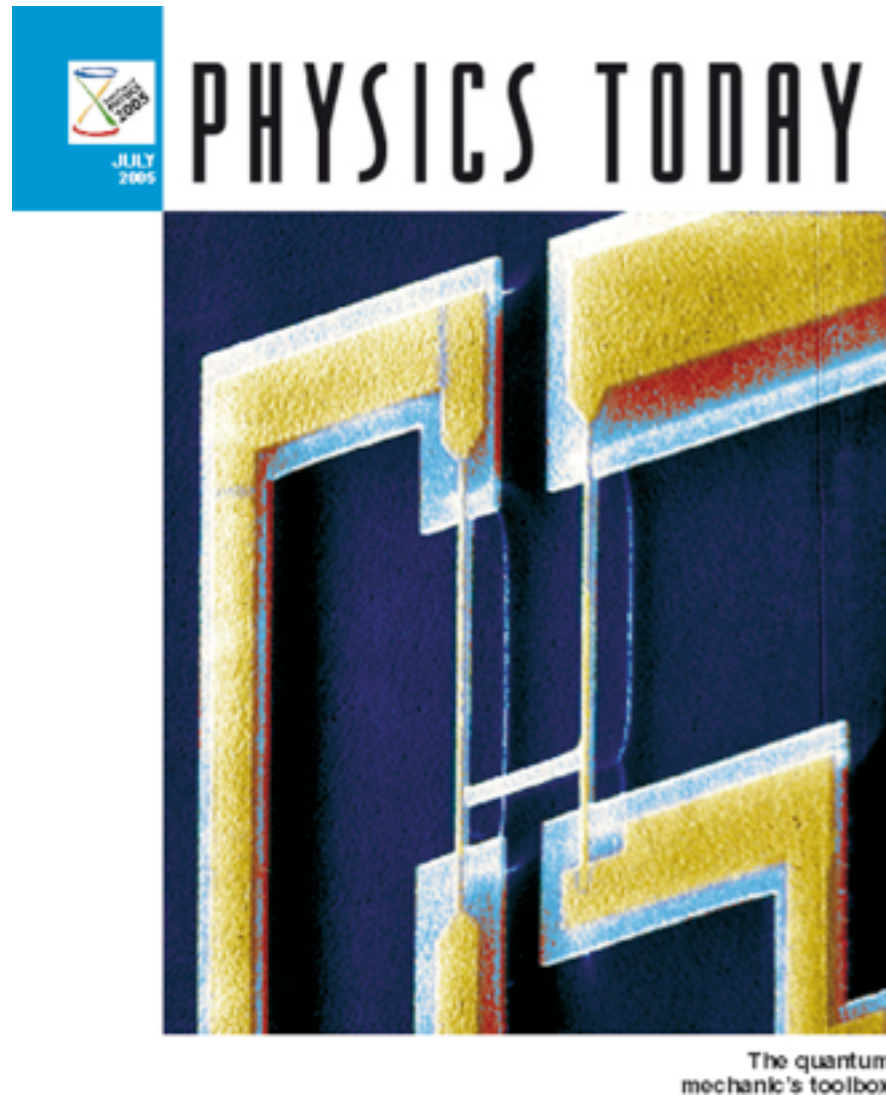
**Precision measurements**  
small displacements, masses, forces, and accelerations



**Optomechanical circuits & arrays**  
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays



# Towards the quantum regime of mechanical motion



## Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

Keith C. Schwab and Michael L. Roukes

**E**verything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element.

achieved to read out those devices, now bring us to the realm of quantum mechanical systems.

### The quantum realm

What conditions are required to observe the quantum properties of a mechanical structure, and what can we learn when we encounter them? Such questions have received

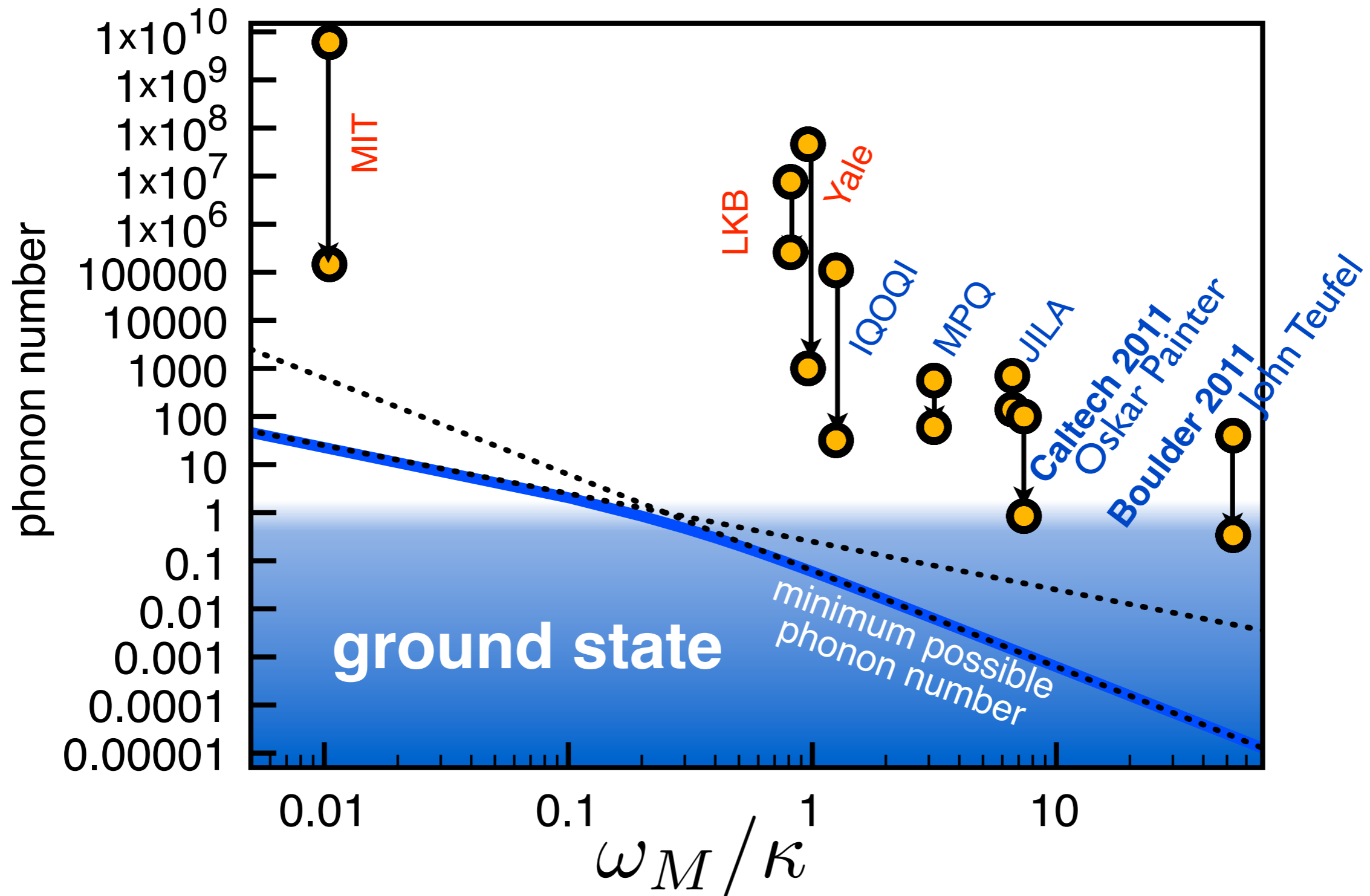
**Schwab and Roukes, Physics Today 2005**

- nano-electro-mechanical systems

Superconducting qubit coupled to nanoresonator: Cleland & Martinis 2010

- optomechanical systems

# Laser-cooling towards the ground state

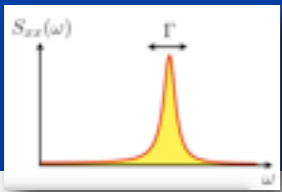


analogy to (cavity-assisted)  
laser cooling of atoms

FM et al., PRL **93**, 093902 (2007)

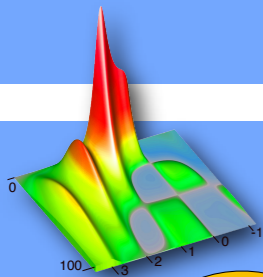
Wilson-Rae et al., PRL **99**, 093901 (2007)

# Optomechanics (Outline)

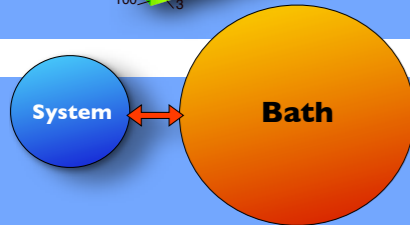


Displacement detection

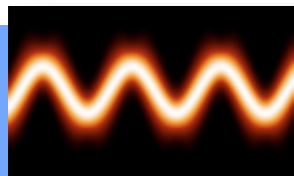
Linear optomechanics



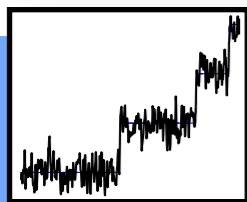
Nonlinear dynamics



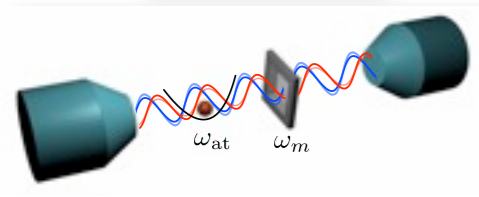
Quantum theory of cooling



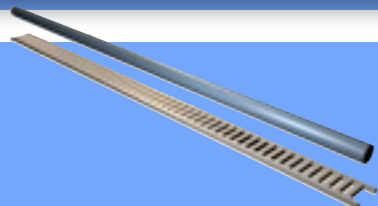
Interesting quantum states



Towards Fock state detection

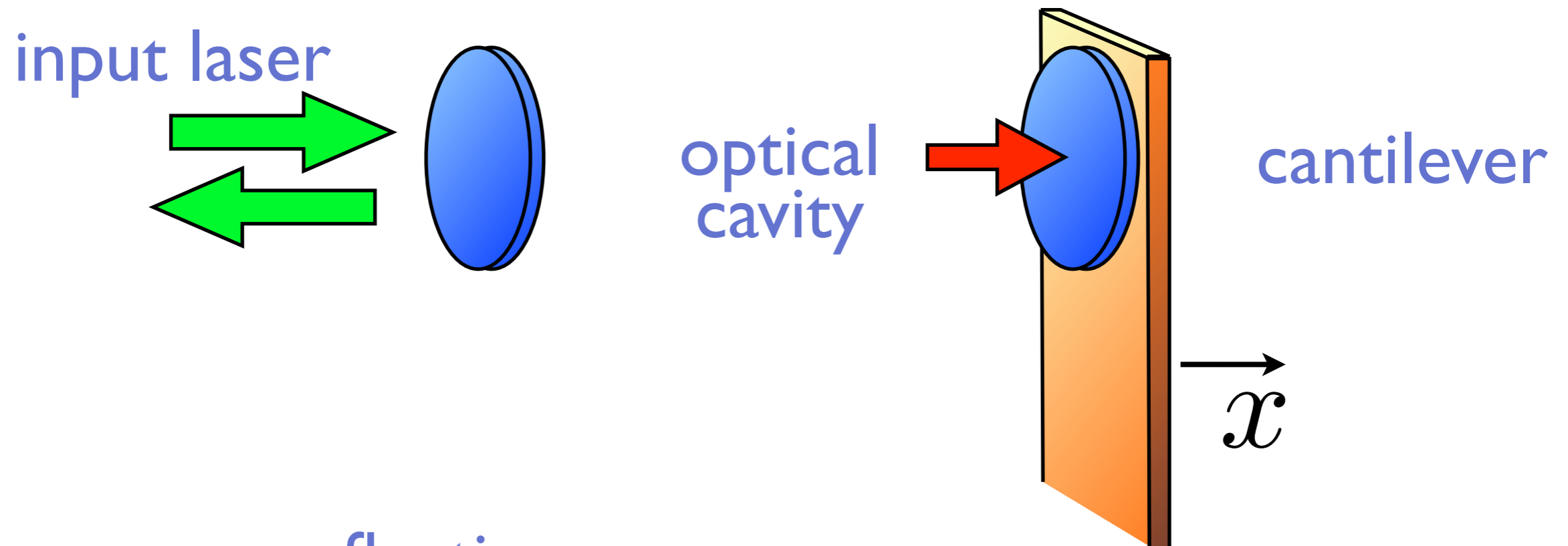


Hybrid systems: coupling to atoms

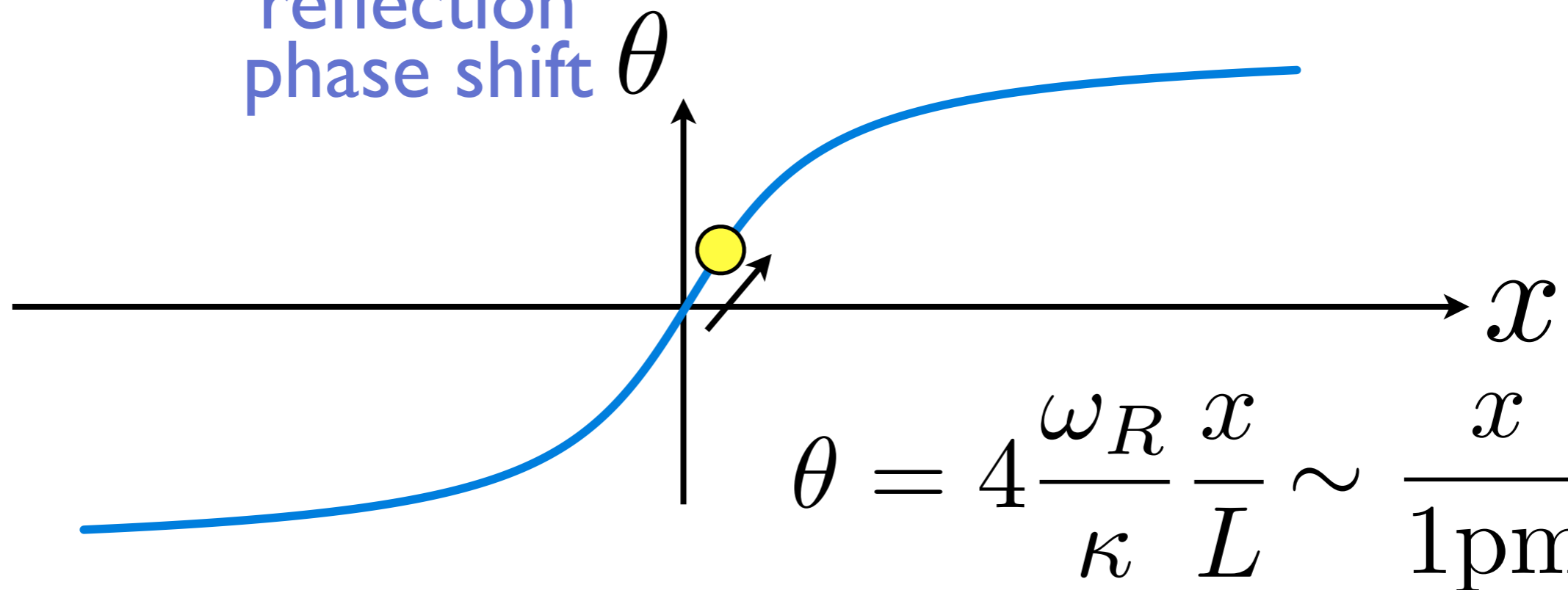


Optomechanical crystals & arrays

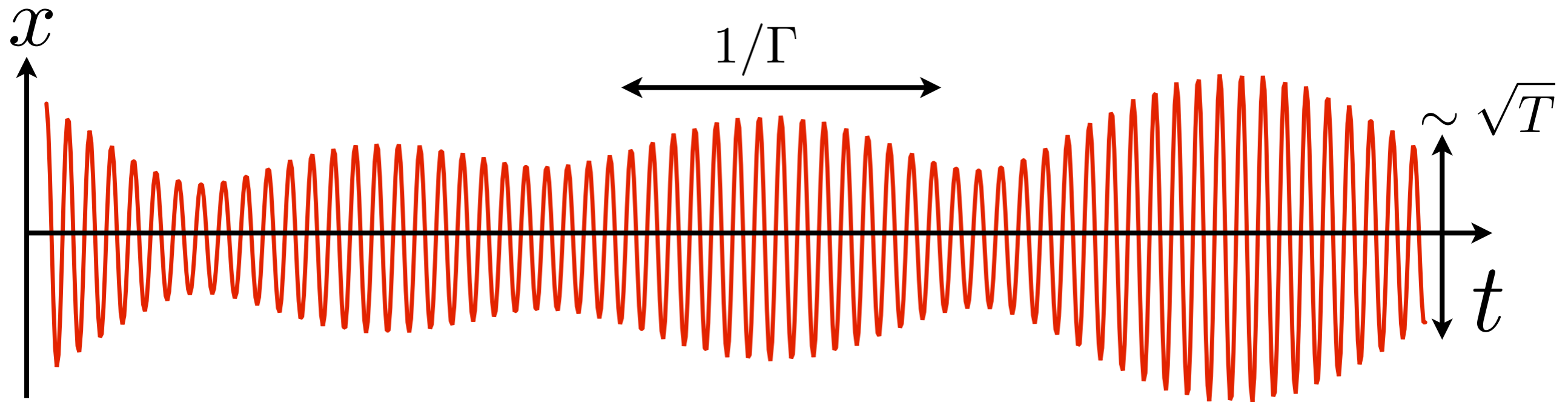
# Optical displacement detection



reflection phase shift  $\theta$



# Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2} \langle x^2 \rangle = \frac{k_B T}{2} \Rightarrow \langle x^2 \rangle = \frac{k_B T}{m\omega_M^2} \text{ extract temperature!}$$

Possibilities:

- Direct time-resolved detection
- Analyze **fluctuation spectrum of  $x$**

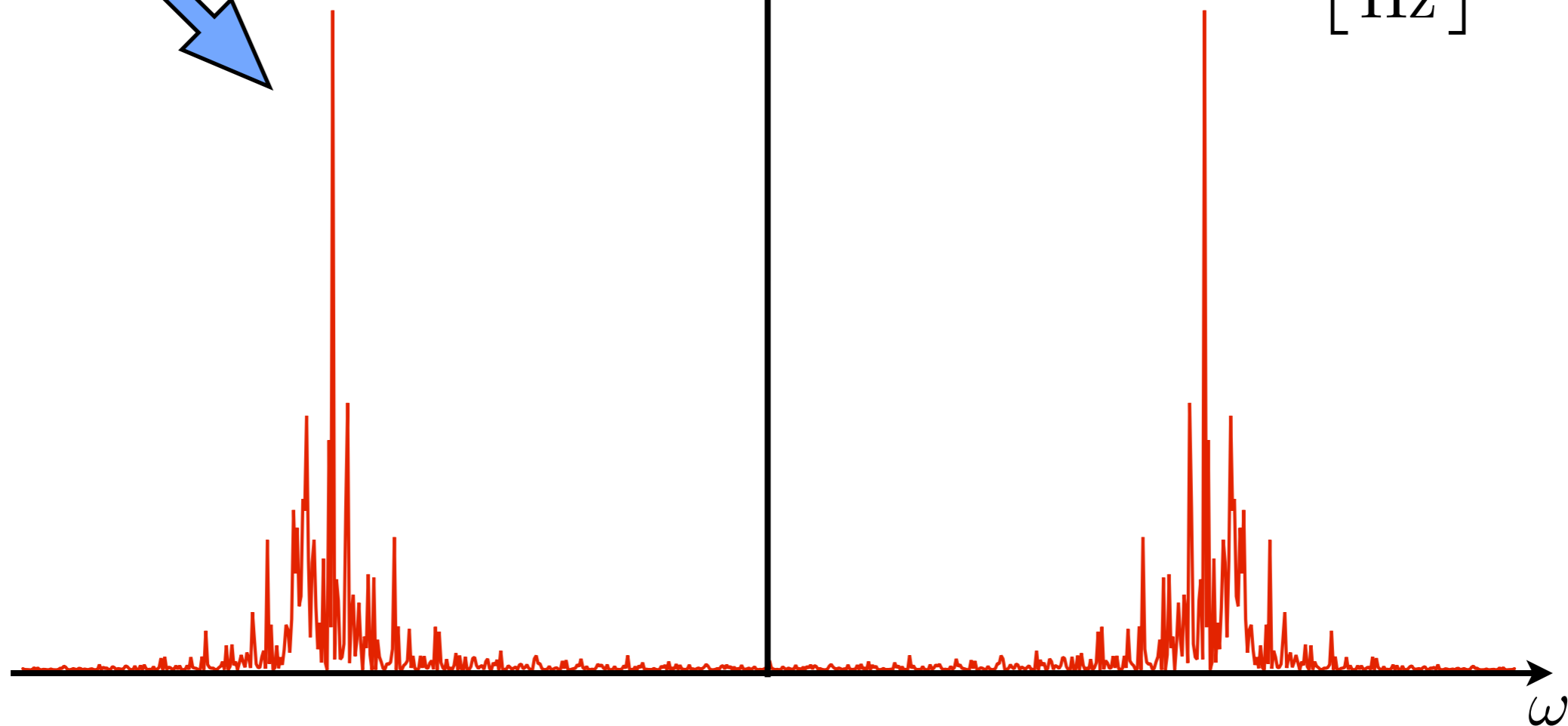
# Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^{\tau} dt e^{i\omega t} x(t)$$

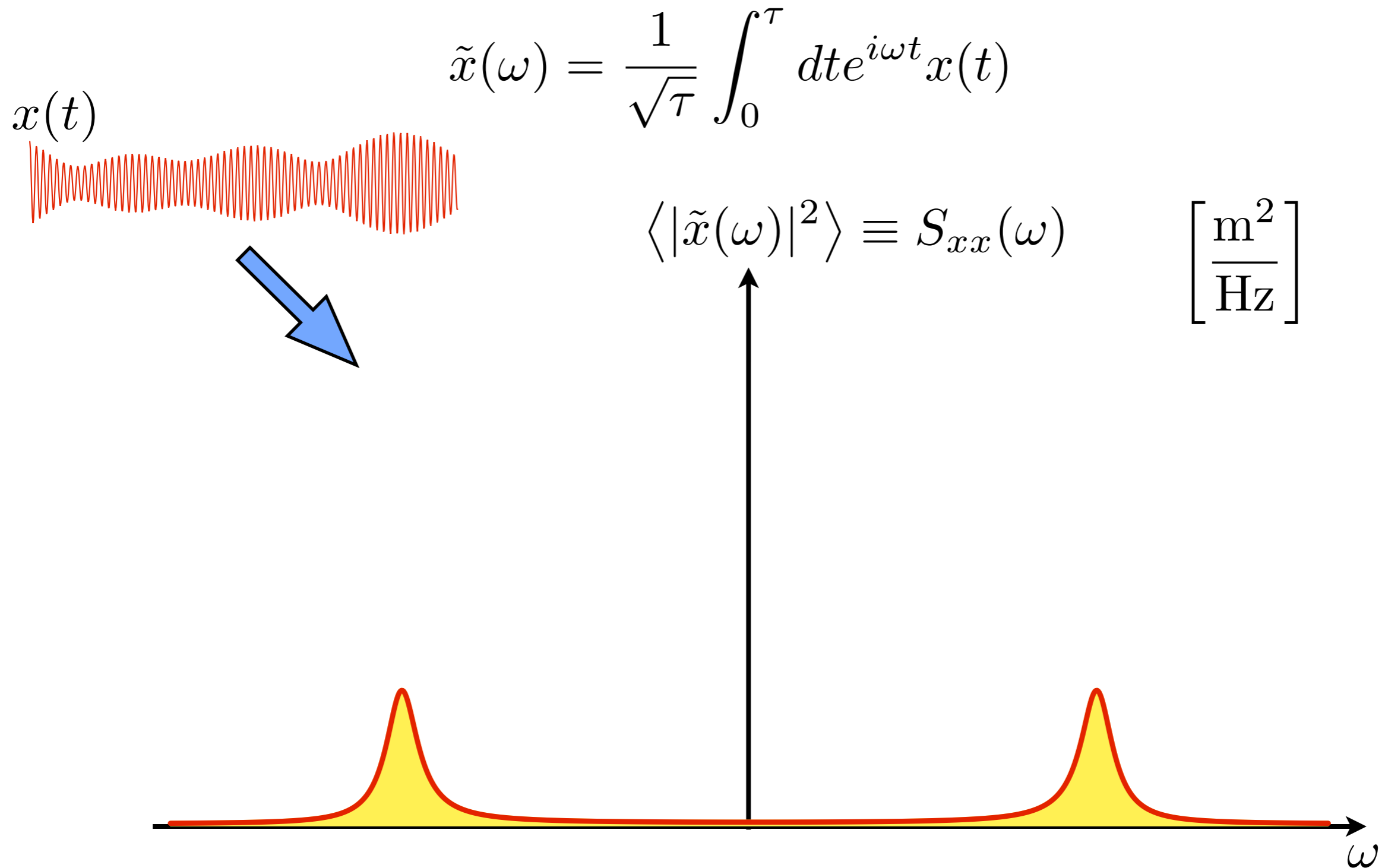
$x(t)$

$|\tilde{x}(\omega)|^2$

$\left[ \frac{\text{m}^2}{\text{Hz}} \right]$



# Fluctuation spectrum



# Fluctuation-dissipation theorem

## General relation between noise spectrum and linear response susceptibility

$$\langle \delta x \rangle (\omega) = \chi_{xx}(\omega) F(\omega)$$

**susceptibility**

$$S_{xx}(\omega) = \frac{2k_B T}{\omega} \text{Im} \chi_{xx}(\omega) \quad (\text{classical limit})$$



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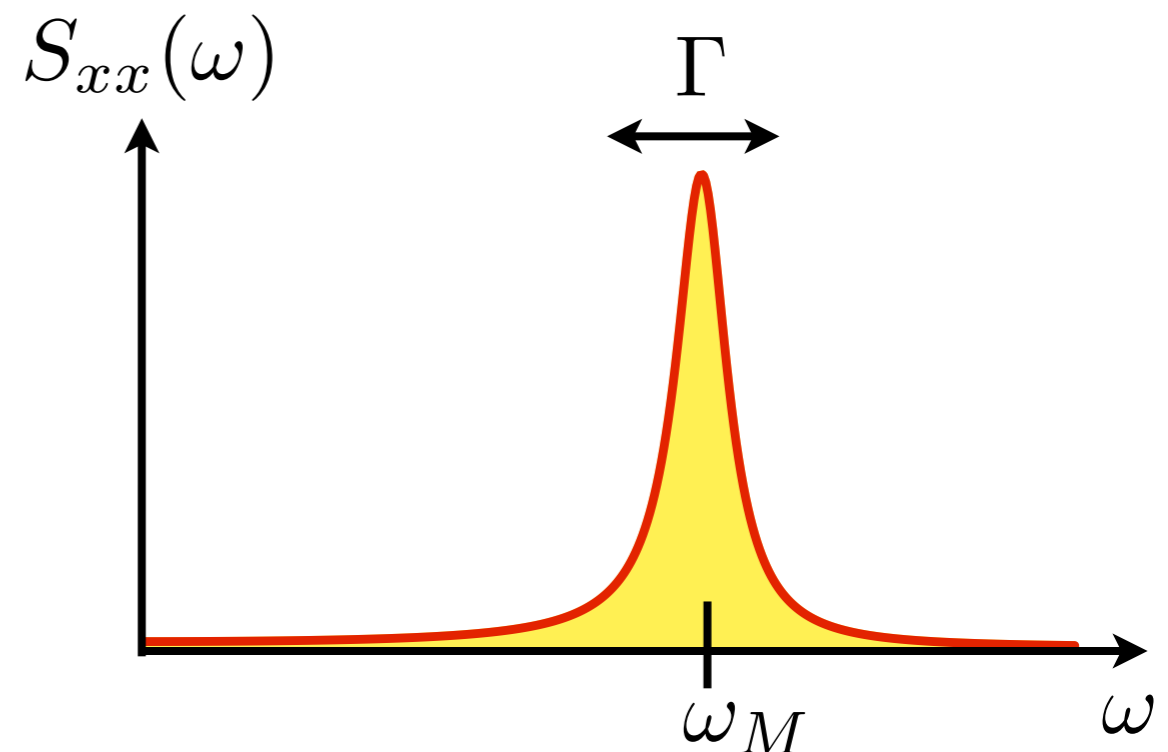
**susceptibility**

$$S_{xx}(\omega) = \frac{2k_B T}{\omega} \text{Im} \chi_{xx}(\omega) \quad (\text{classical limit})$$

for the damped oscillator:

$$m\ddot{x} + m\omega_M^2 x + m\Gamma\dot{x} = F$$

$$x(\omega) = \frac{1}{\underbrace{m(\omega_M^2 - \omega^2) - im\Gamma\omega}_{\chi_{xx}(\omega)}} F(\omega)$$



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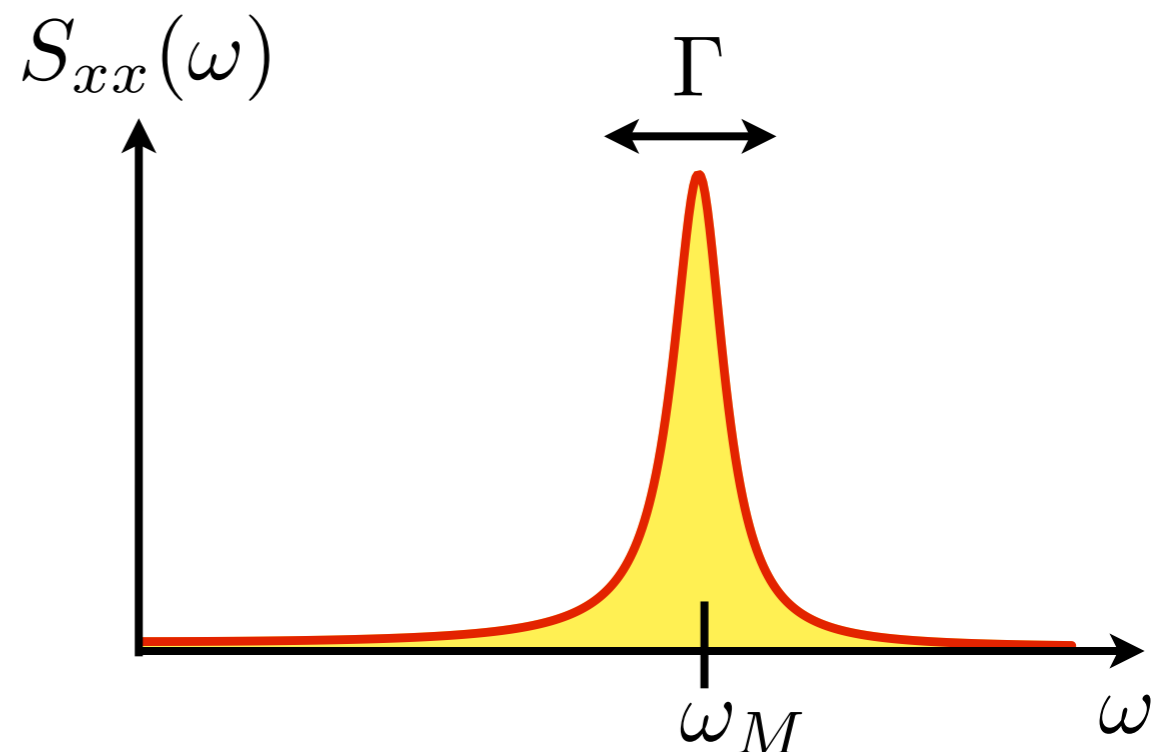
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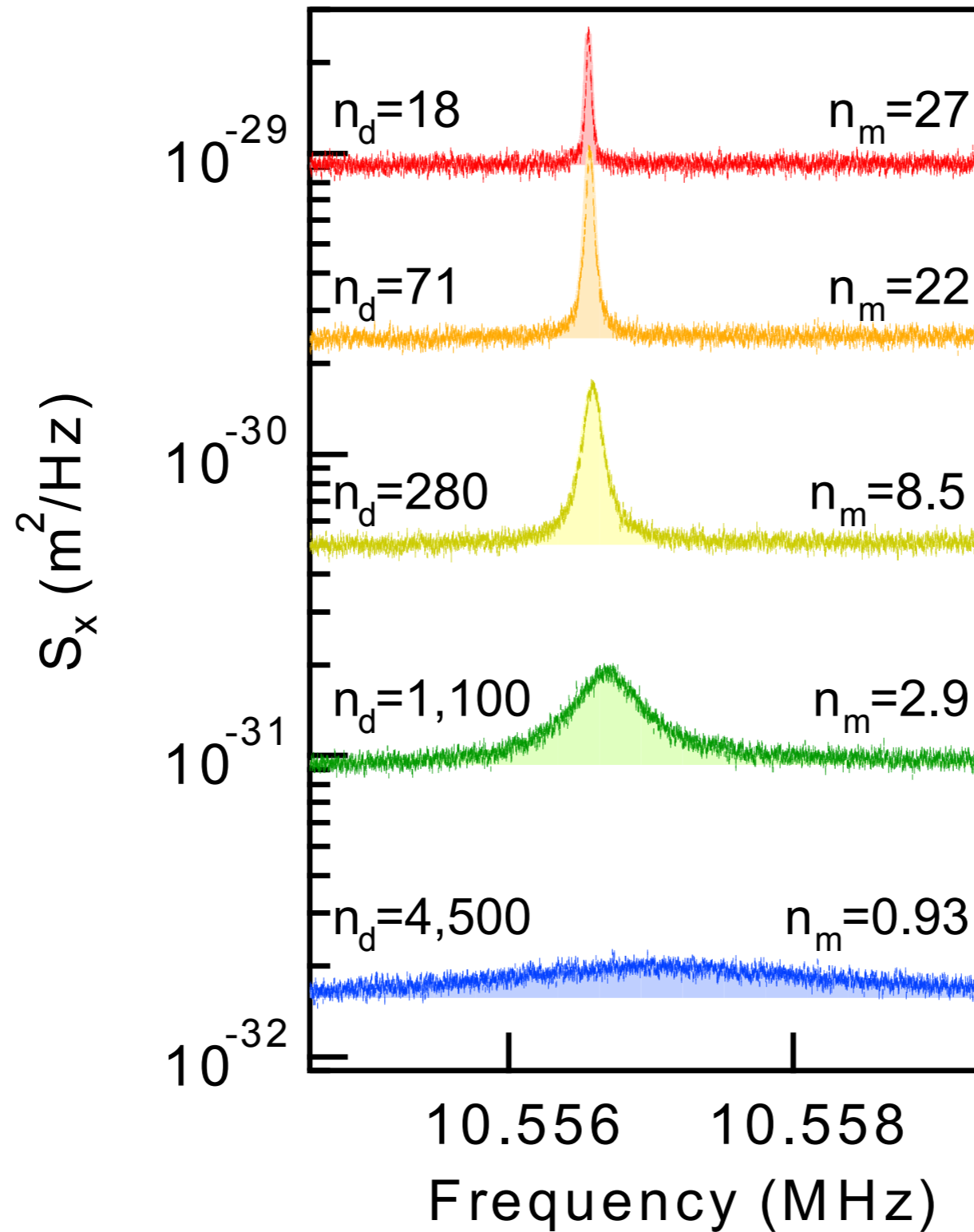


**area yields  
variance of x:**

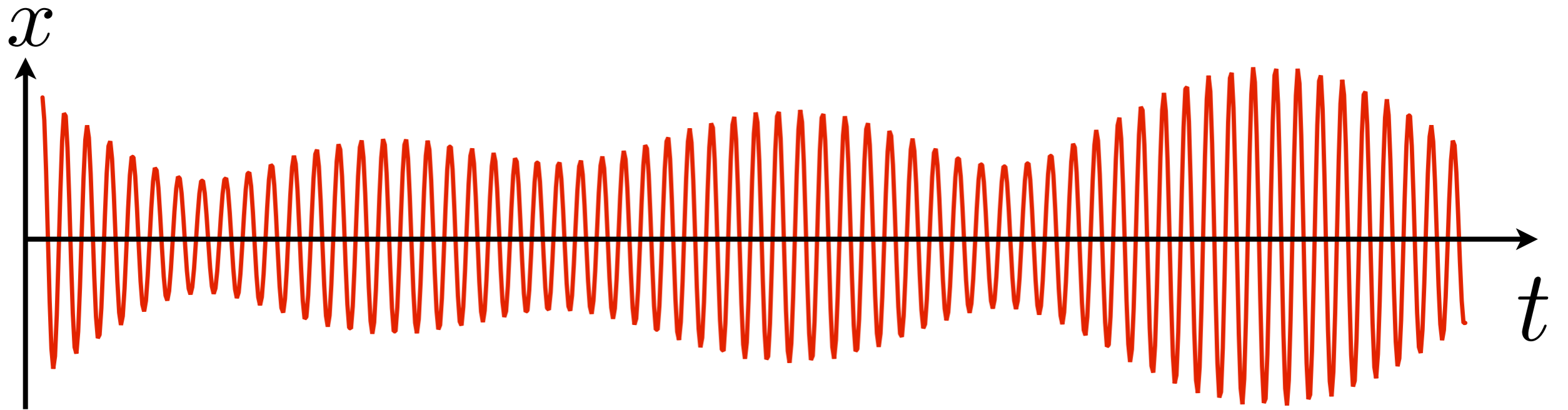
$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

**...yields  
temperature!**

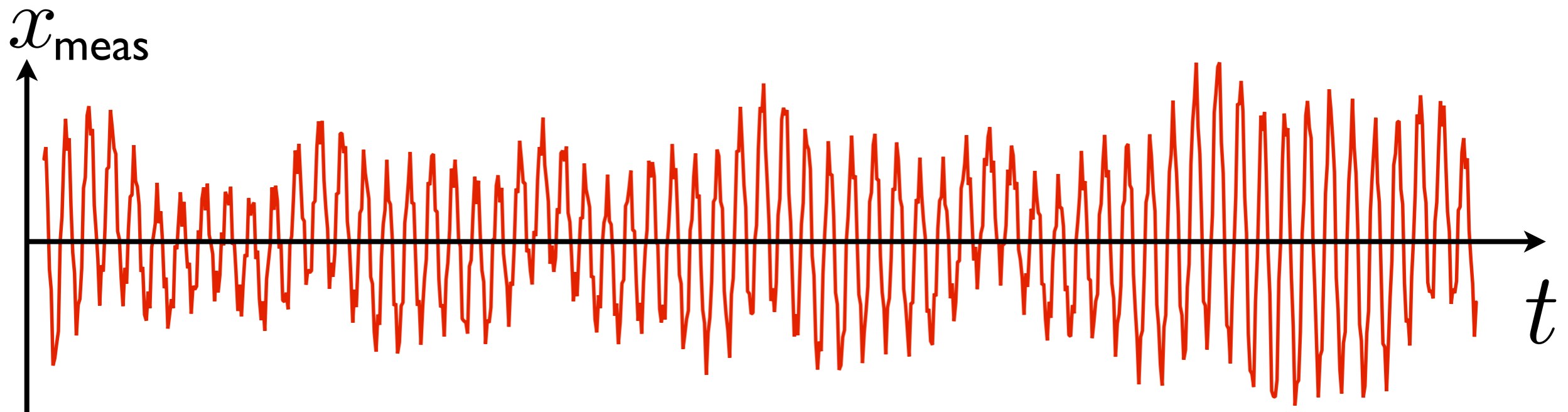
# Displacement spectrum



# Measurement noise



# Measurement noise



$$x_{\text{meas}}(t) = x(t) + x_{\text{noise}}(t)$$

Two contributions to  $x_{\text{noise}}(t)$

1. measurement imprecision

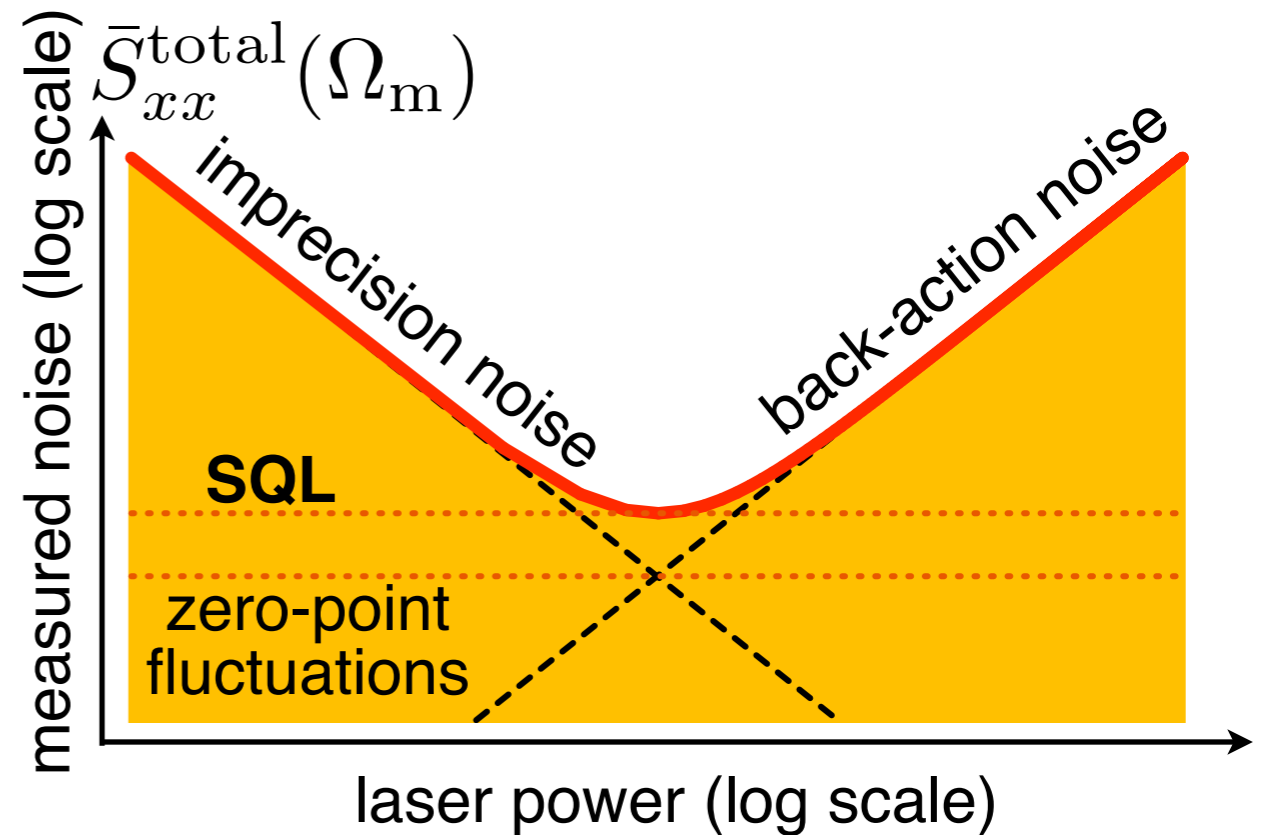
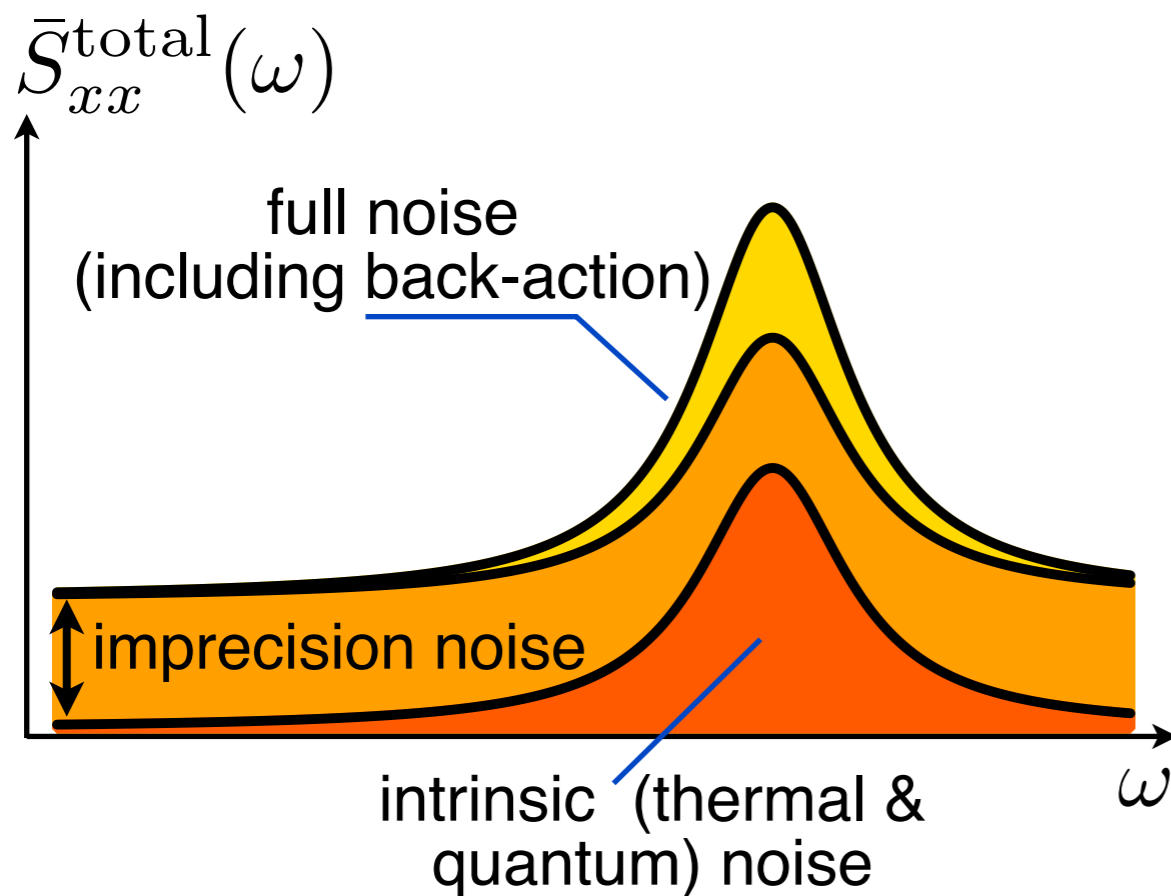
2. measurement back-action:

fluctuating force on system

phase noise of  
laser beam (shot  
noise limit!)

noisy radiation  
pressure force

# “Standard Quantum Limit”



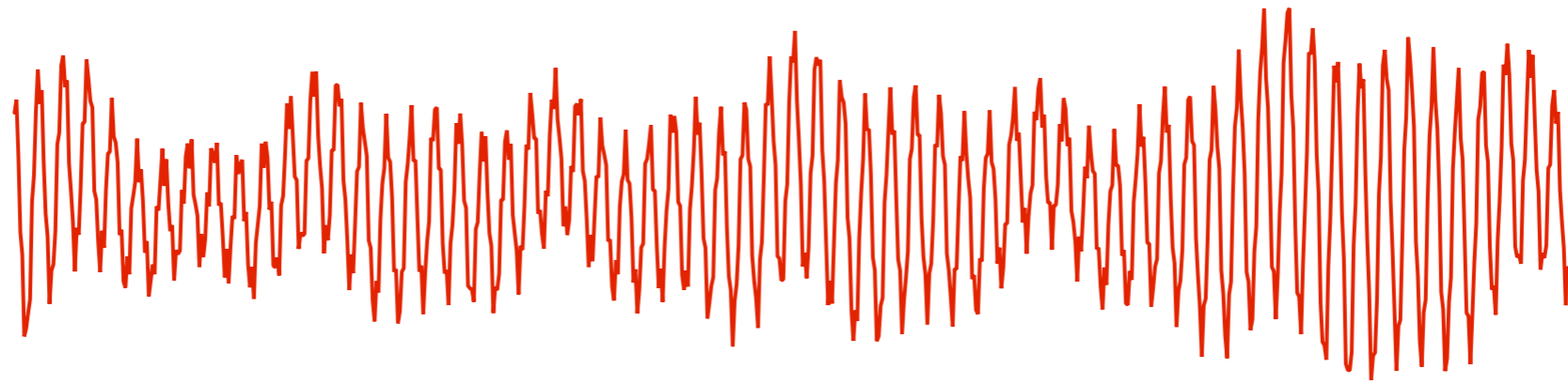
Best case allowed by quantum mechanics:

$$S_{xx}^{(\text{meas})}(\omega) \geq 2 \cdot S_{xx}^{T=0}(\omega)$$

“Standard quantum limit (SQL) of displacement detection”

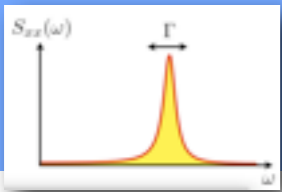
...as if adding the zero-point fluctuations a second time: “adding half a photon”

# Notes on the SQL



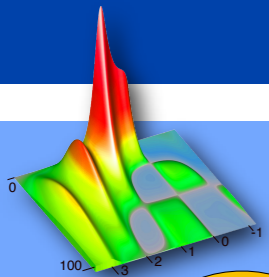
- “**weak measurement**”: integrating the signal over time to suppress the noise
- trying to detect slowly varying “quadratures of motion”:  $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$   
 $[\hat{X}_1, \hat{X}_2] = 2x_{\text{ZPF}}^2$  **Heisenberg is the reason for SQL!**  
**no limit for instantaneous measurement of  $x(t)$ !**
- SQL means: detect  $\hat{X}_{1,2}$  down to  $x_{\text{ZPF}}$  on a time scale  $1/\Gamma$  **Impressive:  $x_{\text{ZPF}} \sim 10^{-15} m$ !**

# Optomechanics (Outline)

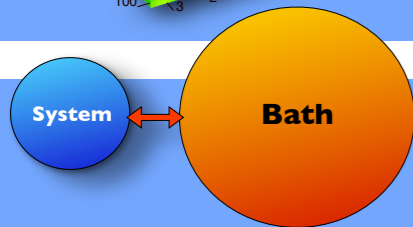


Displacement detection

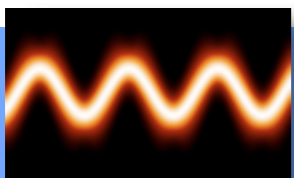
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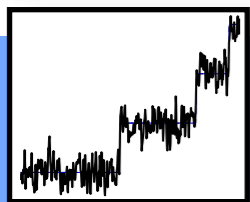
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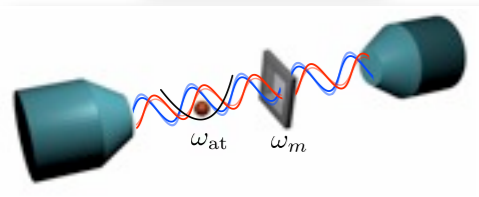
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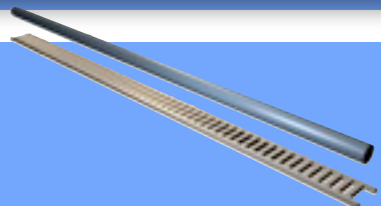
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Towards Fock state detection



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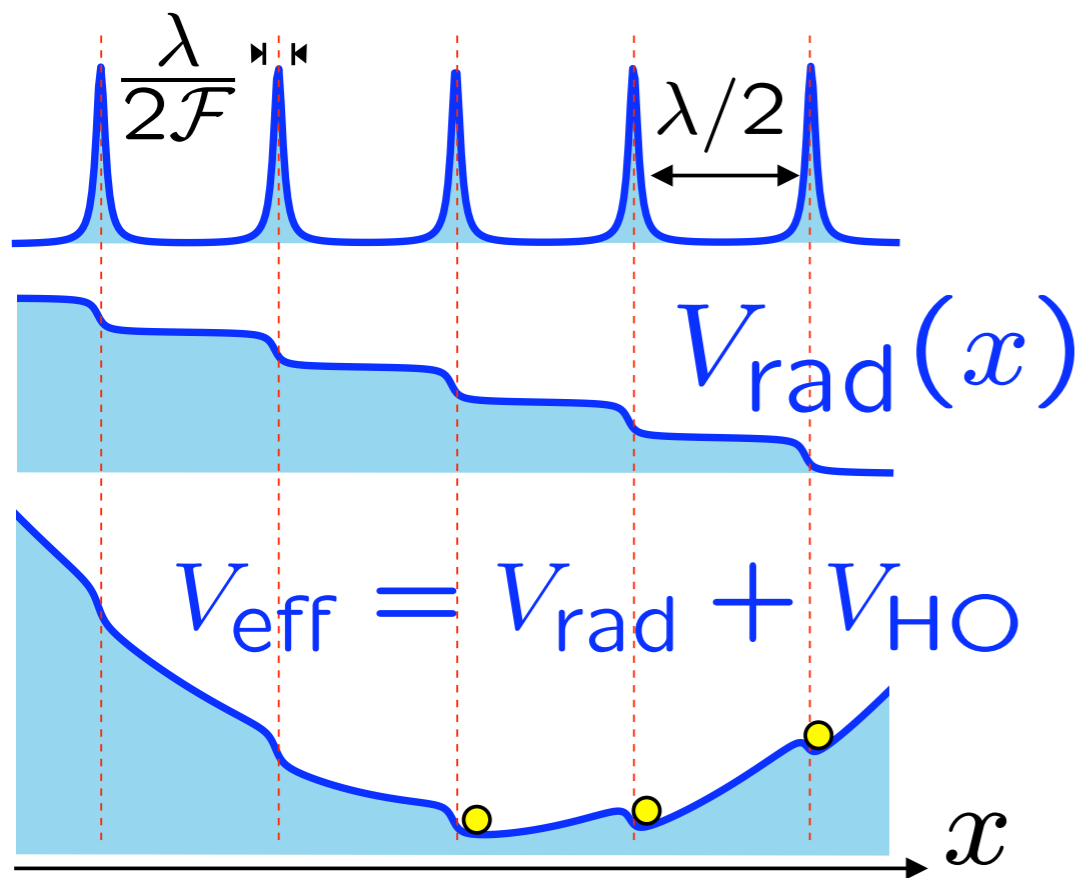


Optomechanical crystals & arrays



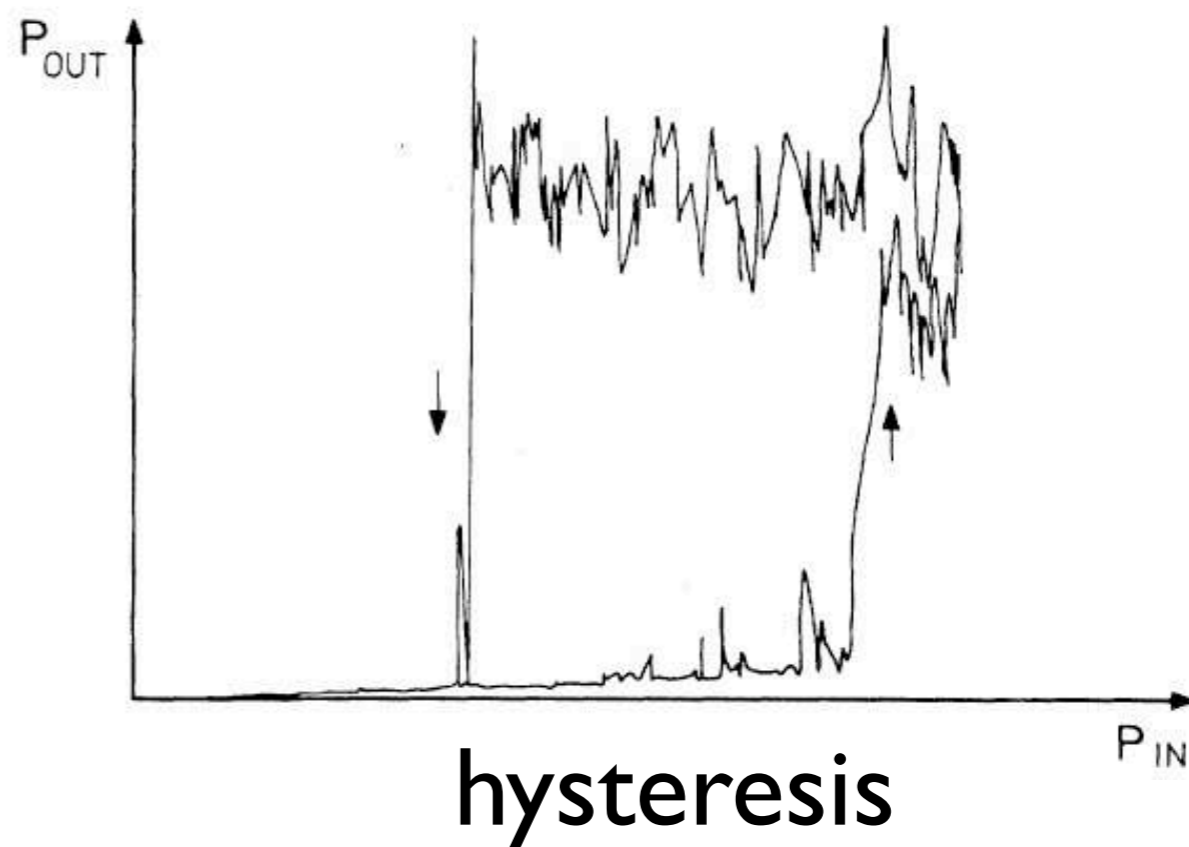
# Basic physics: Statics

$$F_{\text{rad}}(x) = 2I(x)/c$$

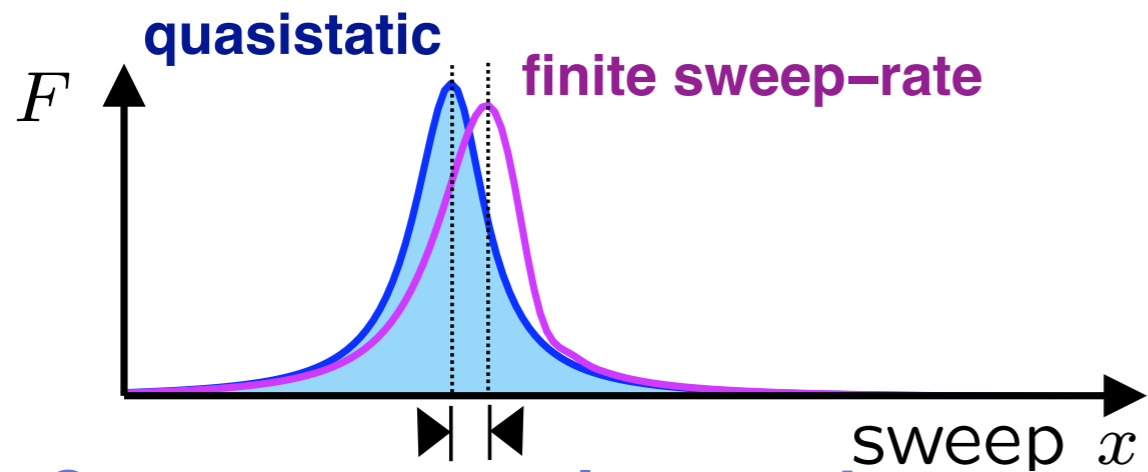


Experimental proof of static bistability:

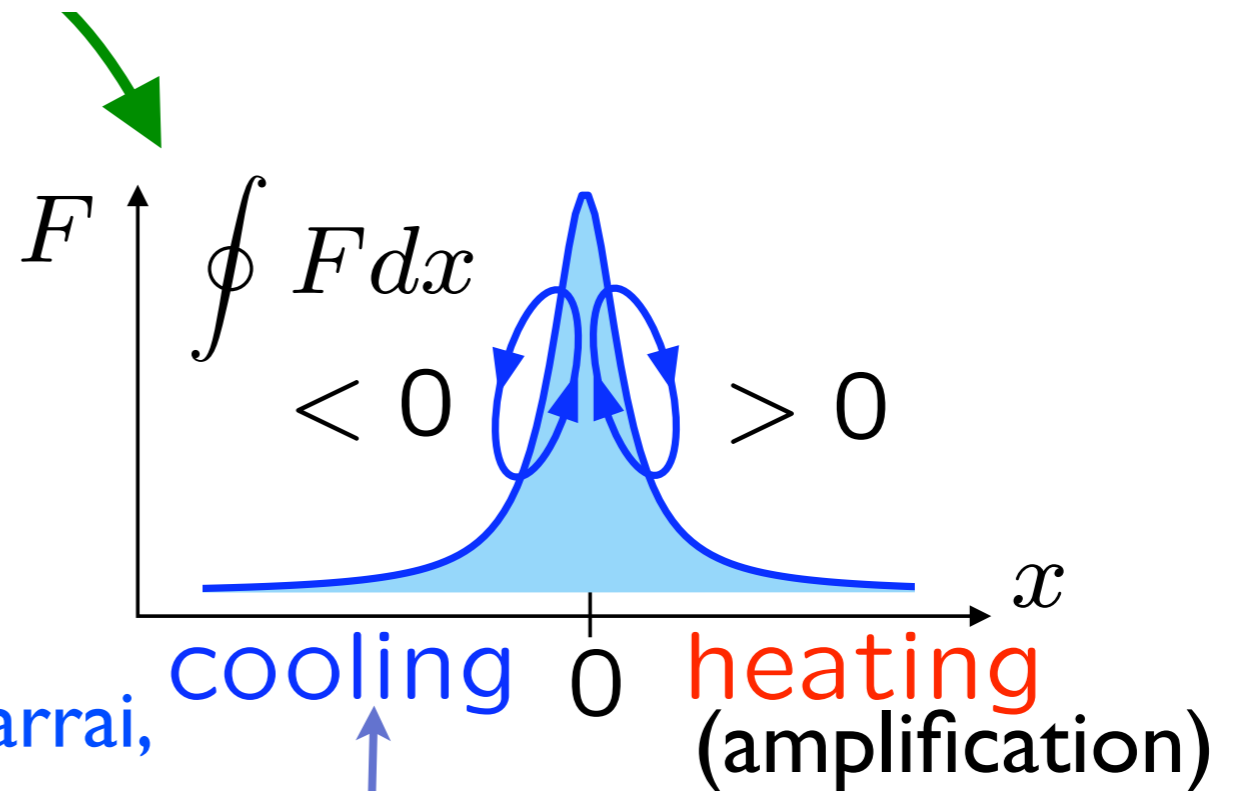
A. Dorsel, J. D. McCullen, P. Meystre,  
E. Vignes and H. Walther:  
Phys. Rev. Lett. 51, 1550 (1983)



# Basic physics: dynamics

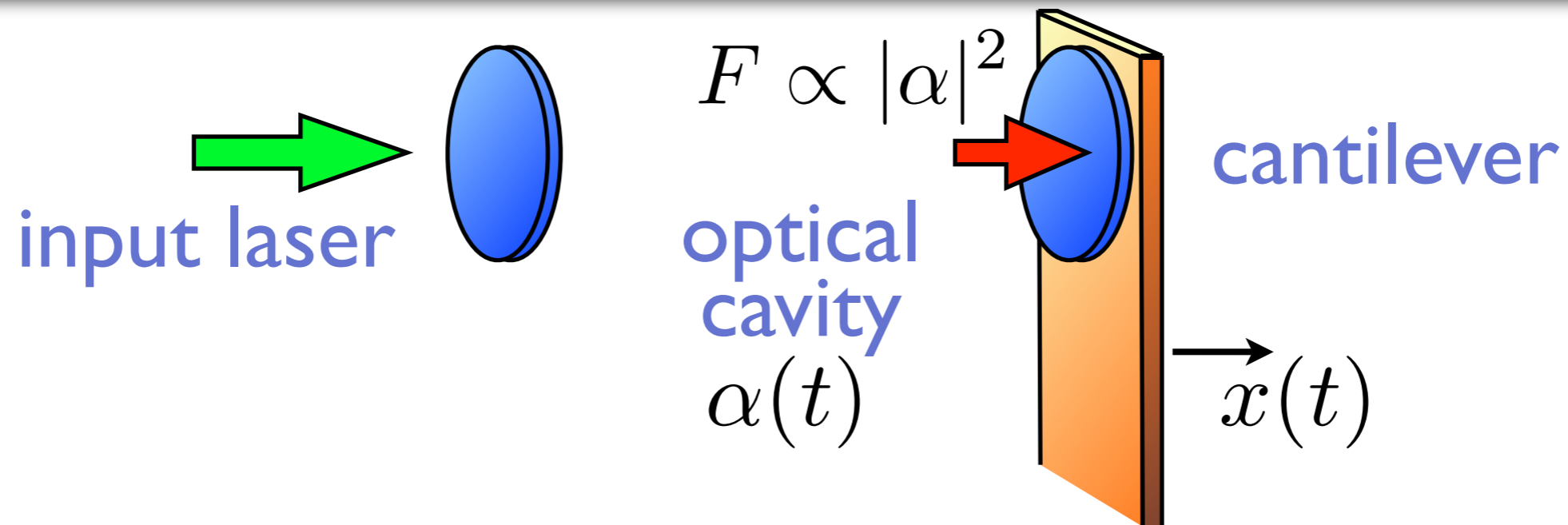


finite optical ringdown time  $\kappa^{-1}$  –  
delayed response to cantilever motion

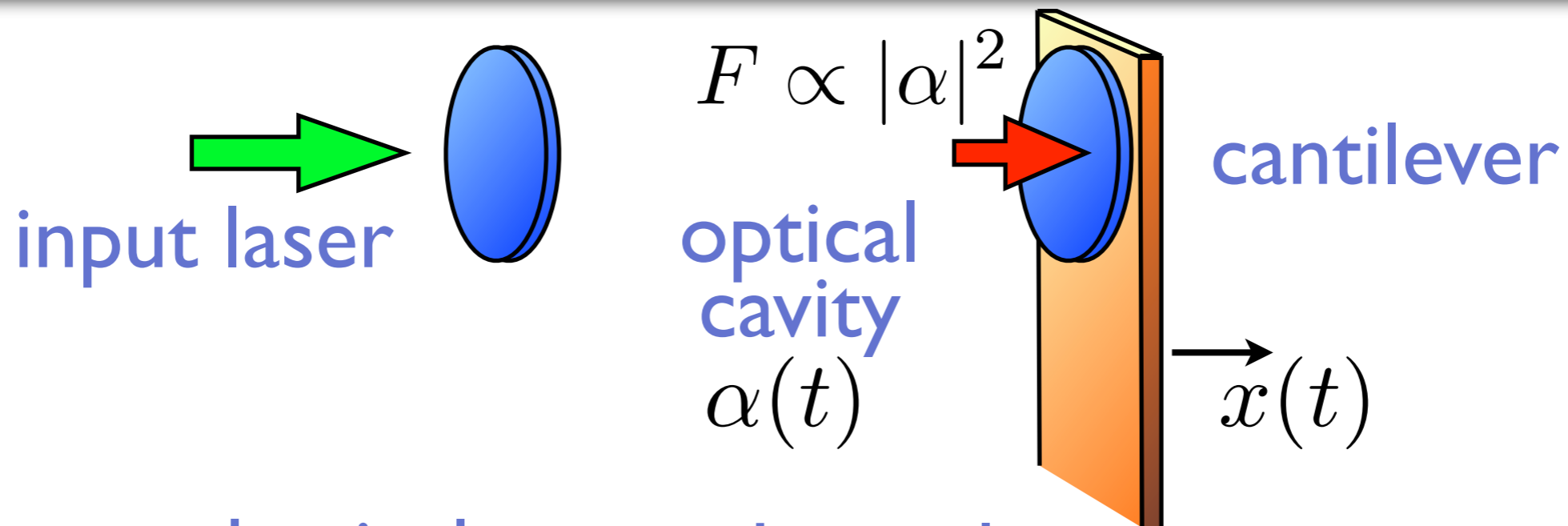


Höhberger-Metzger and Karrai,  
Nature **432**, 1002 (2004):  
300K to 17K [photothermal force]

# Equations of motion



# Equations of motion



$$\ddot{x} = -\omega_M^2 (x - x_0) - \Gamma \dot{x} + F/m$$

mechanical frequency

mechanical damping

radiation pressure

equilibrium position

$$F = \frac{\hbar \omega_R}{L} |\alpha|^2$$

$$\dot{\alpha} = i\omega_R \frac{x}{L} \alpha - \frac{\kappa}{2} (\alpha - \alpha_{\text{in}})$$

detuning from resonance

cavity decay rate

laser amplitude

# Linearized optomechanics

$$\alpha(t) = \bar{\alpha} + \delta\alpha(t)$$

$$x(t) = \bar{x} + \delta x(t)$$

$\Rightarrow \dots \Rightarrow$

(solve for arbitrary  $F_{\text{ext}}(\omega)$ )

$$\delta x(\omega) = \frac{1}{\underbrace{m(\omega_M^2 - \omega^2) - im\omega\Gamma + \Sigma(\omega)}_{\chi_{xx}^{\text{eff}}(\omega)}} F_{\text{ext}}(\omega)$$

$$\delta\omega_M^2 = \frac{1}{m} \text{Re}\Sigma(\omega_M)$$

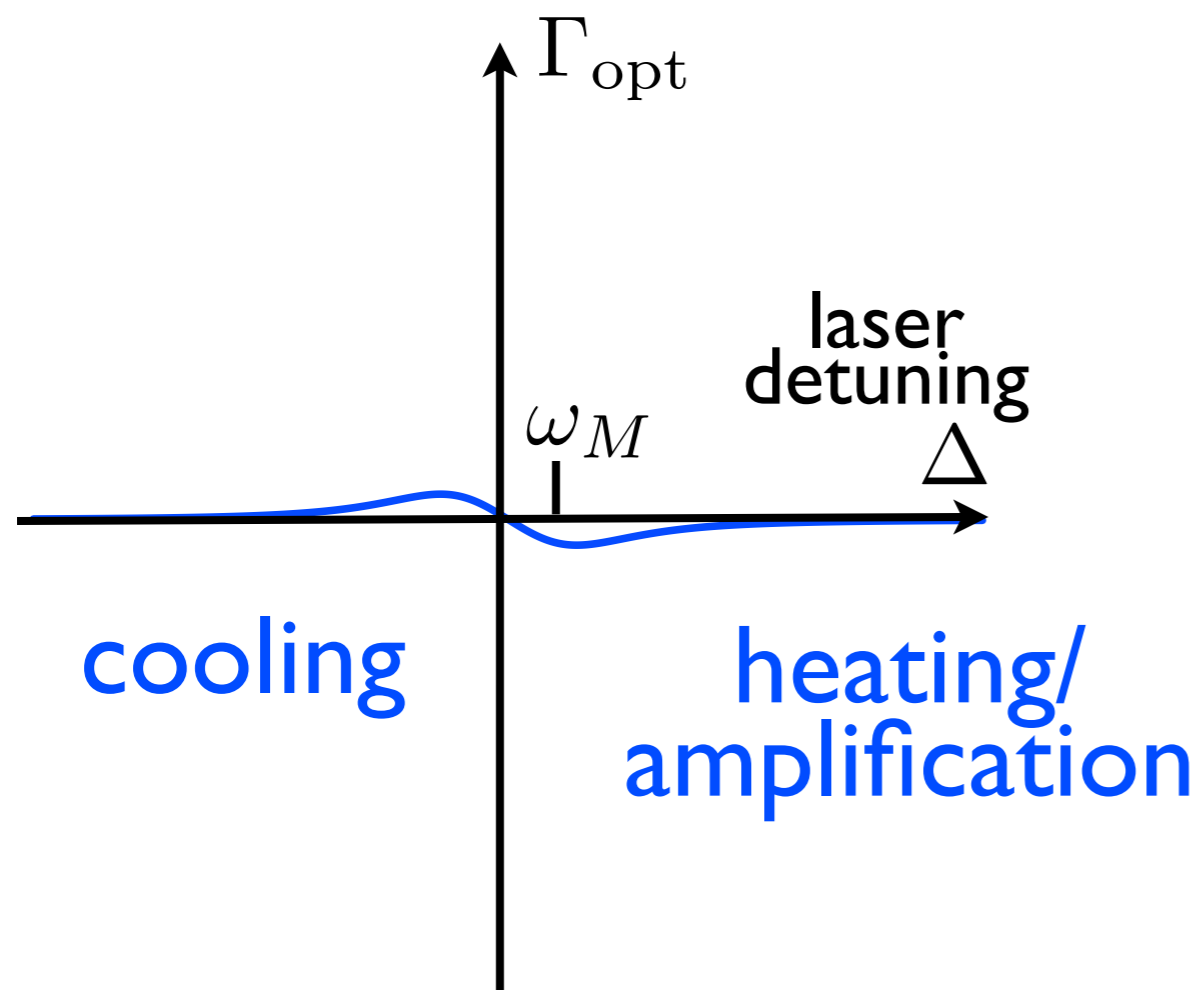
Optomechanical  
frequency shift  
("optical spring")

$$\Gamma_{\text{opt}} = -\frac{1}{m\omega_M} \text{Im}\Sigma(\omega_M)$$

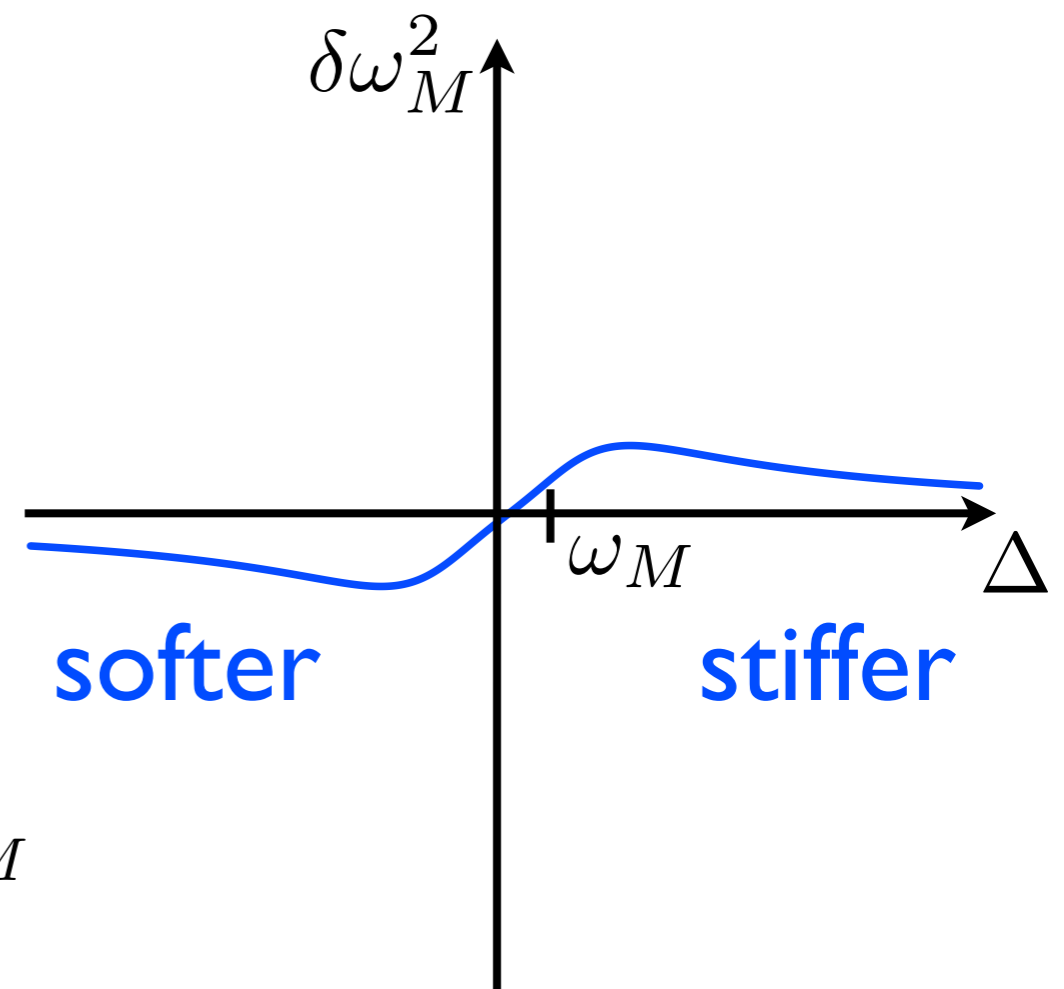
Effective  
optomechanical  
damping rate

# Linearized dynamics

Effective  
optomechanical  
damping rate



Optomechanical  
frequency shift  
("optical spring")

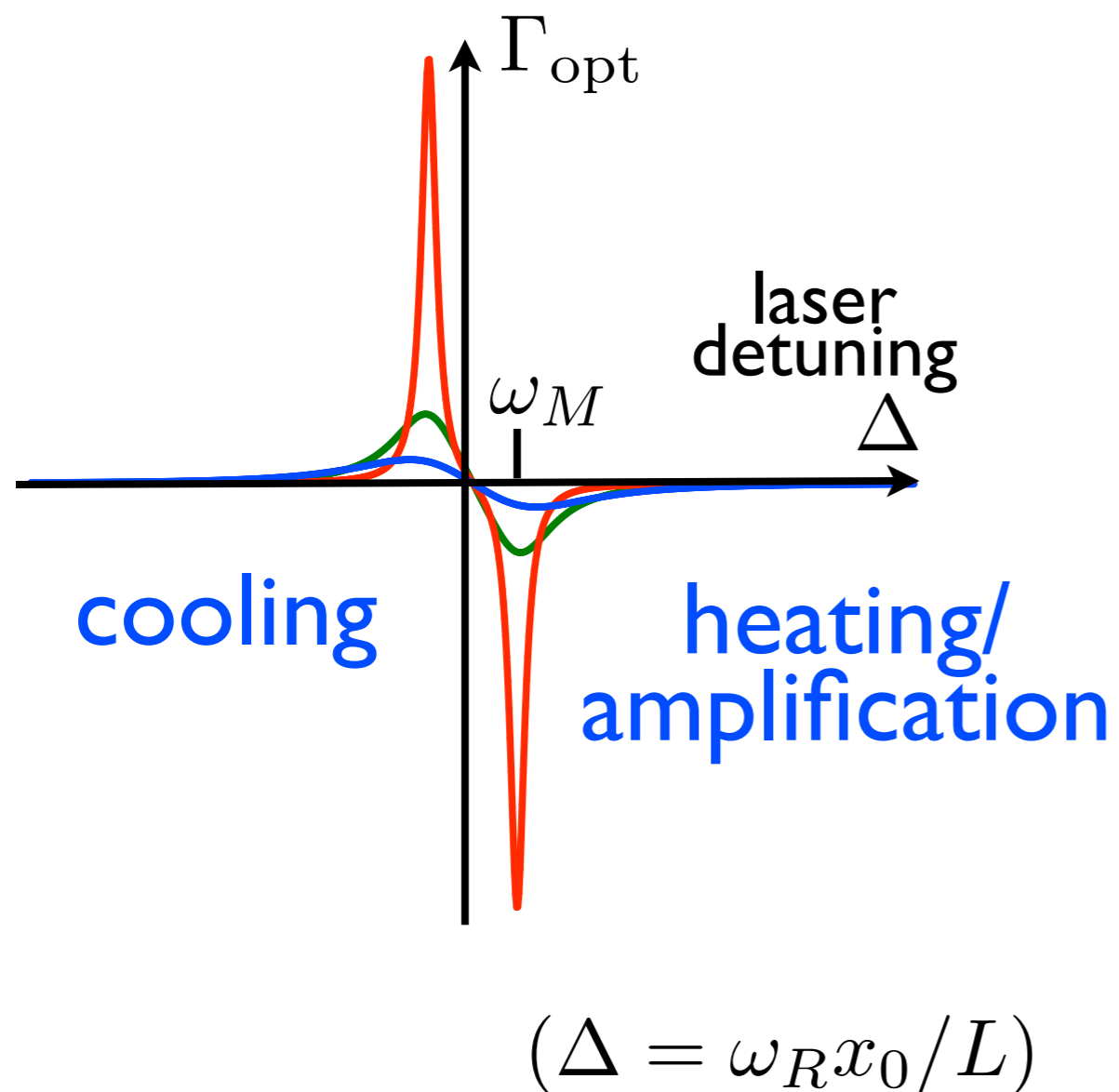


$\kappa/\omega_M$   
■ 2

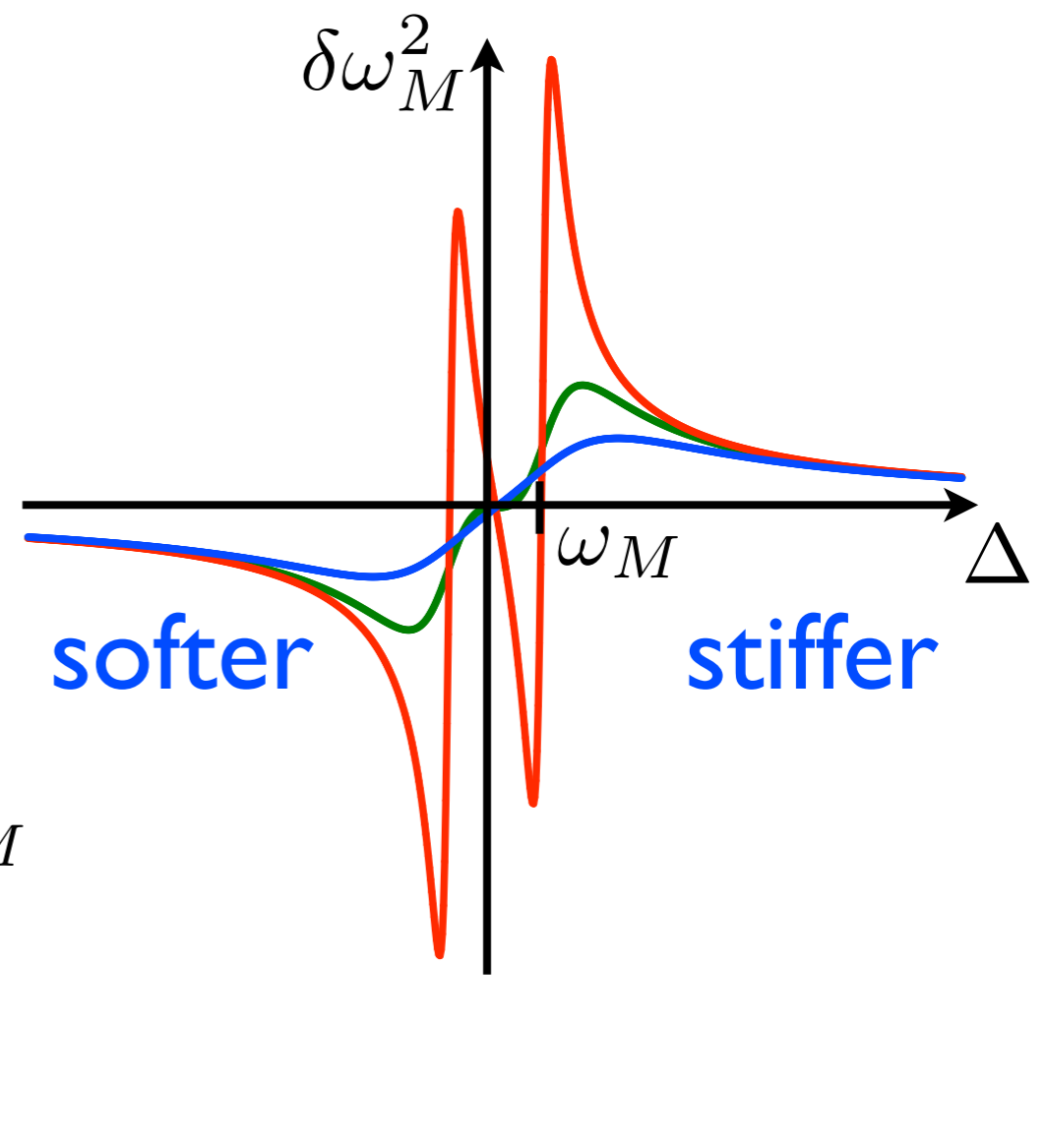
$(\Delta = \omega_R x_0 / L)$

# Linearized dynamics

## Effective optomechanical damping rate

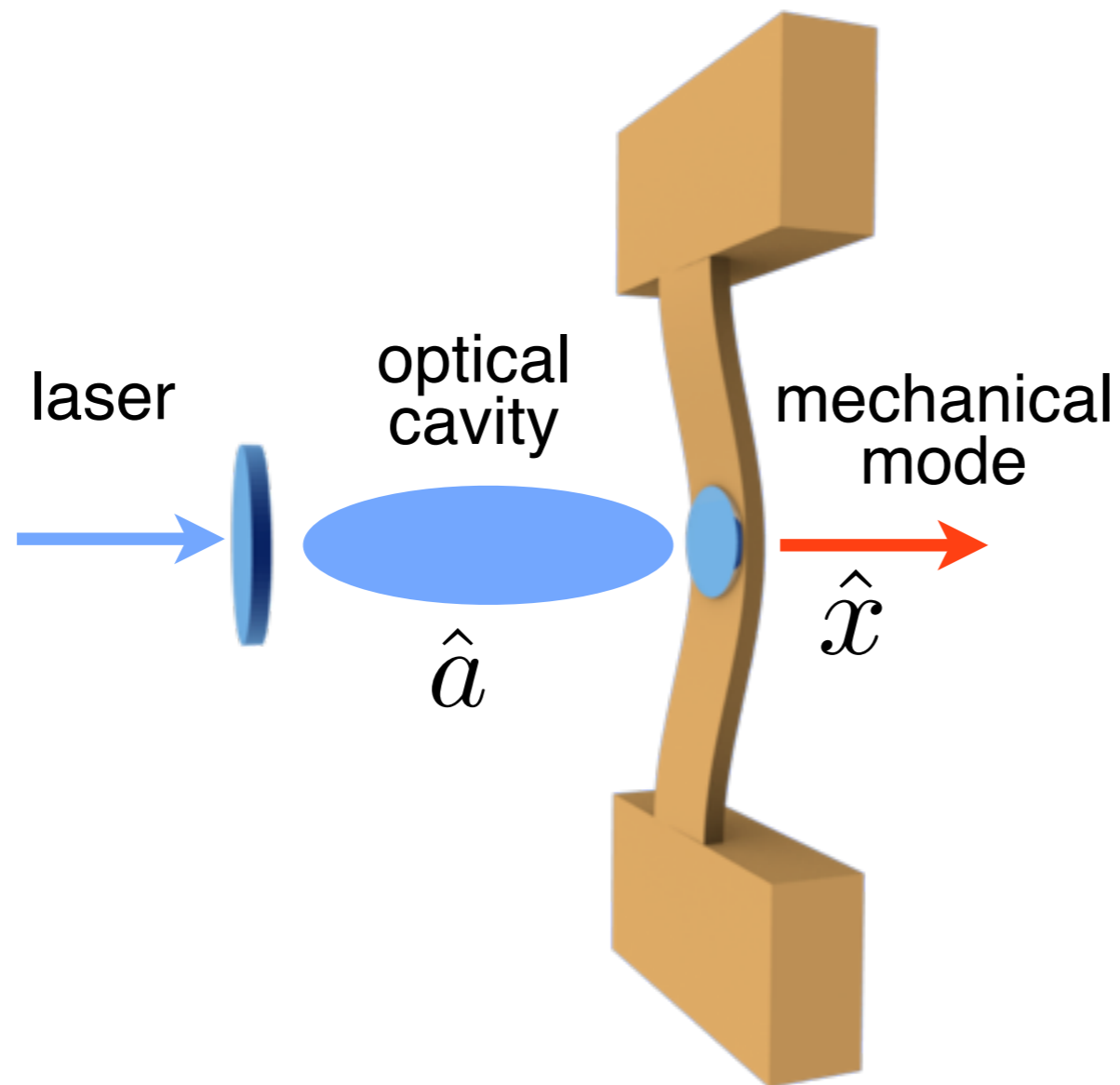


## Optomechanical frequency shift ("optical spring")



- $\kappa/\omega_M$
- 2
  - 1
  - 0.2

# Optomechanical Hamiltonian



$$g_0 \sim \text{Hz} - \text{MHz}$$

$$\hat{H} = -(\Delta + g_0(\hat{b} + \hat{b}^\dagger))\hat{a}^\dagger\hat{a} + \Omega\hat{b}^\dagger\hat{b} + \dots$$

laser detuning  
 $\Delta = \omega_L - \omega_{\text{cav}}$

optomech.  
coupling

$$\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger)$$
$$x_{\text{ZPF}} = \sqrt{\hbar/2m\Omega}$$



# Quantum optomechanics: Linearized Hamiltonian

$$\hat{a} = \alpha + \delta\hat{a}$$

large amplitude  
(laser drive)      quantum fluctuations

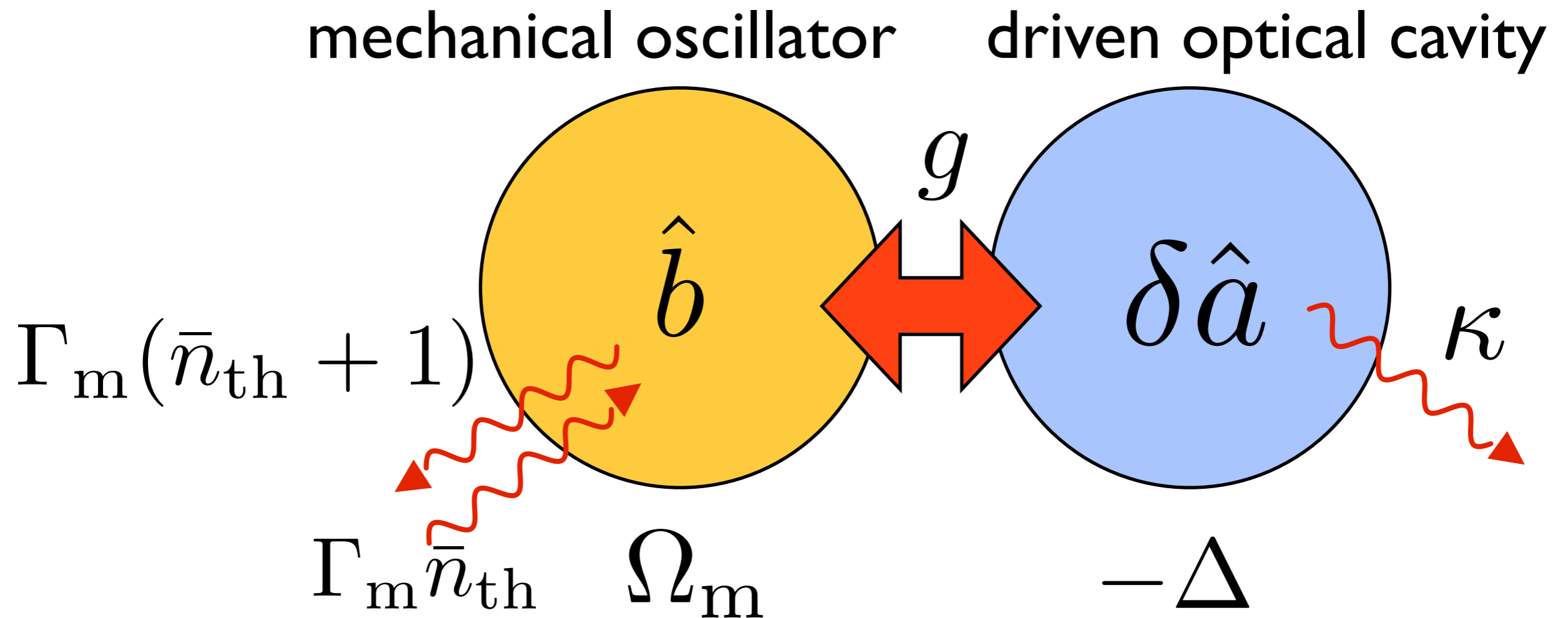
$$g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger) \mapsto \underbrace{g_0 \alpha}_{g} (\delta\hat{a} + \delta\hat{a}^\dagger) (\hat{b} + \hat{b}^\dagger)$$

$g$  bilinear interaction  
tunable coupling!

Sufficient to explain (almost) all current  
optomechanical experiments in the quantum regime

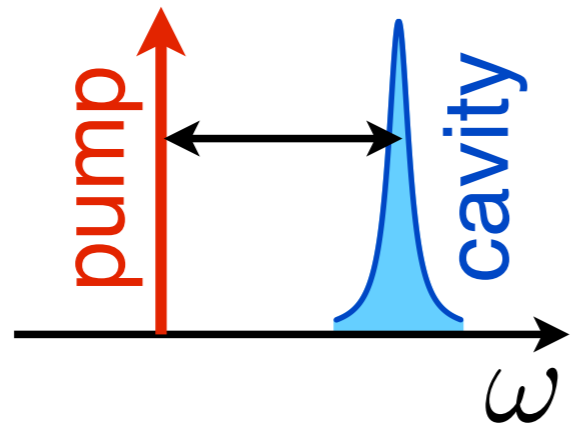
# Mechanics & Optics

After linearization: two linearly coupled harmonic oscillators!



# Different regimes

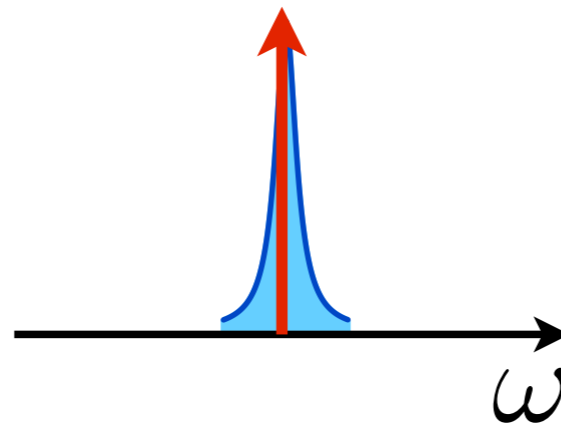
red-detuned



$$\Delta = -\Omega_m$$

beam-splitter  
(cooling)

$$\delta \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \delta \hat{a}$$

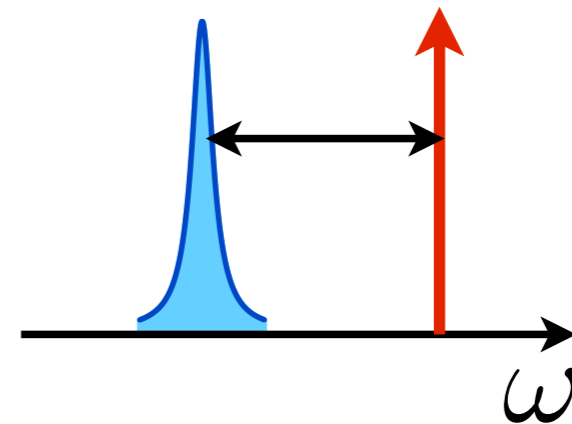


$$\Delta = 0$$

QND

$$\hat{x}_a \hat{x}_b$$

blue-detuned

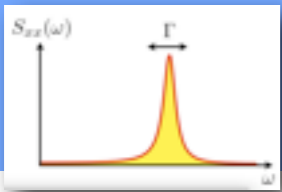


$$\Delta = +\Omega_m$$

squeezer  
(entanglement)

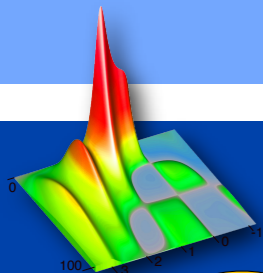
$$\delta \hat{a}^\dagger \hat{b}^\dagger + \hat{b} \delta \hat{a}$$

# Optomechanics (Outline)

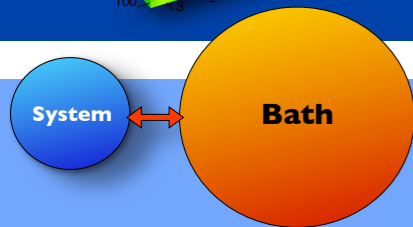


Displacement detection

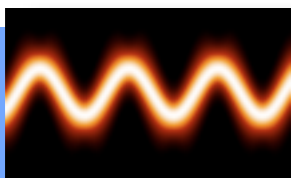
Linear optomechanics



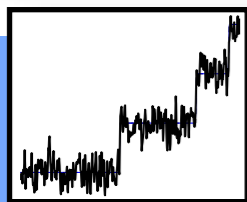
Nonlinear dynamics



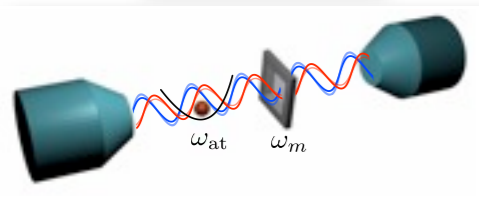
Quantum theory of cooling



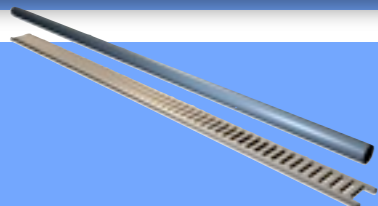
Interesting quantum states



Towards Fock state detection

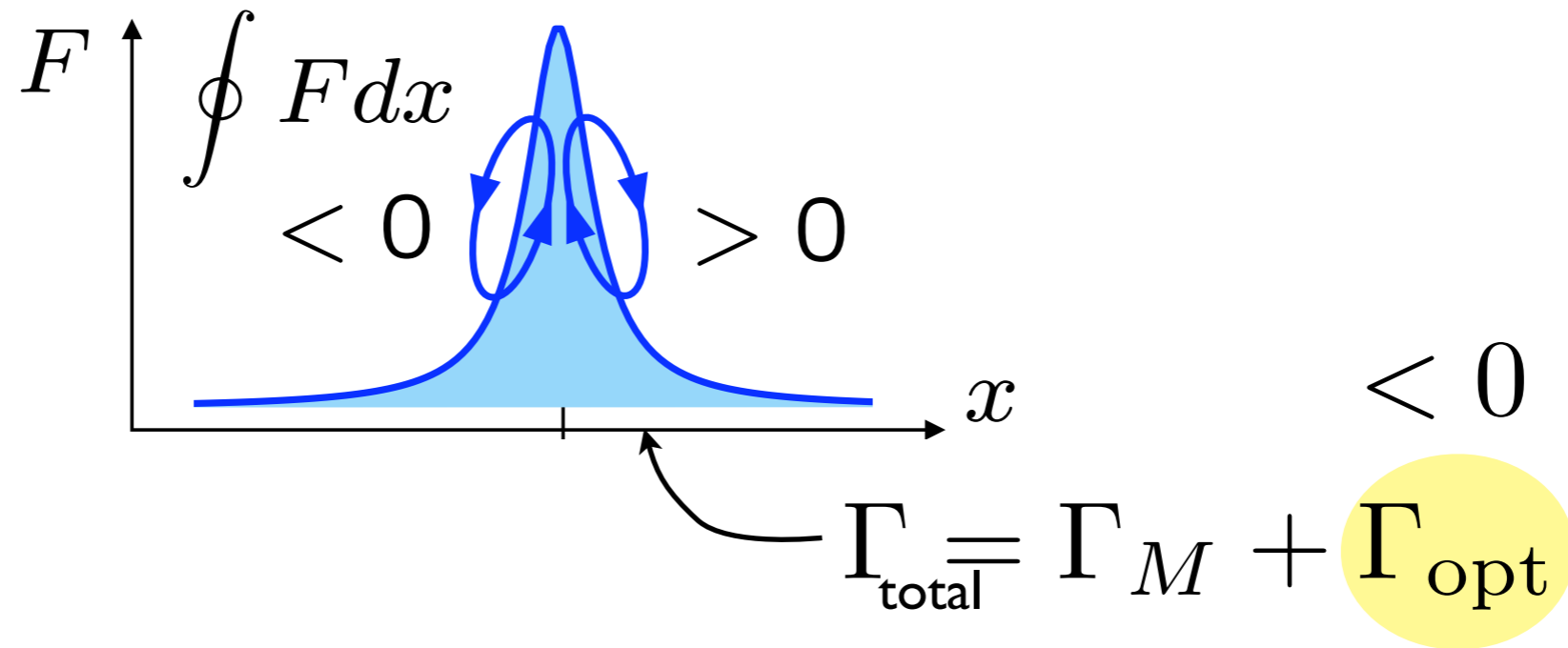


Hybrid systems: coupling to atoms

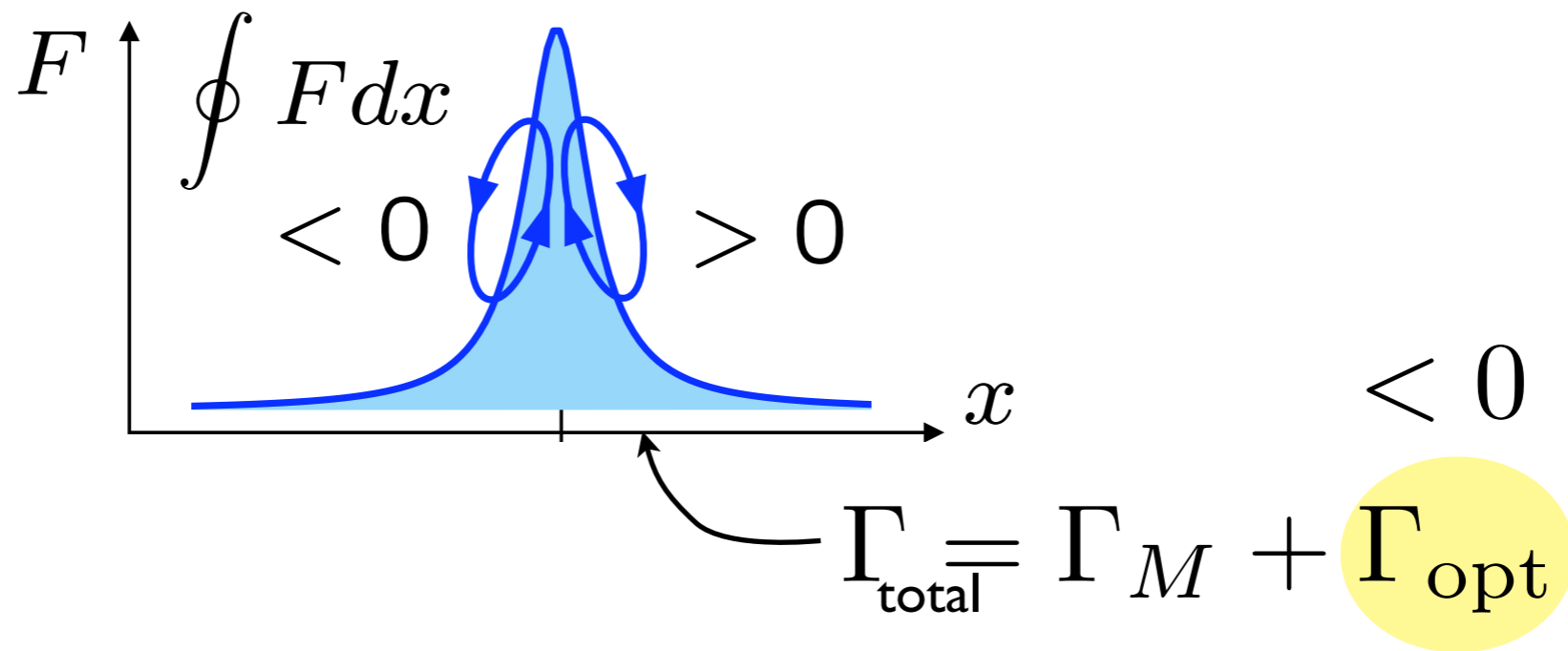


Optomechanical crystals & arrays

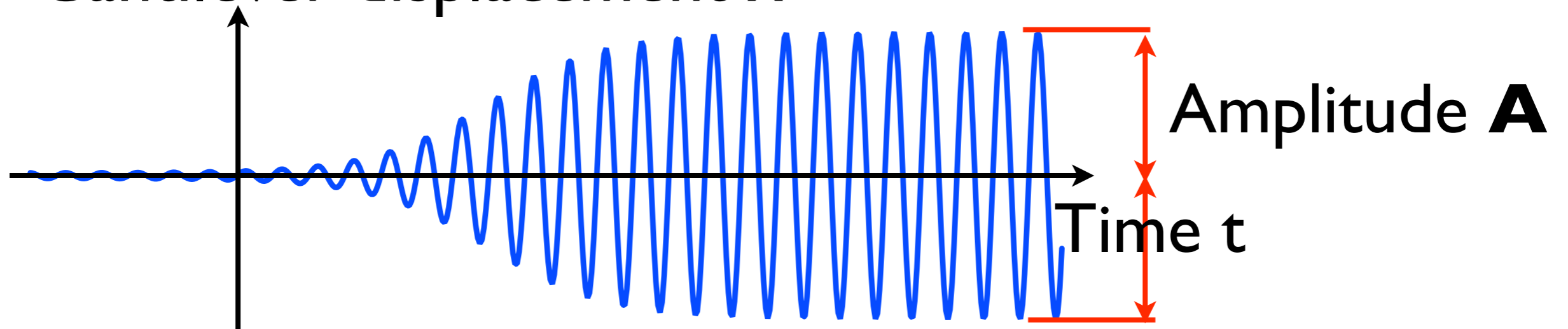
# Self-induced oscillations



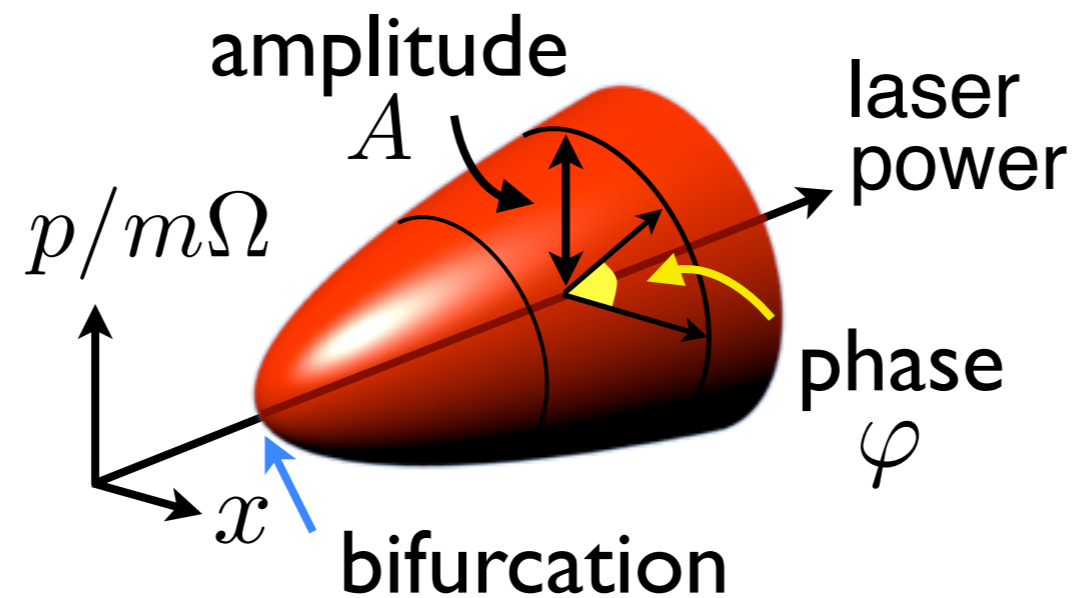
# Self-induced oscillations



Beyond some laser input power threshold: instability  
Cantilever displacement  $x$

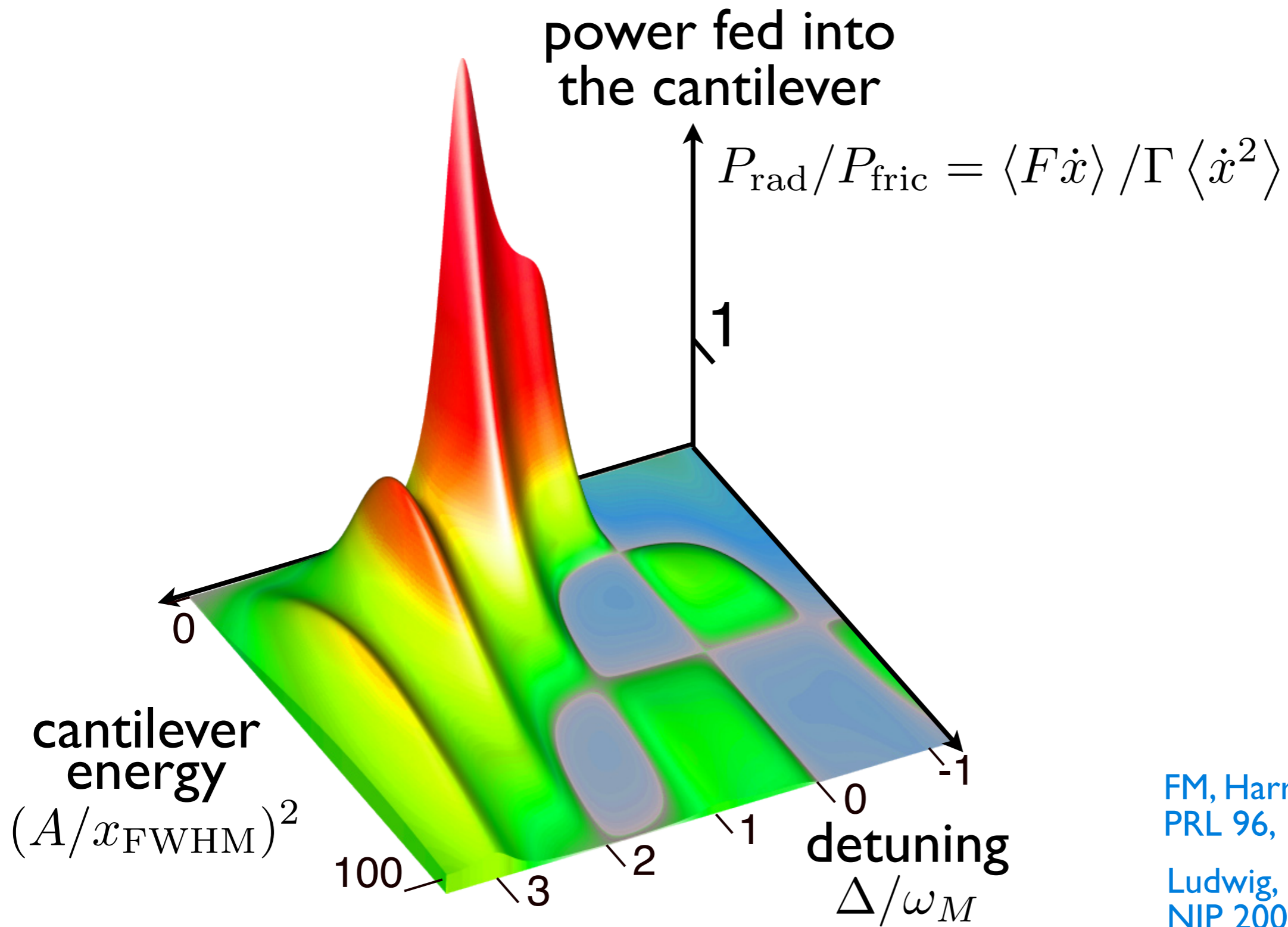


# An optomechanical cell as a Hopf oscillator



Amplitude fixed, phase undetermined!

# Attractor diagram

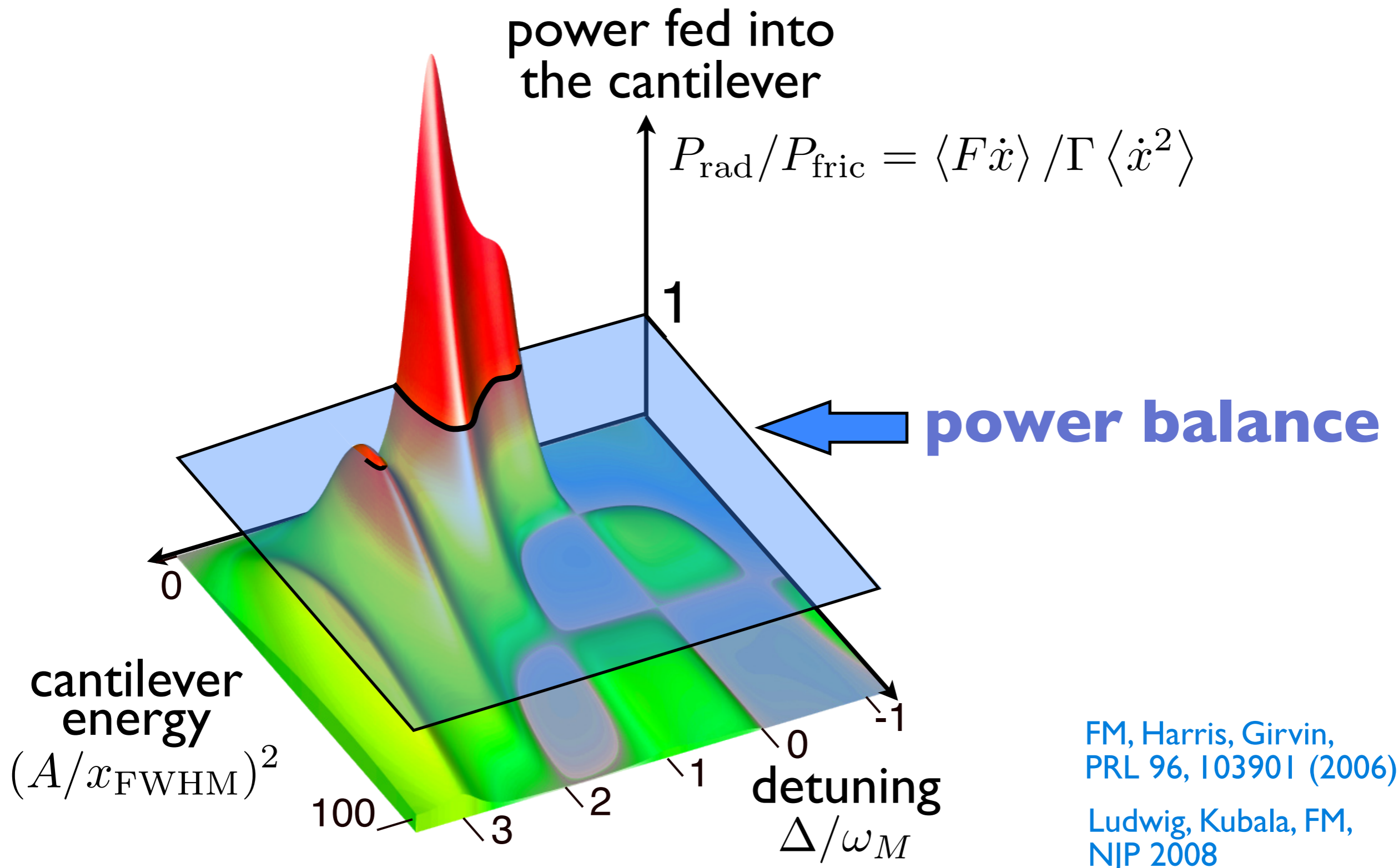


FM, Harris, Girvin,  
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,  
NJP 2008



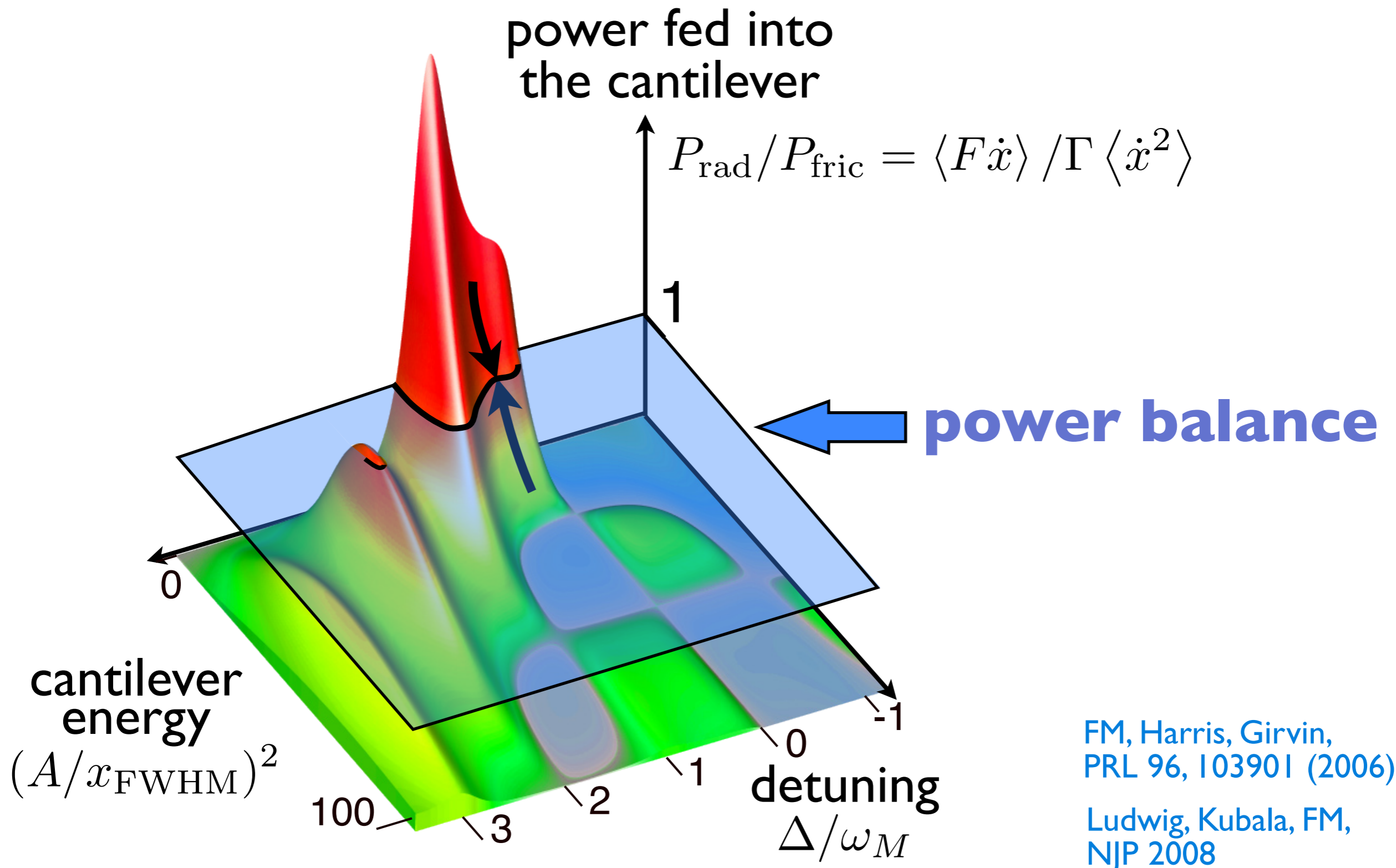
# Attractor diagram



FM, Harris, Girvin,  
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,  
NJP 2008

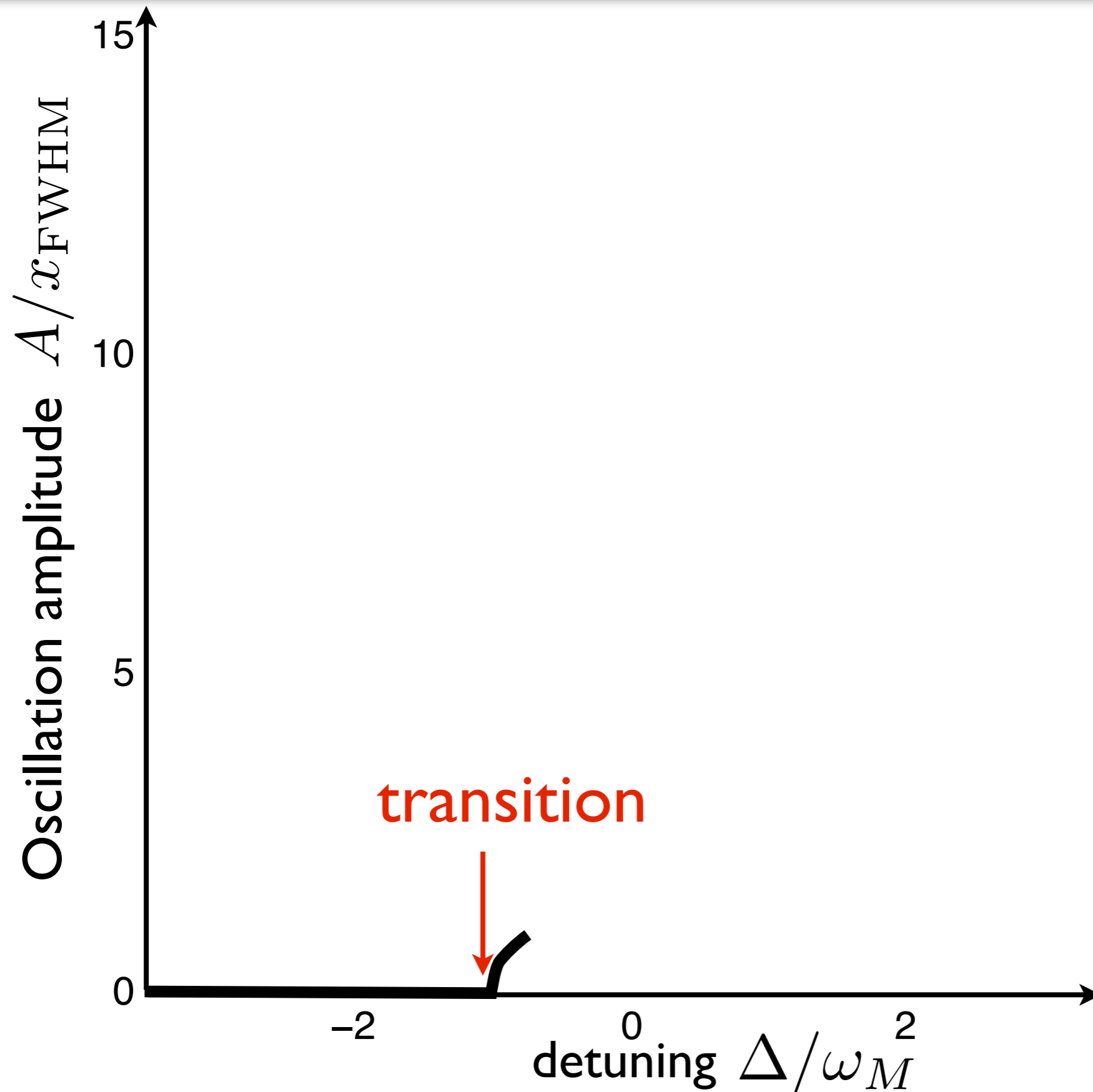
# Attractor diagram



FM, Harris, Girvin,  
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,  
NJP 2008

# Attractor diagram



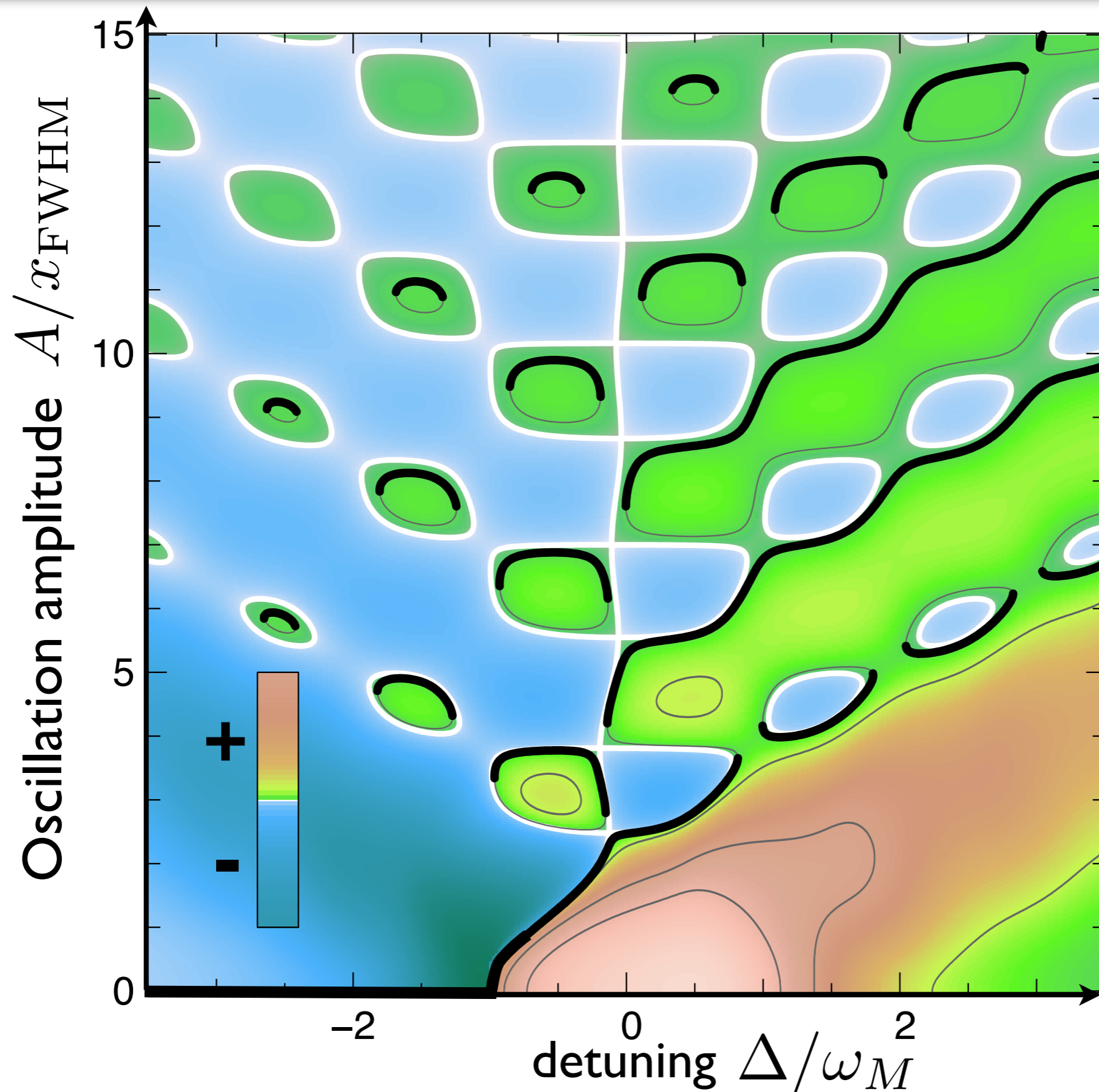
Höhberger, Karrai, IEEE  
proceedings 2004

Carmon, Rokhsari,  
Yang, Kippenberg,  
Vahala, PRL 2005

FM, Harris, Girvin,  
PRL 2006

Metzger et al.,  
PRL 2008

# Attractor diagram



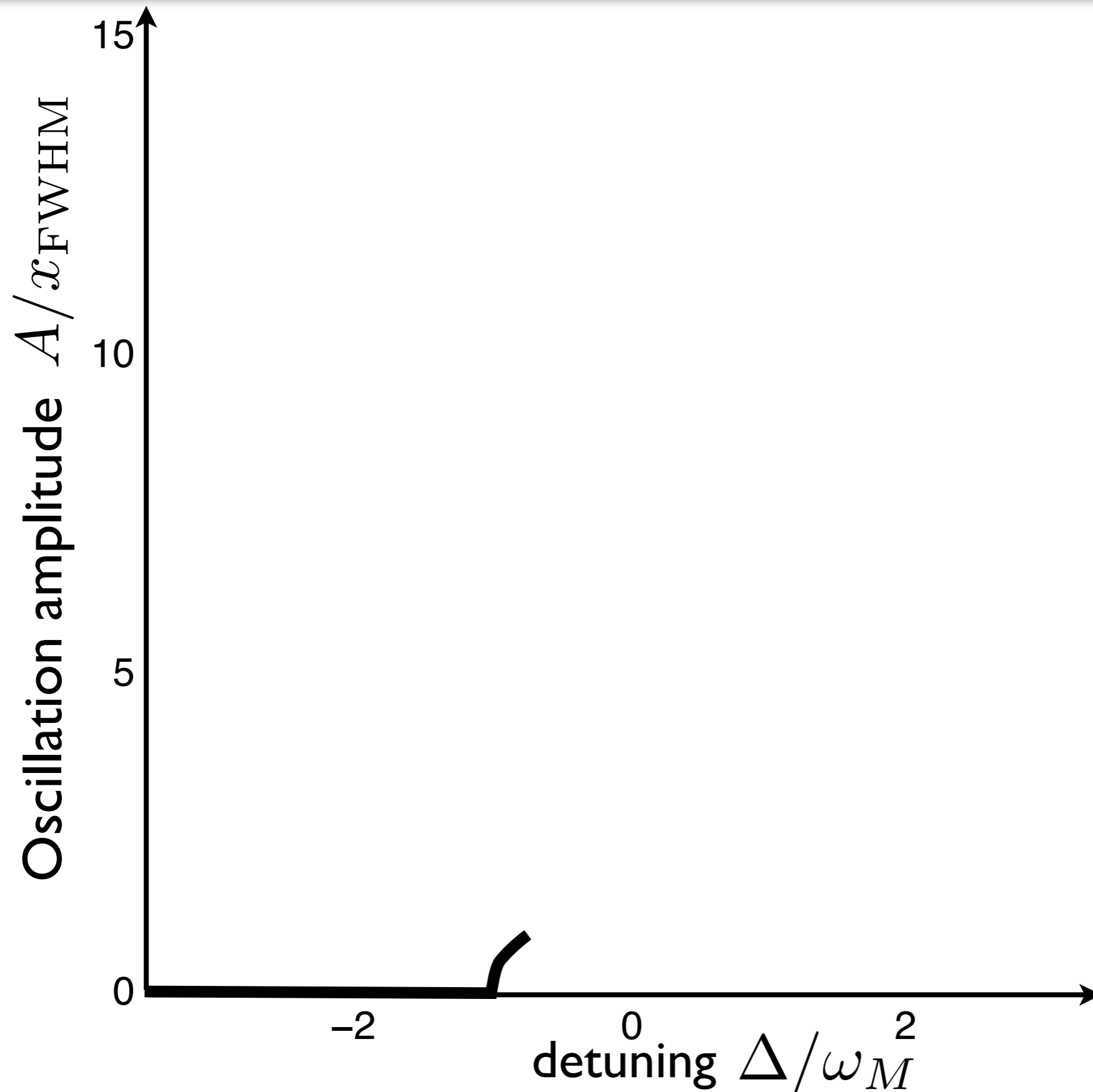
Höhberger, Karrai, IEEE proceedings 2004

Carmon, Rokhsari, Yang, Kippenberg, Vahala, PRL 2005

FM, Harris, Girvin, PRL 2006

Metzger et al., PRL 2008

# Attractor diagram



Höhberger, Karrai, IEEE  
proceedings 2004

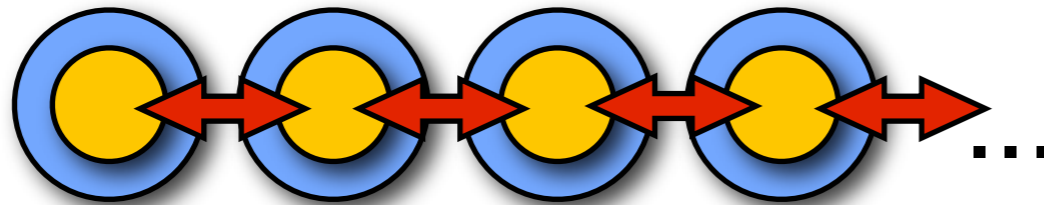
Carmon, Rokhsari,  
Yang, Kippenberg,  
Vahala, PRL 2005

FM, Harris, Girvin,  
PRL 2006

Metzger et al.,  
PRL 2008

# Coupled oscillators?

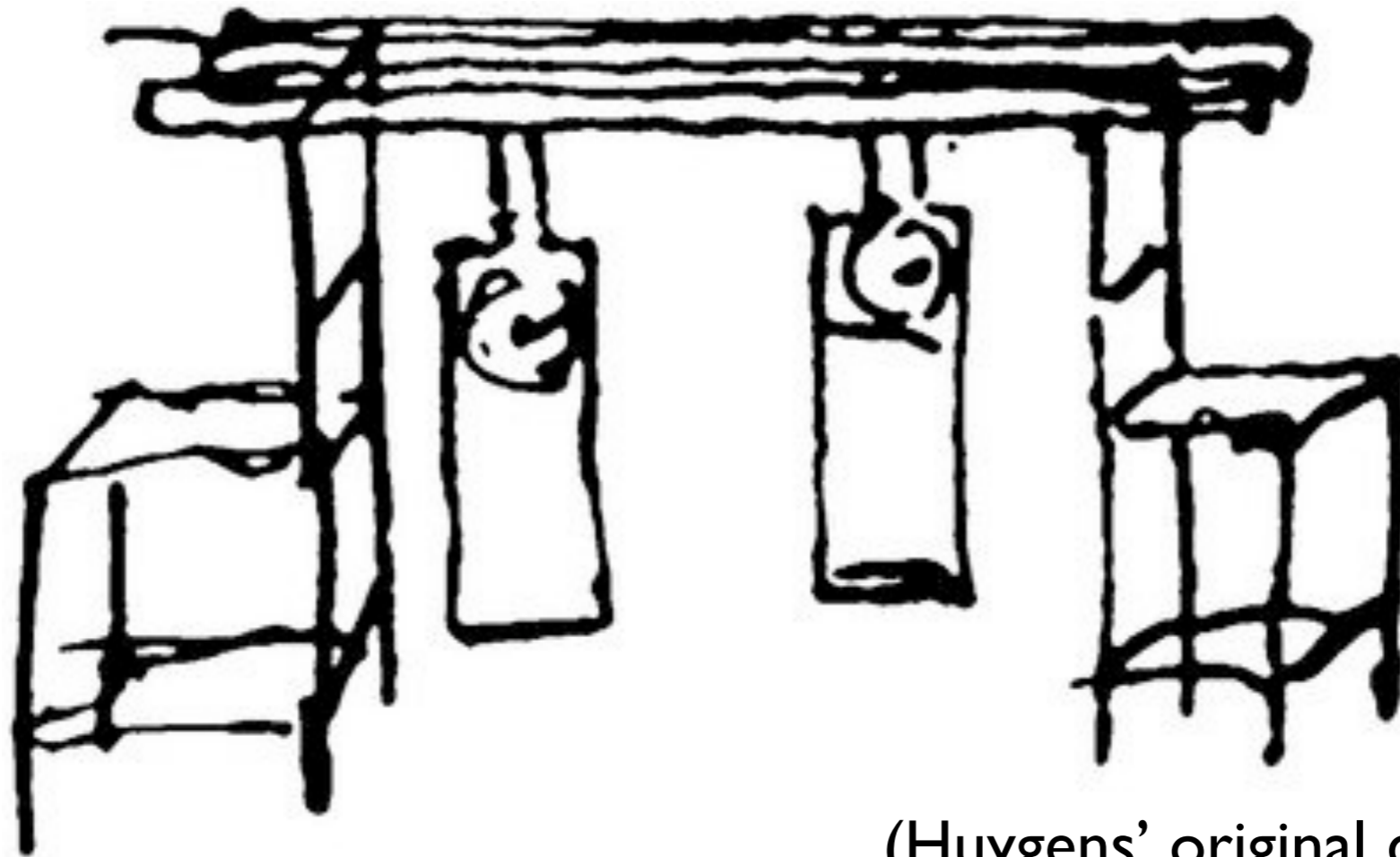
# Coupled oscillators?



Collective dynamics in an array of coupled cells?

Phase-locking: **synchronization!**

# Synchronization: Huygens' observation



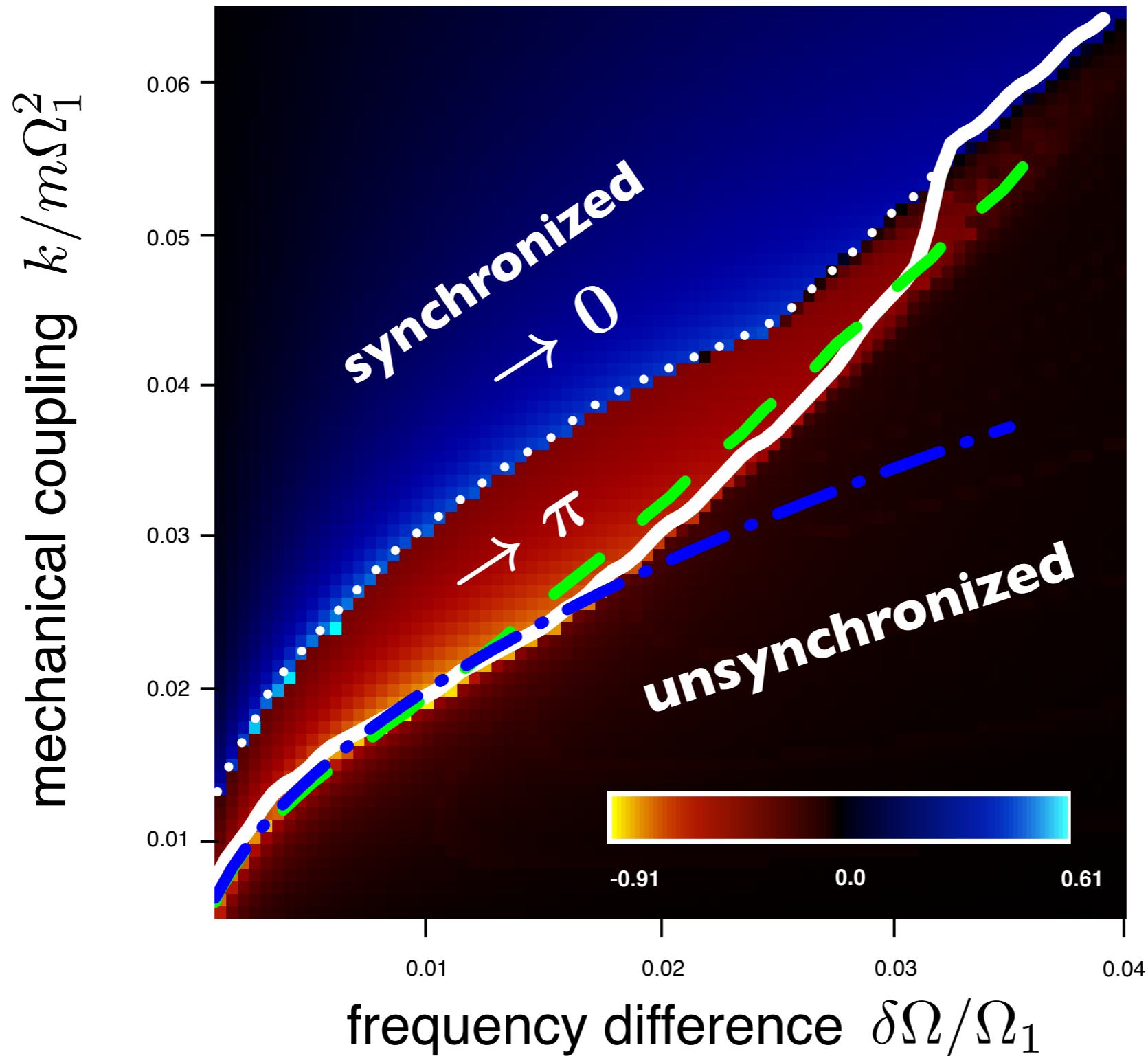
(Huygens' original drawing!)

Coupled pendula synchronize...

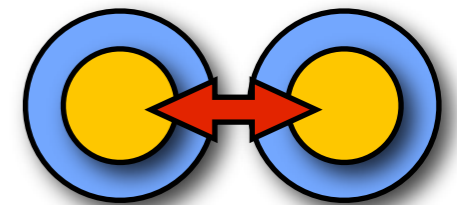
...even though frequencies slightly different



# Classical nonlinear collective dynamics: Synchronization in an optomechanical array

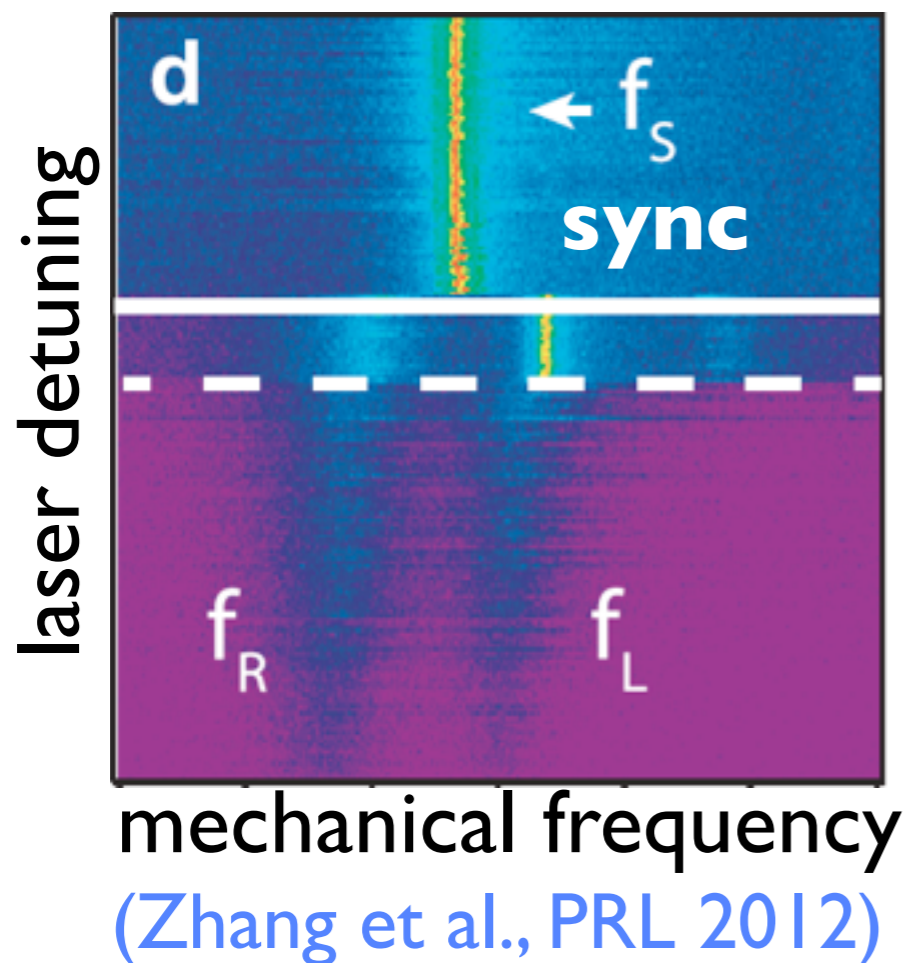
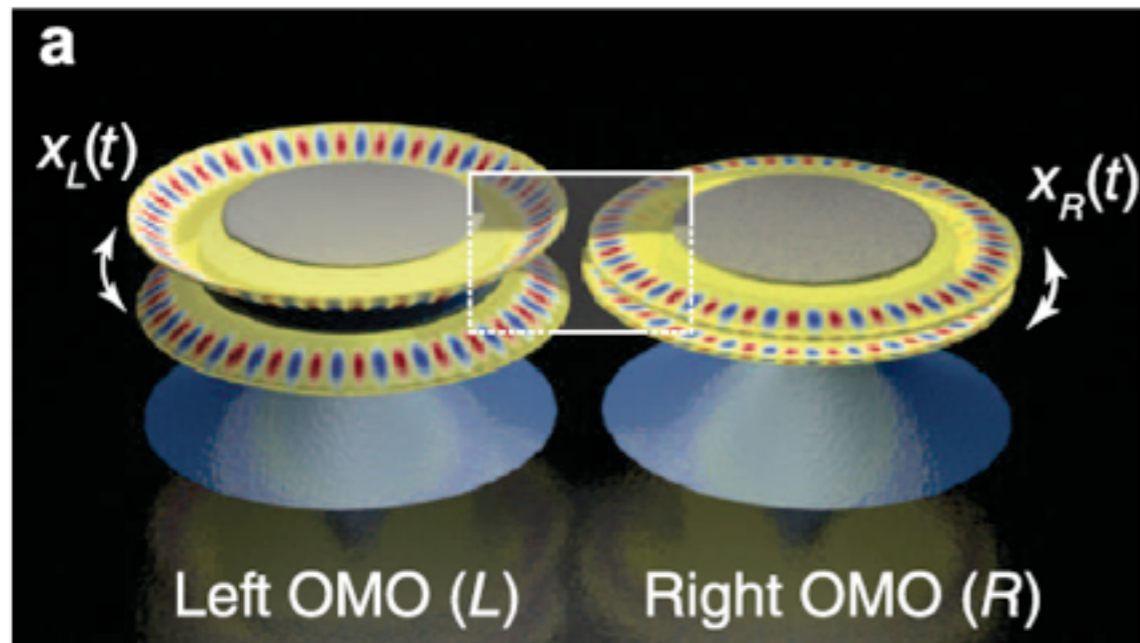


two coupled cells

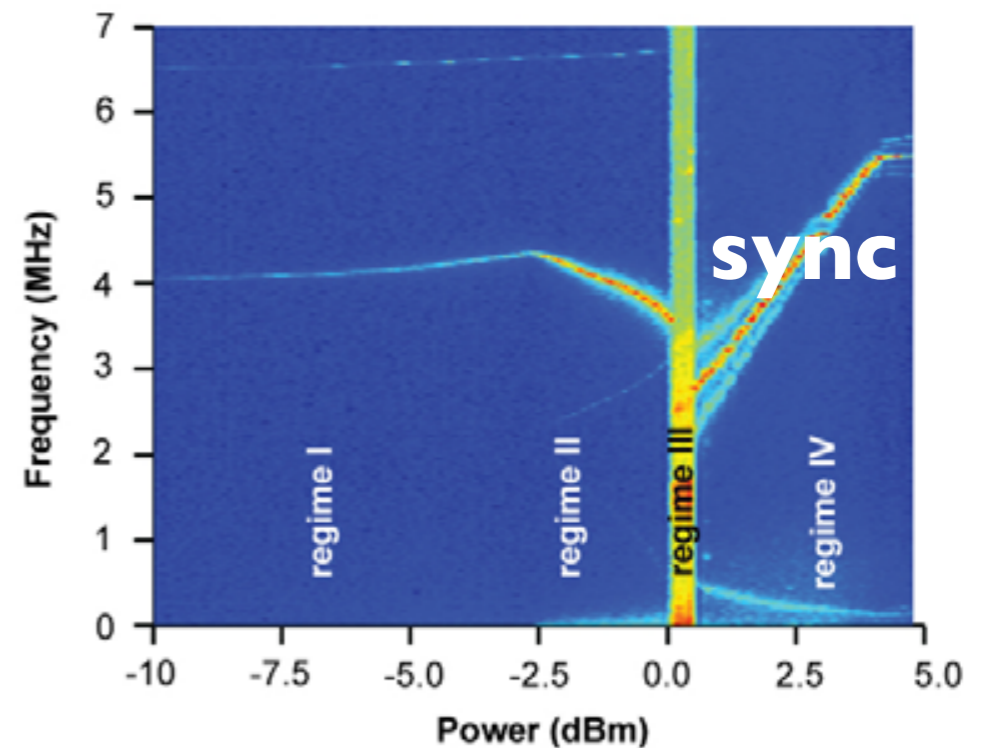
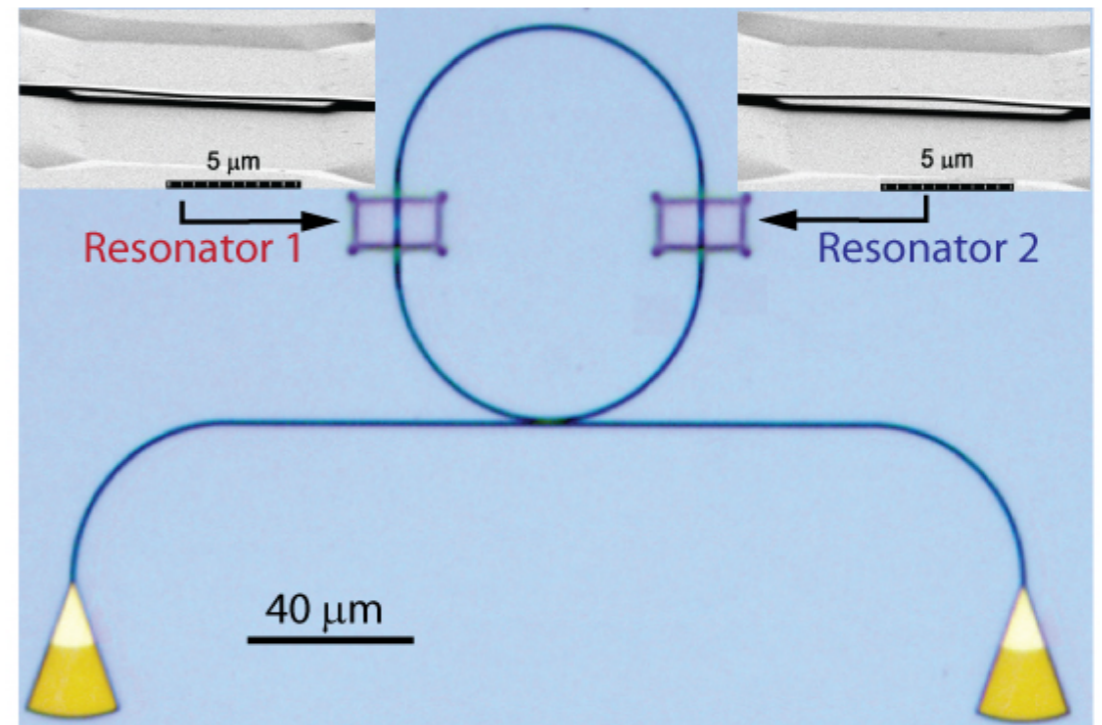


# Experiments (two cells, joint optical mode)

## Michal Lipson lab, Cornell

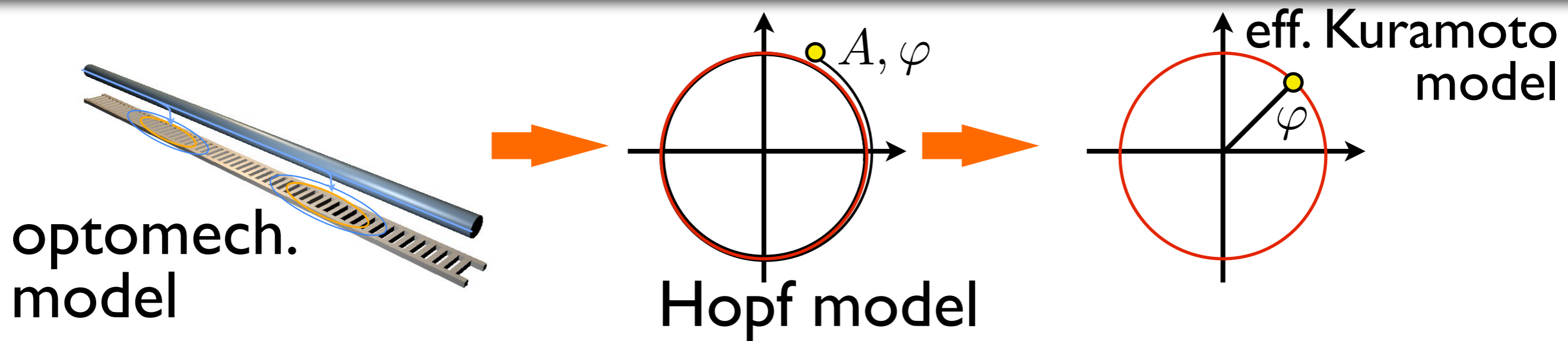


## Hong Tang lab, Yale

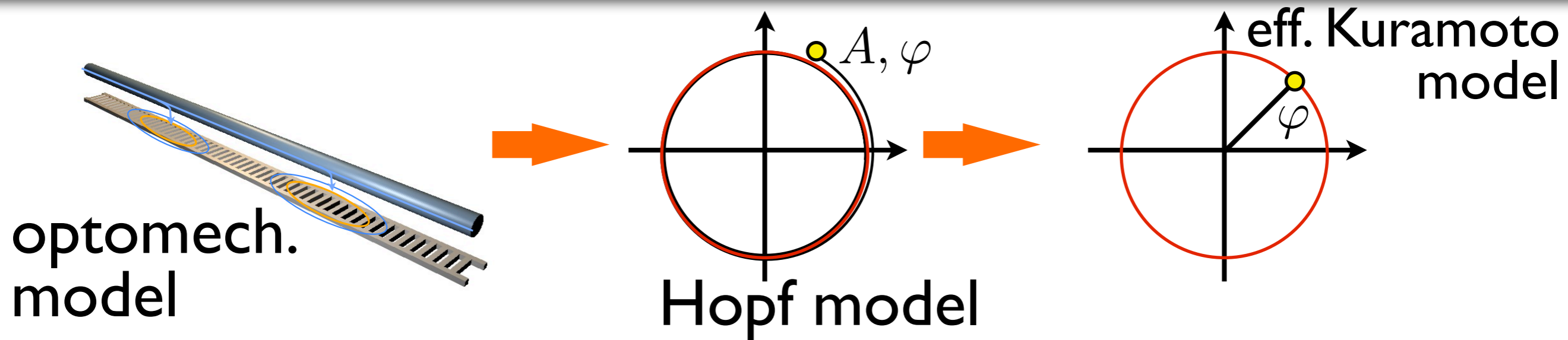


(Bagheri, Poot, FM, Tang; PRL 2013)

# Effective Kuramoto model



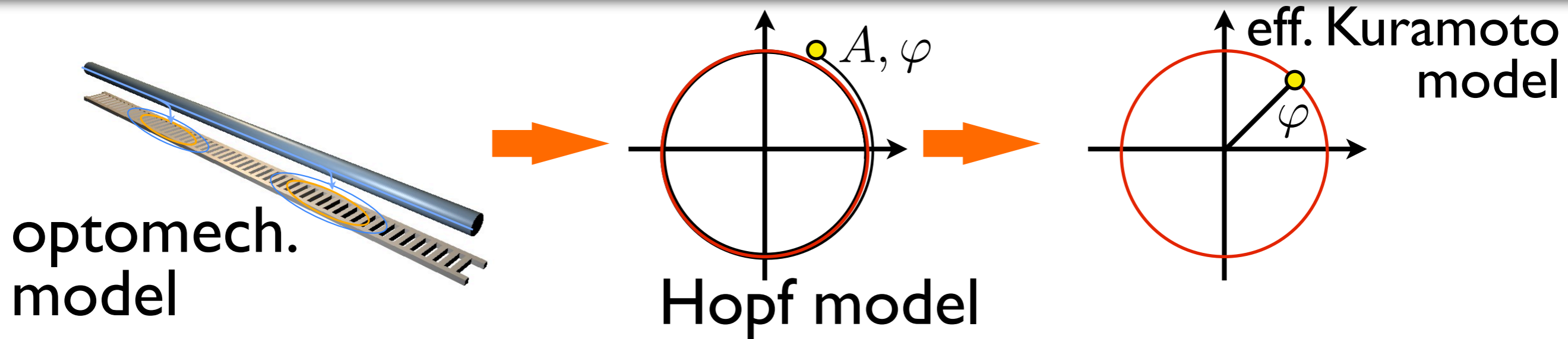
# Effective Kuramoto model



Effective Kuramoto-type model  
for coupled Hopf oscillators:

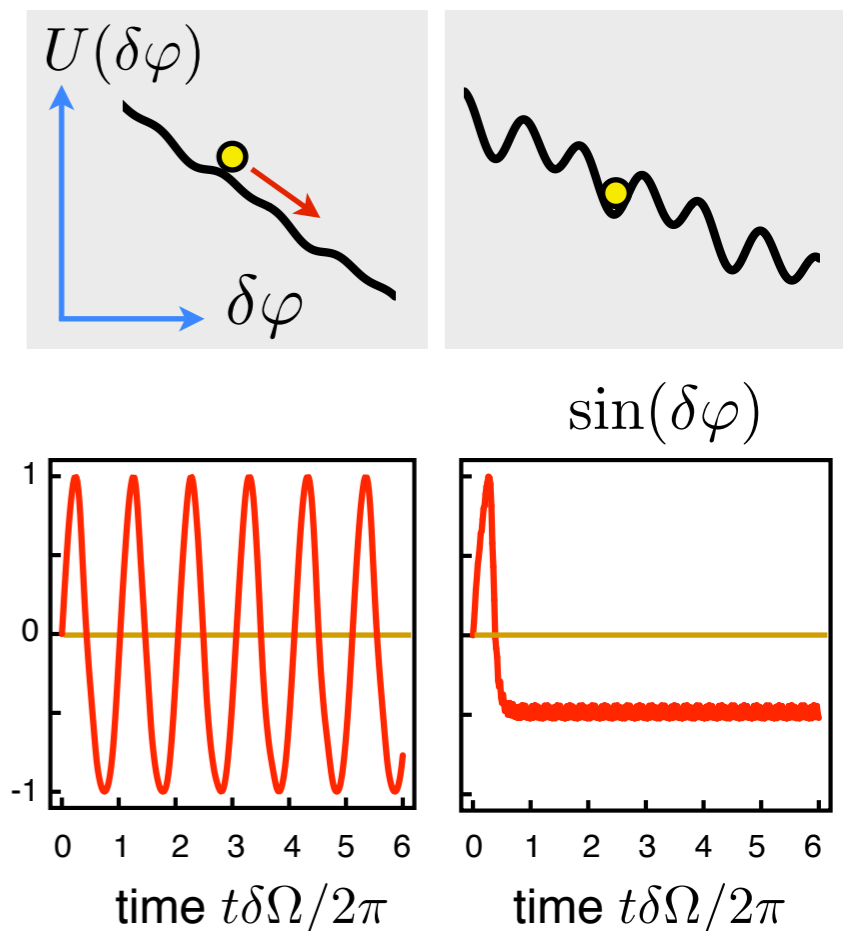
$$\delta\dot{\varphi} = \delta\Omega - 2K_s \sin(2\delta\varphi) - 2K_c \cos(2\delta\varphi)$$

# Effective Kuramoto model

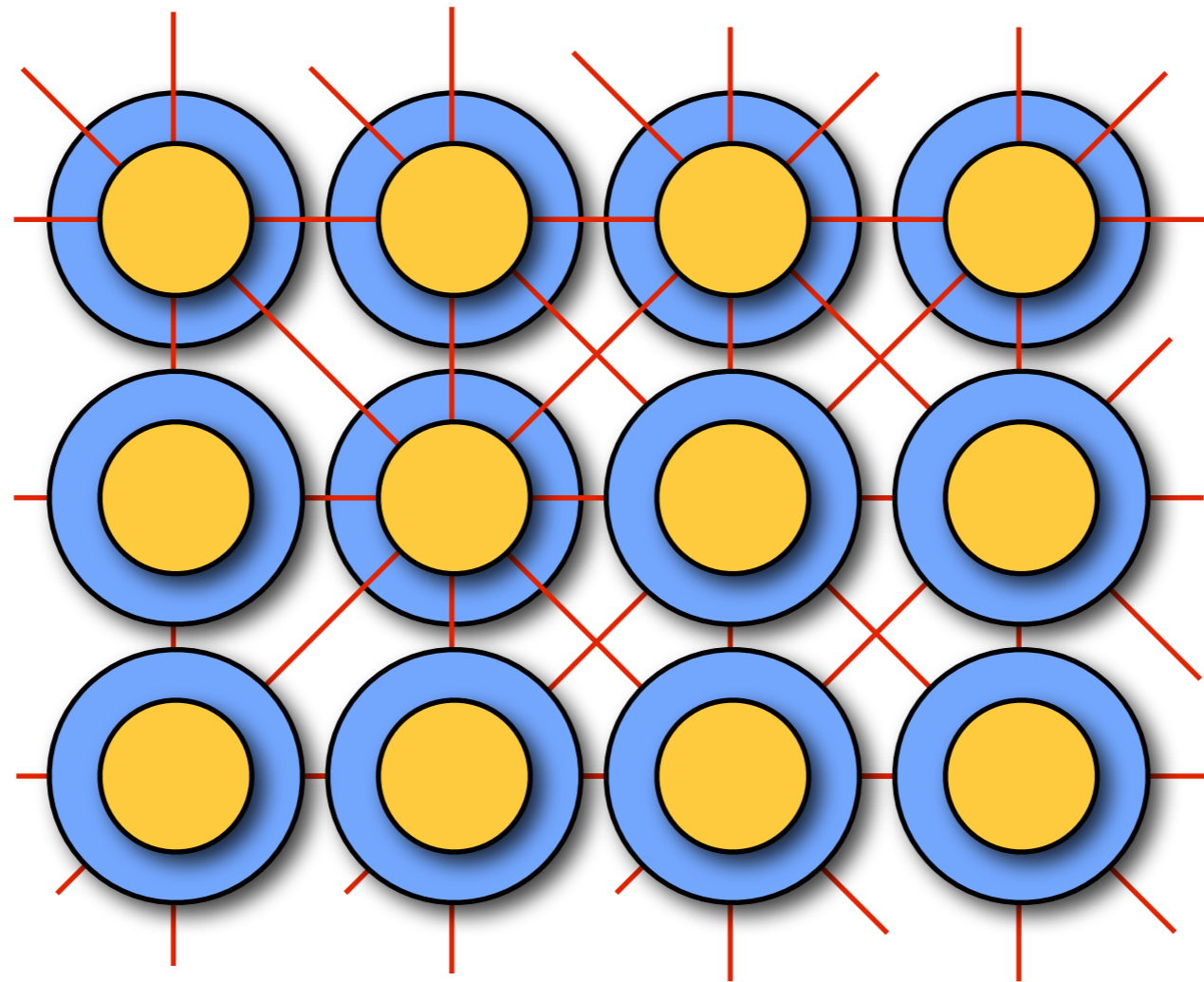


Effective Kuramoto-type model for coupled Hopf oscillators:

$$\delta\dot{\varphi} = \delta\Omega - 2K_s \sin(2\delta\varphi) - 2K_c \cos(2\delta\varphi)$$

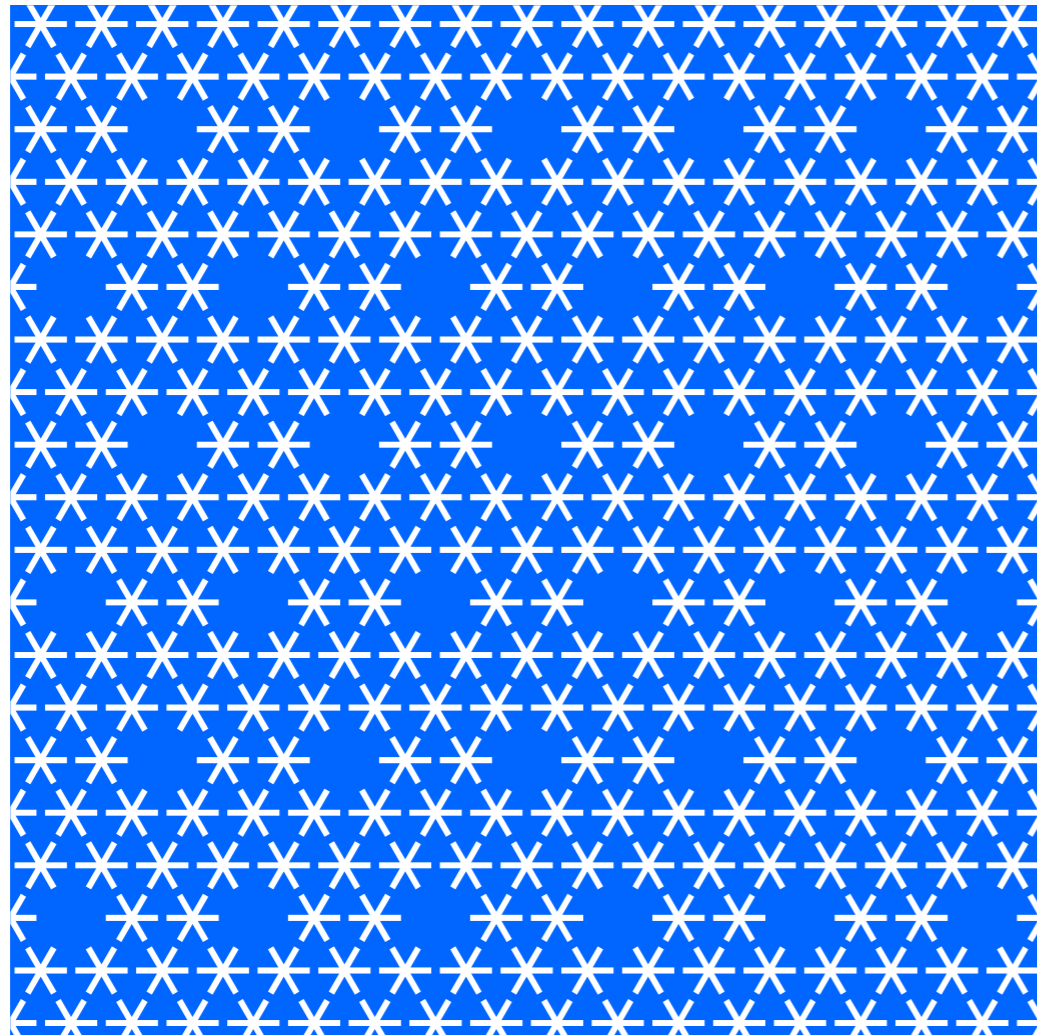


# Synchronization in optomechanical arrays

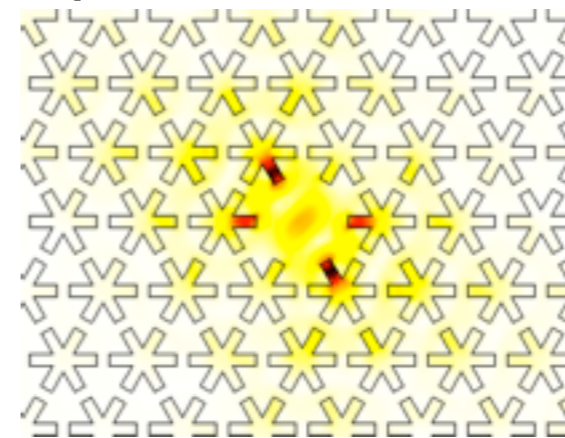


# Optomechanical arrays

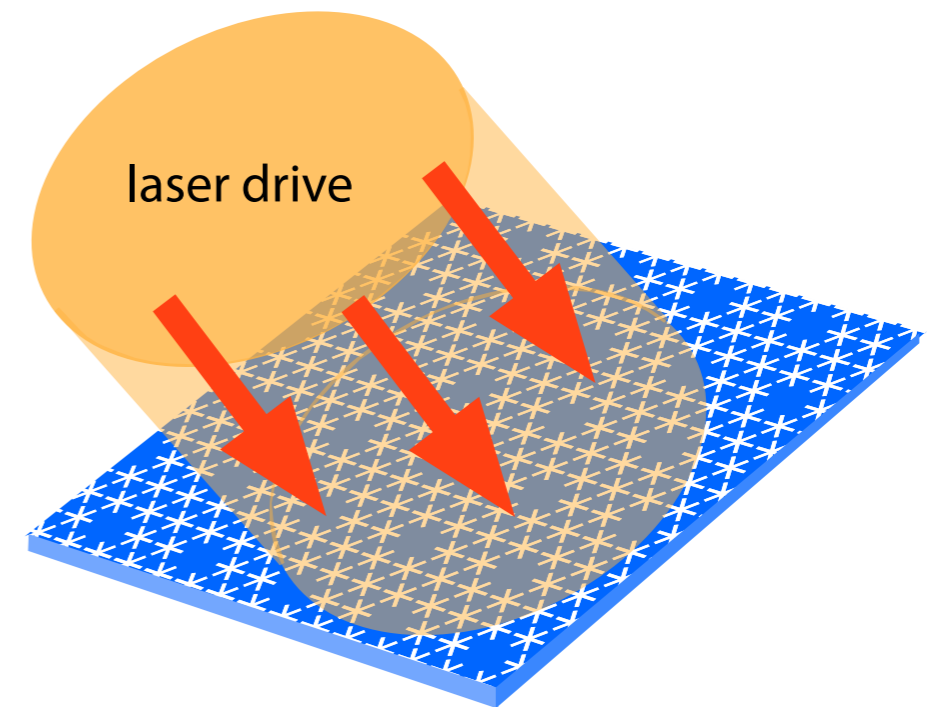
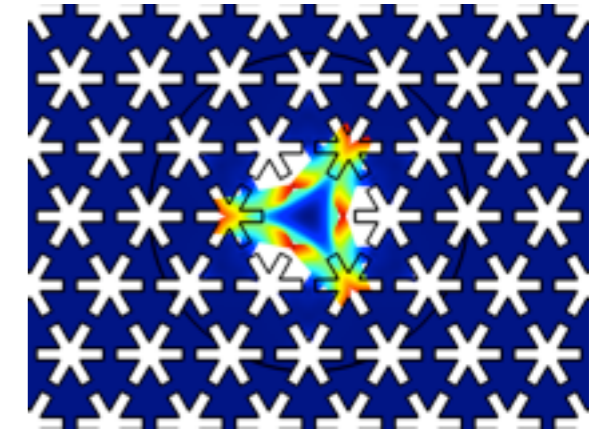
Optomechanical array: Many coupled optomechanical cells



optical mode



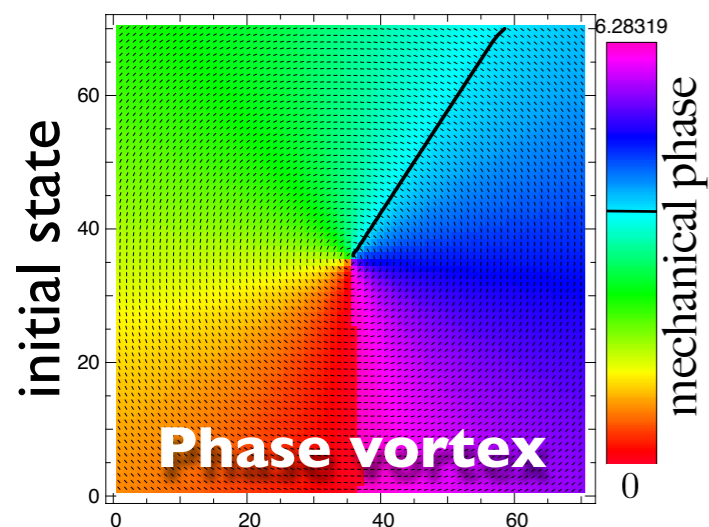
mechanical mode



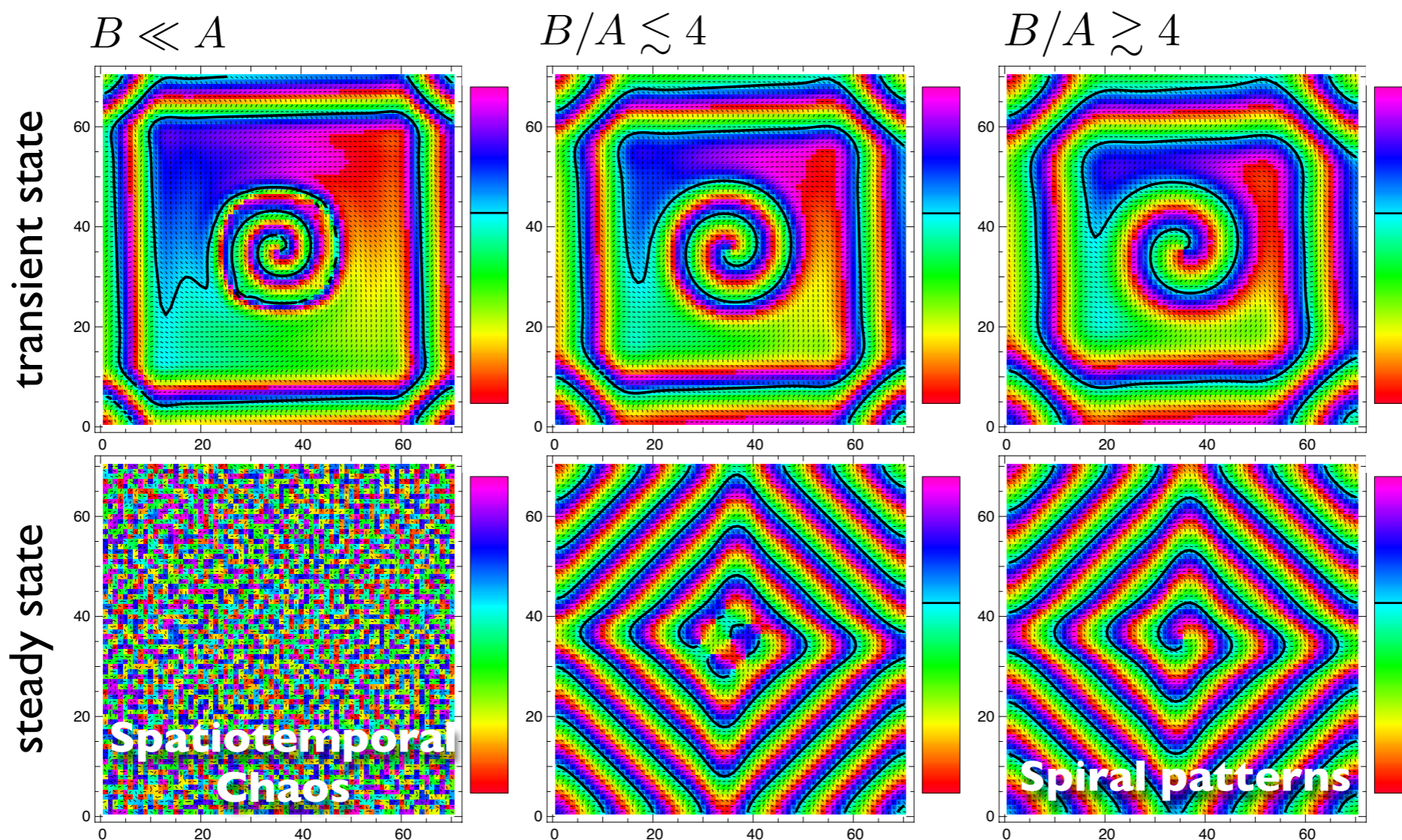
Possible design based on “snowflake” 2D optomechanical crystal (Painter group), here: with suitable defects forming a superlattice (array of cells)

# Pattern formation in optomechanical arrays

work with Christian Brendel, Roland Lauter, Steve Habrake, Max Ludwig



(B,A can be calculated from microscopic parameters)



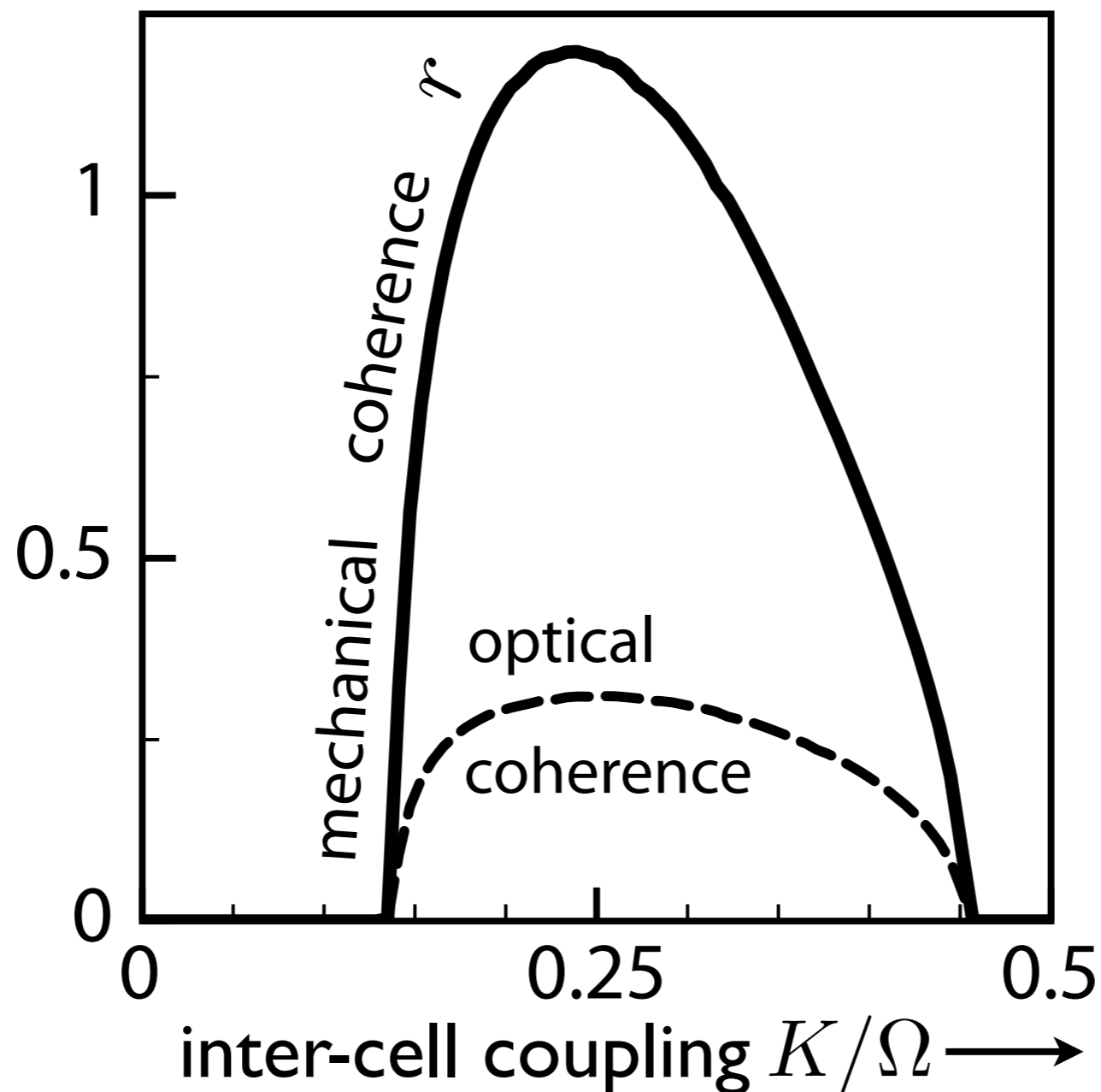


# Transition towards coherent mechanical oscillations

Observation for strong inter-cell coupling:  
spontaneous time-dependence of phonon field

$$\langle \hat{b} \rangle (t) = \bar{b} + r e^{-i\Omega_{\text{eff}} t}$$

“order parameter”  
 (“mechanical coherence”)

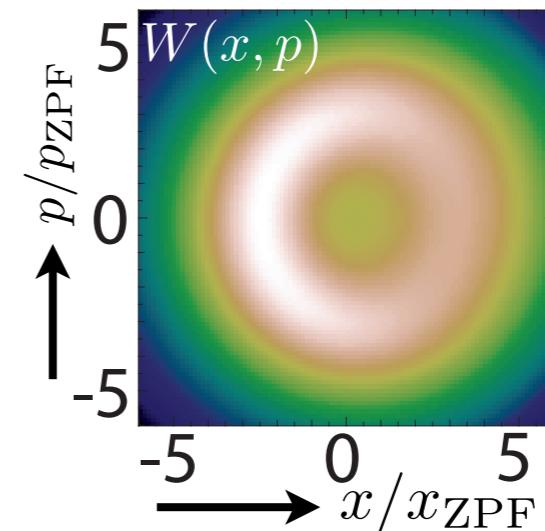


quantum mean-field  
result

# Mechanical quantum states

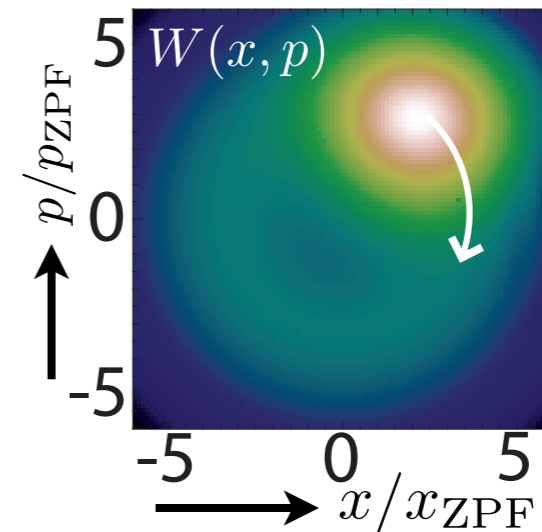
## Incoherent mechan. oscillations (weak inter-cell coupling)

Mechanical Wigner density shows incoherent mixture of all possible oscillation phases



## Coherent mechan. oscillations (strong inter-cell coupling)

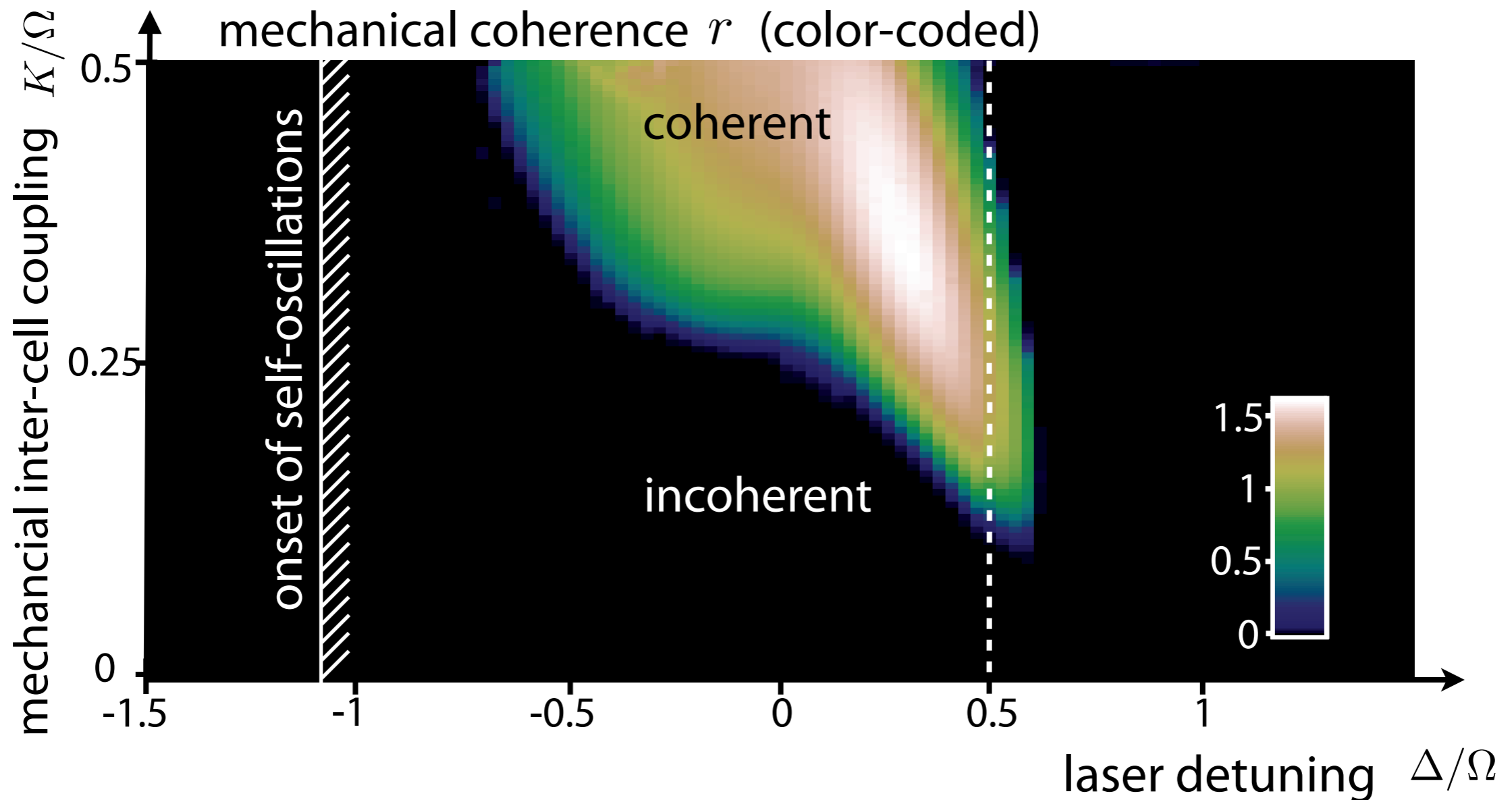
Mechanical Wigner density shows preferred phase (coherent state) – spontaneous symmetry breaking!



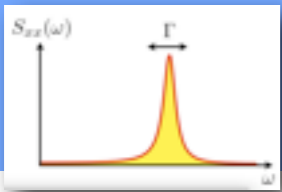
# Transition towards coherent mechanical oscillations

$$\langle \hat{b} \rangle (t) = \bar{b} + r e^{-i\Omega_{\text{eff}} t}$$

“order parameter”  
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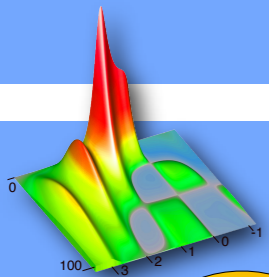


# Optomechanics (Outline)

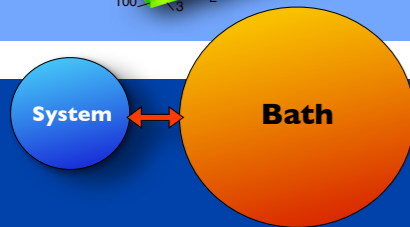


Displacement detection

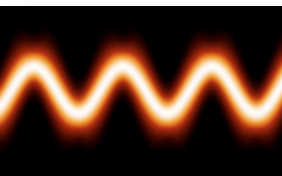
Linear optomechanics



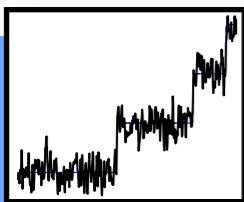
Nonlinear dynamics



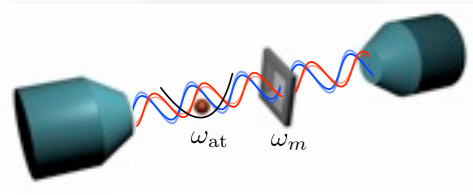
Quantum theory of cooling



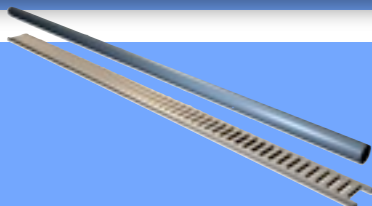
Interesting quantum states



Towards Fock state detection

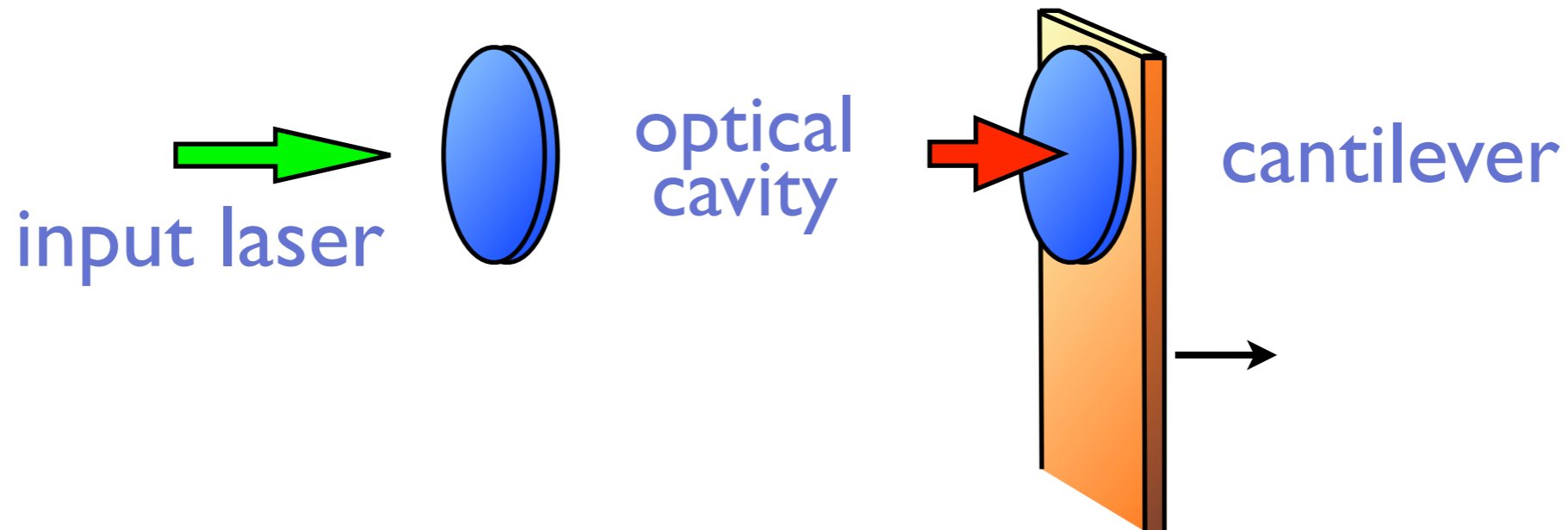


Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

# Cooling with light



**Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever**

$$k_B T_{\text{eff}} \ll \hbar \omega_M$$

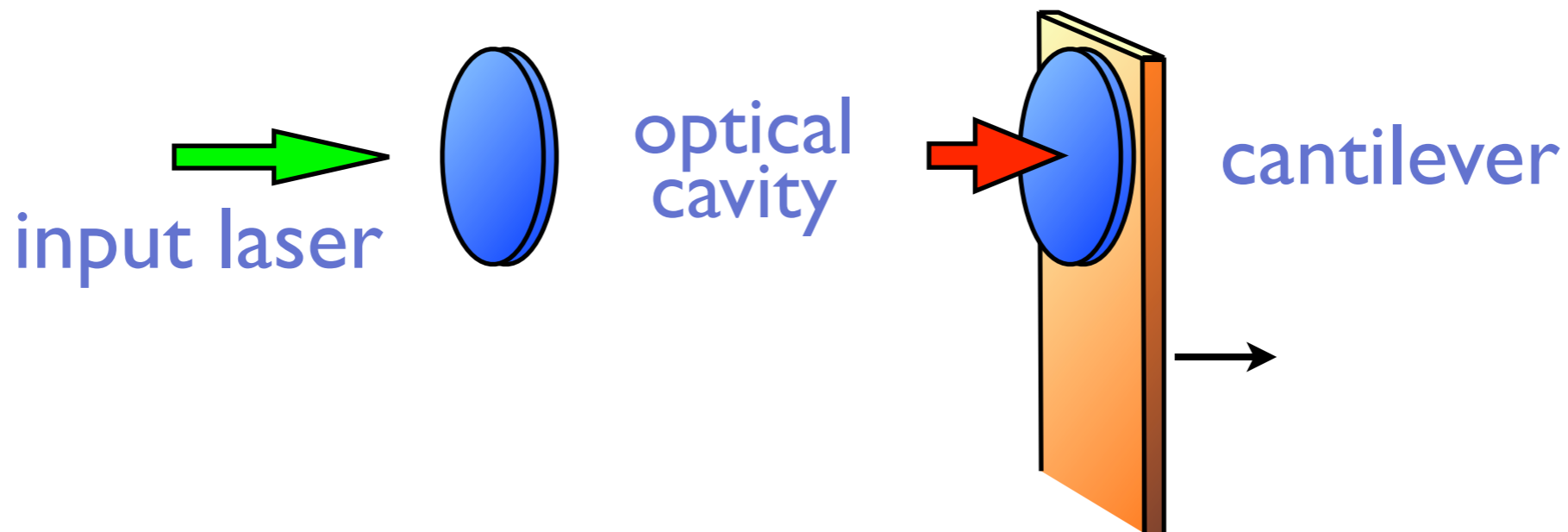
**Classical theory:**

$$T_{\text{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\text{opt}} + \Gamma_M}$$

optomechanical damping rate

Pioneering theory and experiments: **Braginsky**  
(since 1960s)

# Cooling with light



**Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever**

$$k_B T_{\text{eff}} \ll \hbar \omega_M$$

**Classical theory:**

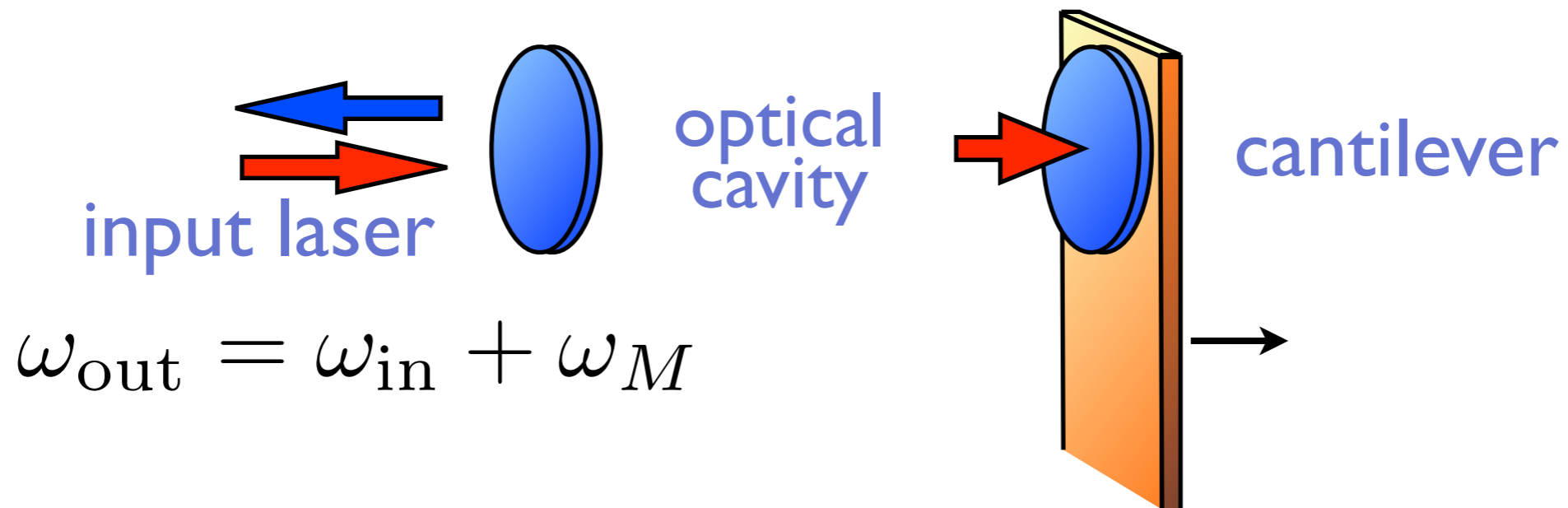
$$T_{\text{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\text{opt}} + \Gamma_M} \rightarrow 0 ?$$

**quantum limit?**  
**shot noise!**

optomechanical damping rate

Pioneering theory and experiments: **Braginsky** (since 1960s)

# Cooling with light



## Quantum picture: Raman scattering – sideband cooling

### Original idea:

Sideband cooling in ion traps – Hänsch, Schawlow / Wineland, Dehmelt 1975

### Similar ideas proposed for nanomechanics:

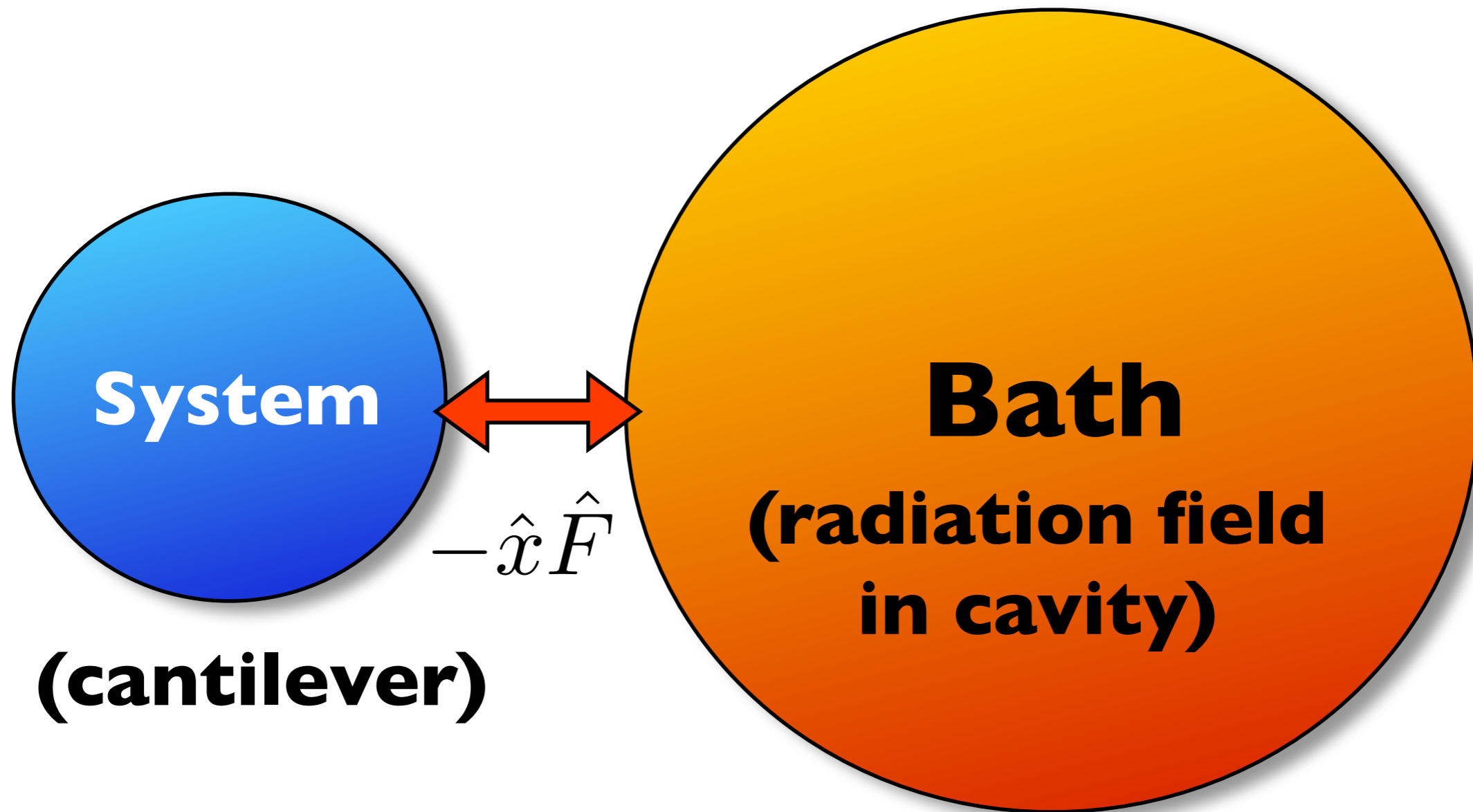
cantilever + quantum dot – Wilson-Rae, Zoller, Imamoglu 2004

cantilever + Cooper-pair box – Martin Shnirman, Tian, Zoller 2004

cantilever + ion – Tian, Zoller 2004

cantilever + supercond. SET – Clerk, Bennett / Blencowe, Imbers, Armour 2005,  
Naik et al. (Schwab group) 2006

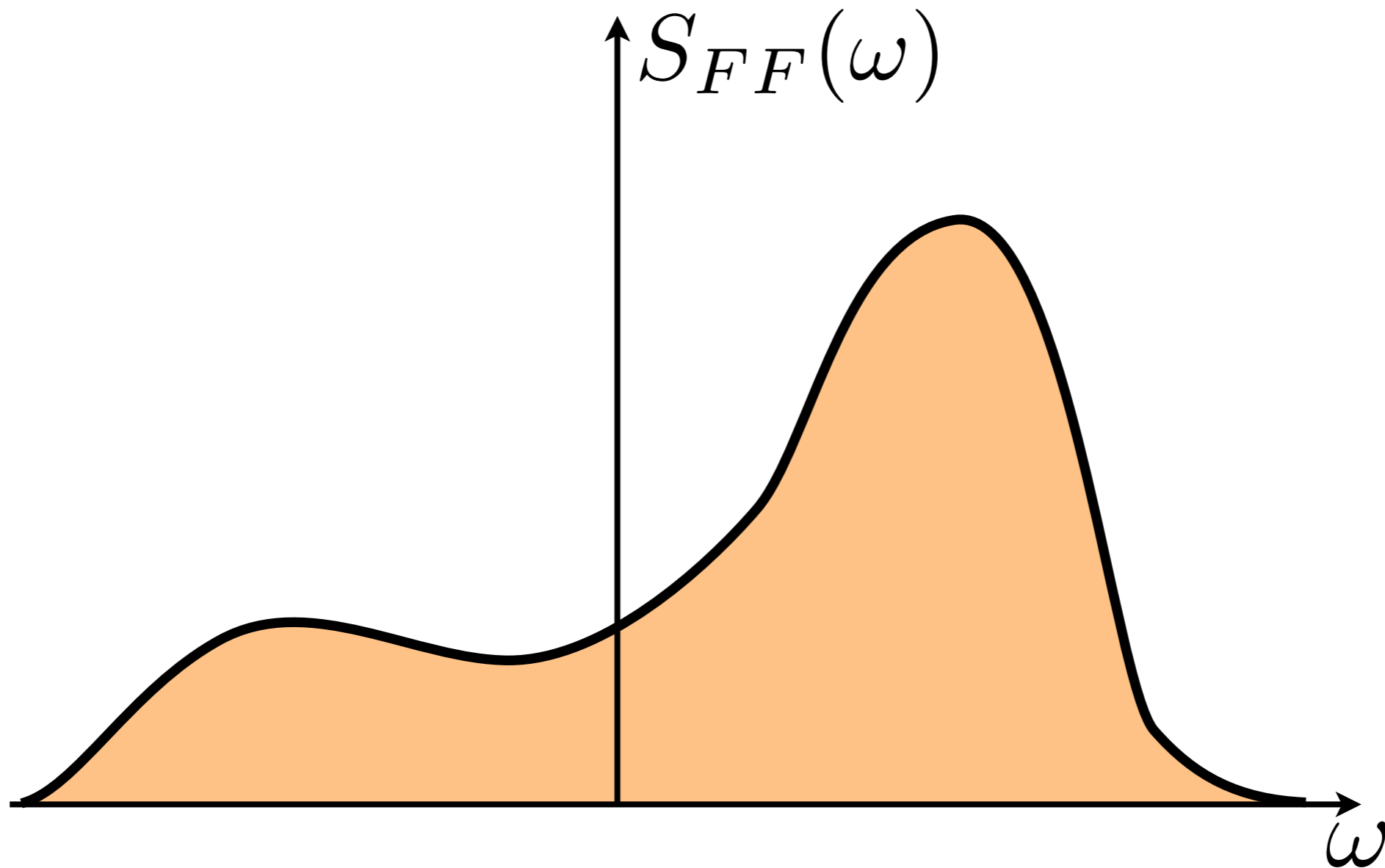
# Quantum noise approach





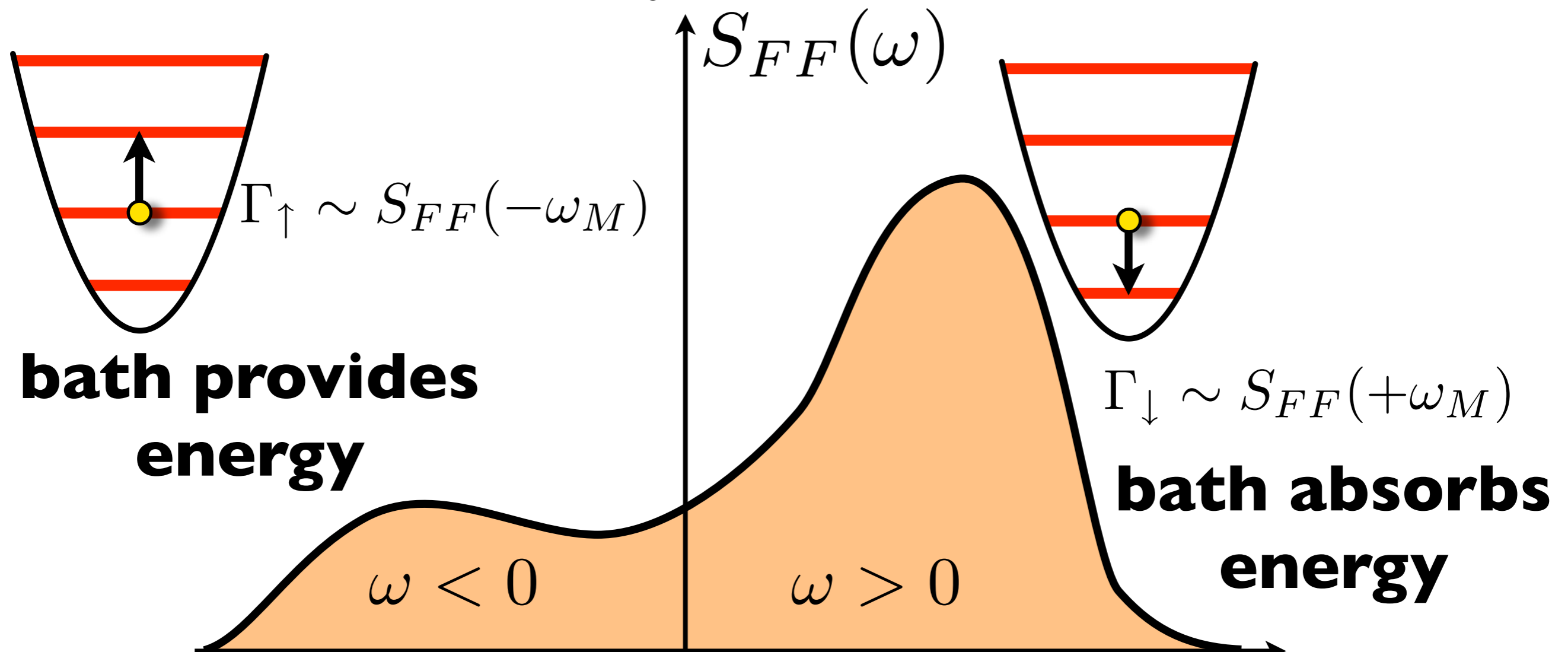
# Quantum noise approach

**spectrum**  $S_{FF}(\omega) = \int e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle dt$



# Quantum noise approach

**spectrum**  $S_{FF}(\omega) = \int e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle dt$



**transition rate**  $\Gamma_{f \leftarrow i} = \frac{|x_{fi}|^2}{\hbar^2} S_{FF} \left( \frac{\epsilon_i - \epsilon_f}{\hbar} \right)$

# Quantum theory of optomechanical cooling

## Spectrum of radiation pressure fluctuations

$$S_{FF}(\omega) = \int e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle dt$$

radiation  
pressure  
force

$$\hat{F} = \left( \frac{\hbar\omega_R}{L} \right) \hat{a}^\dagger \hat{a}$$

photon number

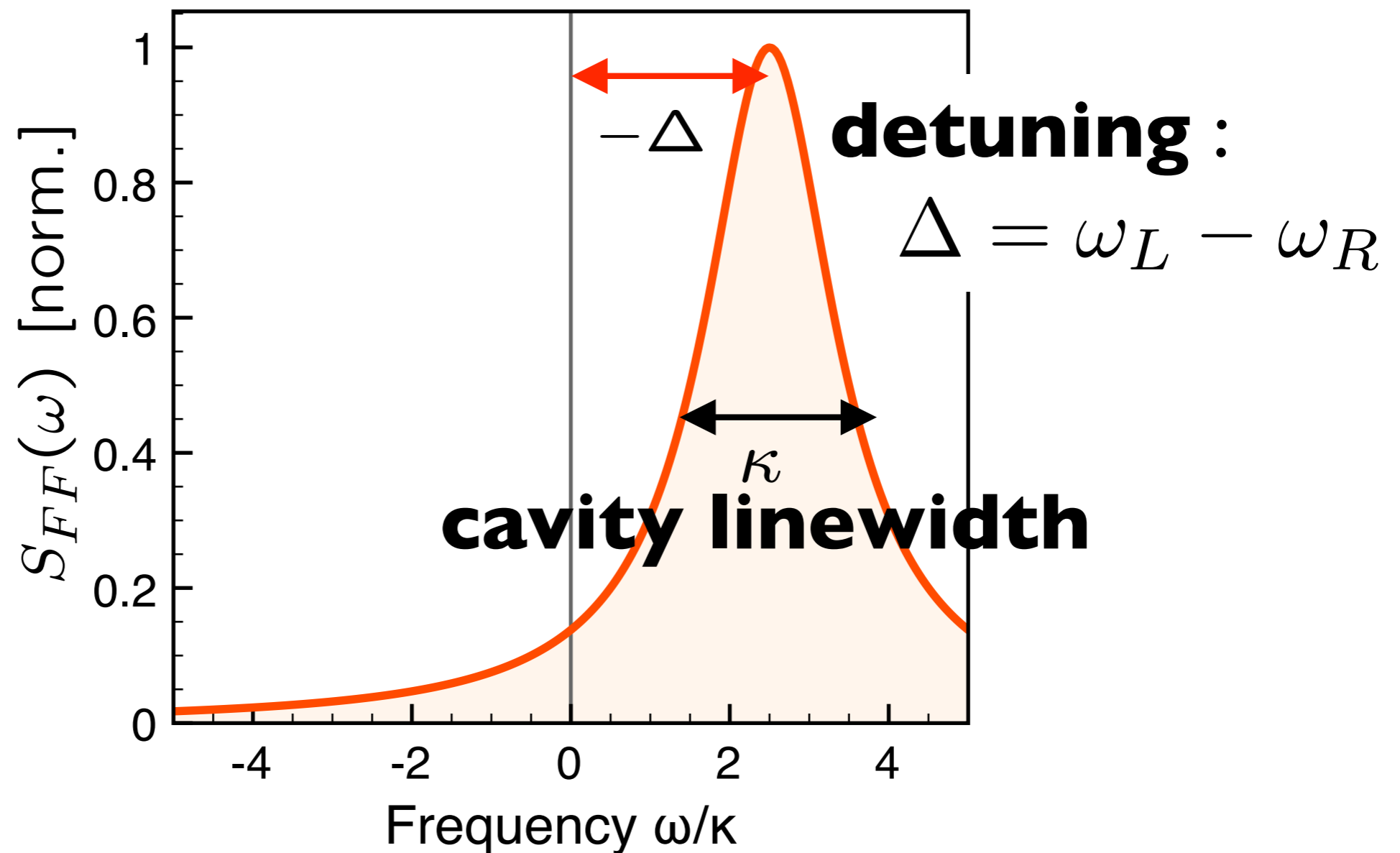
$$S_{FF}(\omega) = \left( \frac{\hbar\omega_R}{L} \right)^2 \bar{n}_p \frac{\kappa}{(\omega + \Delta)^2 + (\kappa/2)^2}$$

photon shot noise spectrum

# Quantum theory of optomechanical cooling

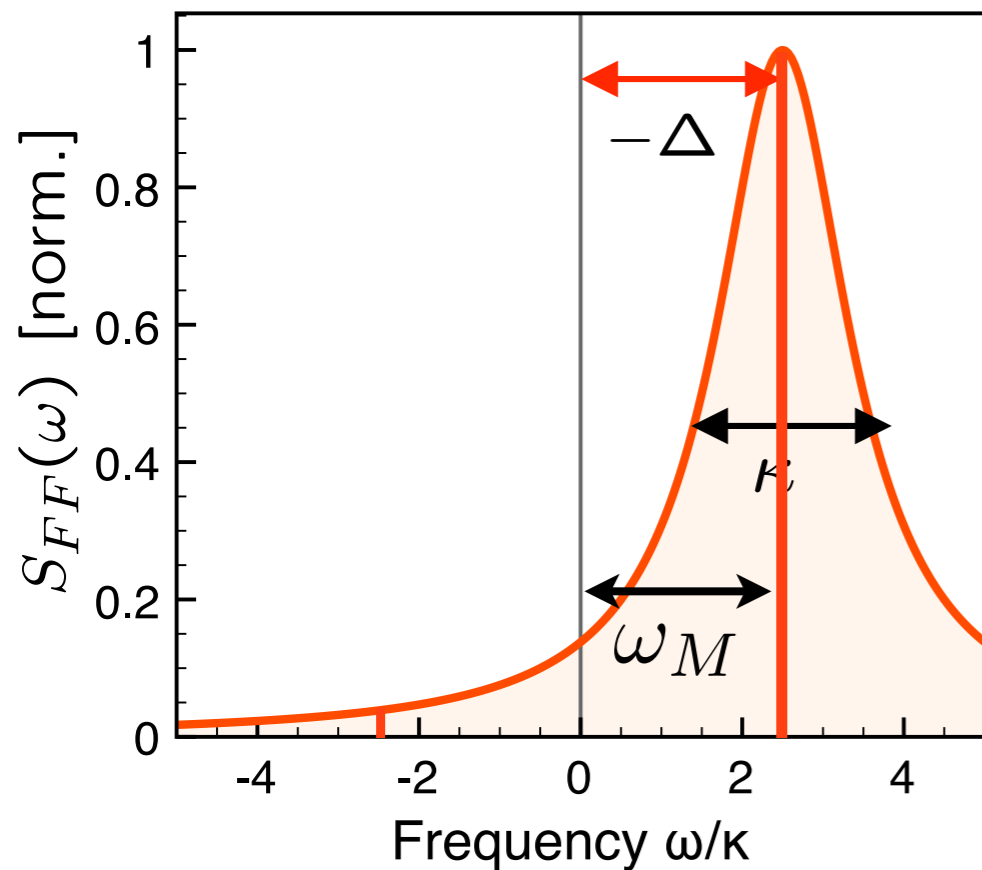
## Spectrum of radiation pressure fluctuations

$$S_{FF}(\omega) = \int e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle dt$$



cavity emits energy / absorbs energy

# Quantum theory of optomechanical cooling



cavity emits energy / absorbs energy

## Cooling rate

$$\Gamma_{\text{opt}} = \frac{x_{\text{ZPF}}^2}{\hbar^2} [S_{FF}(+\omega_M) - S_{FF}(-\omega_M)]$$

## Quantum limit for cantilever phonon number

$$\frac{n_{\text{opt}} + 1}{n_{\text{opt}}} = \frac{S_{FF}(+\omega_M)}{S_{FF}(-\omega_M)}$$

$$\Delta = -\omega_M \Rightarrow n_{\text{opt}} = \left( \frac{\kappa}{4\omega_M} \right)^2$$

## Ground-state cooling

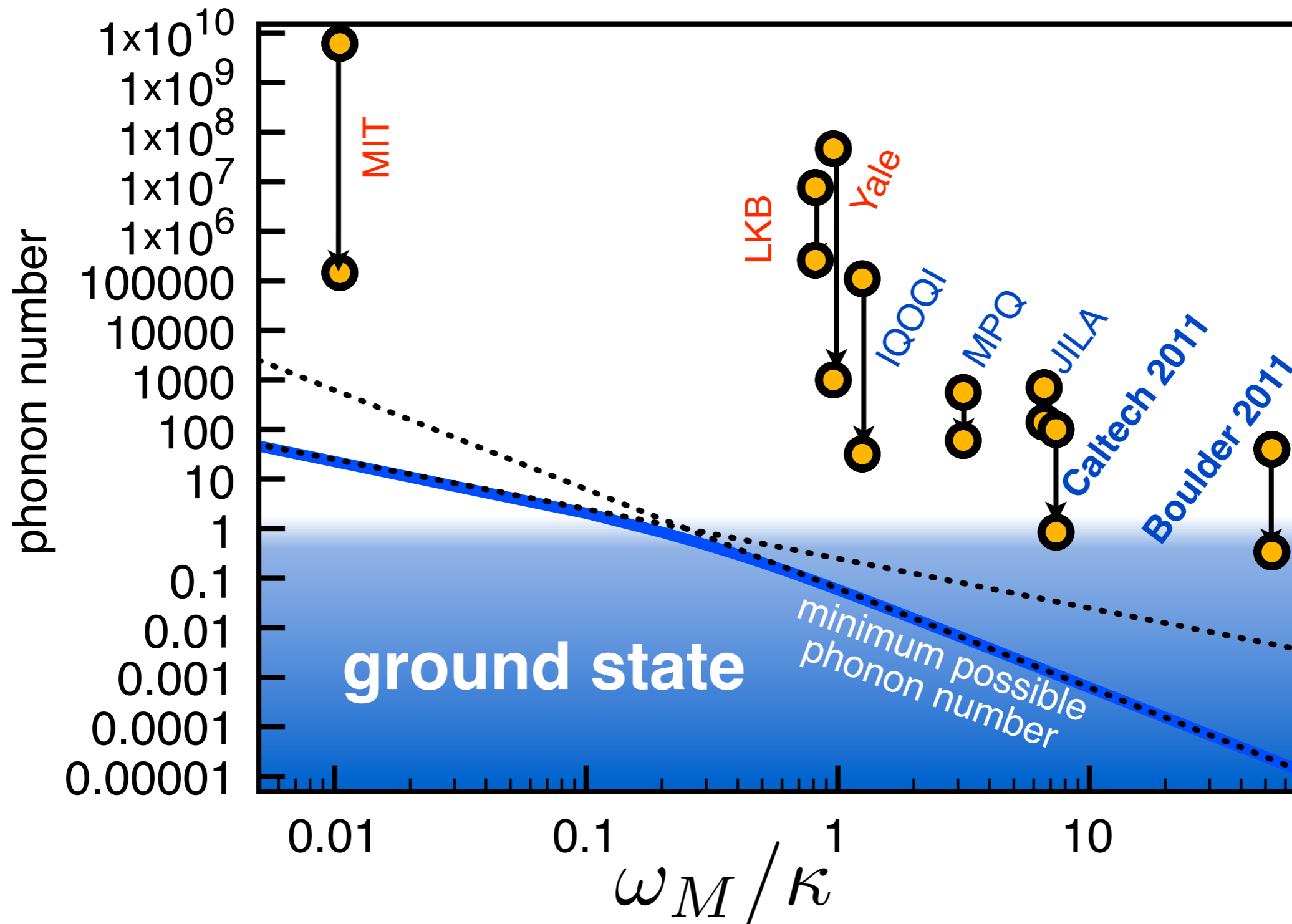
needs: high optical finesse / large mechanical frequency

FM, Chen, Clerk, Girvin,  
PRL **93**, 093902 (2007)

also: Wilson-Rae, Nooshi, Zwerger,  
Kippenberg, PRL **99**, 093901 (2007);  
Genes et al, PRA 2008

experiment with  $\kappa/\omega_M \approx 1/20$   
Kippenberg group 2007

# Laser-cooling towards the ground state

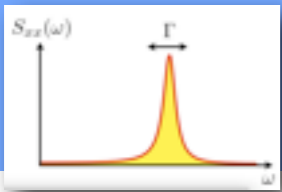


analogy to (cavity-assisted)  
laser cooling of atoms

FM et al., PRL **93**, 093902 (2007)

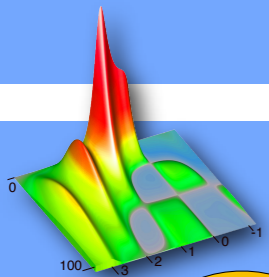
Wilson-Rae et al., PRL **99**, 093901 (2007)

# Optomechanics (Outline)

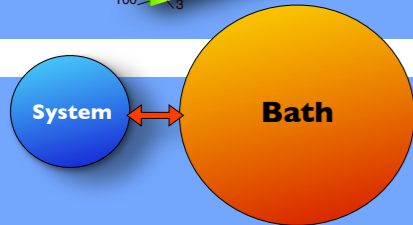


Displacement detection

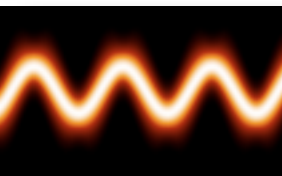
Linear optomechanics



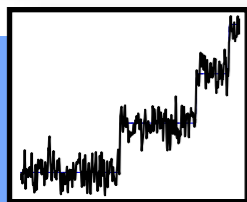
Nonlinear dynamics



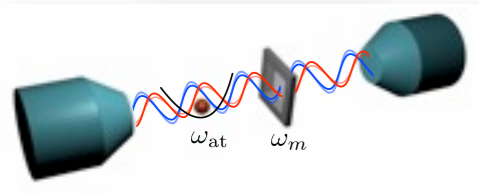
Quantum theory of cooling



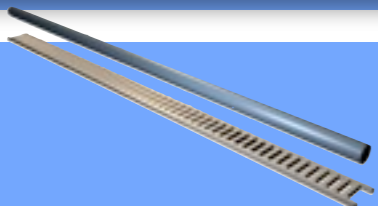
Interesting quantum states



Towards Fock state detection



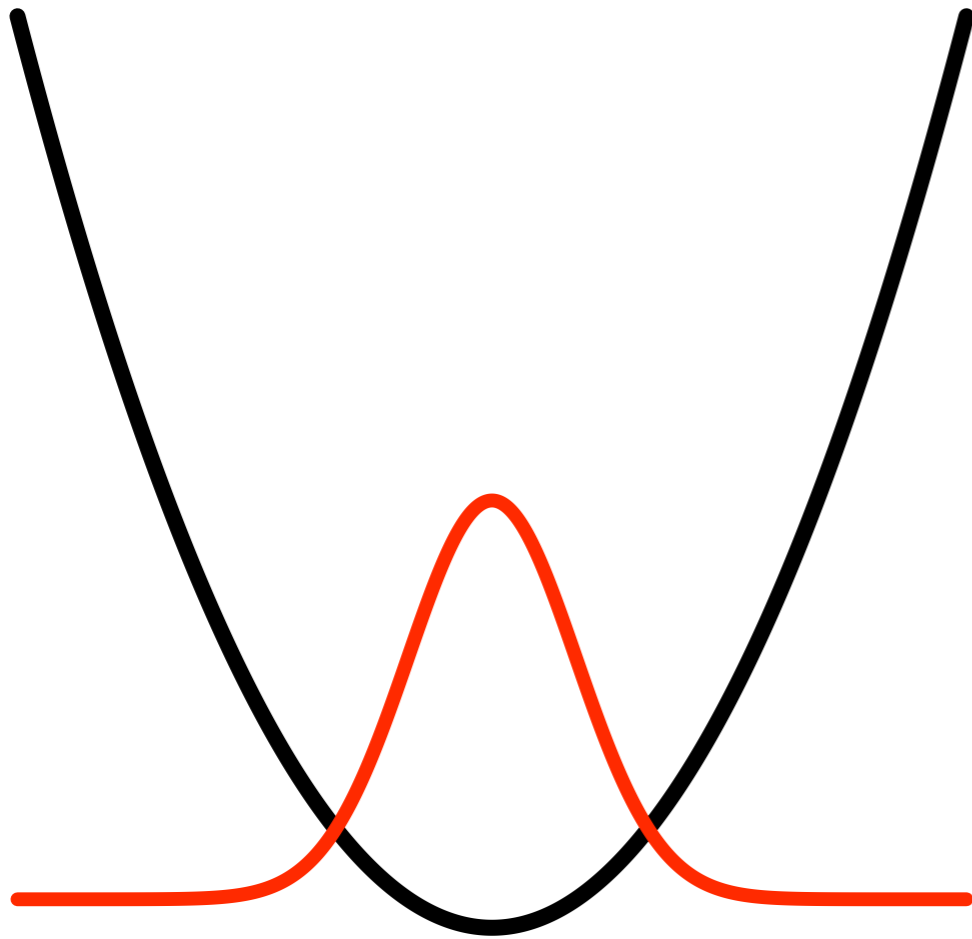
Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

# Squeezed states

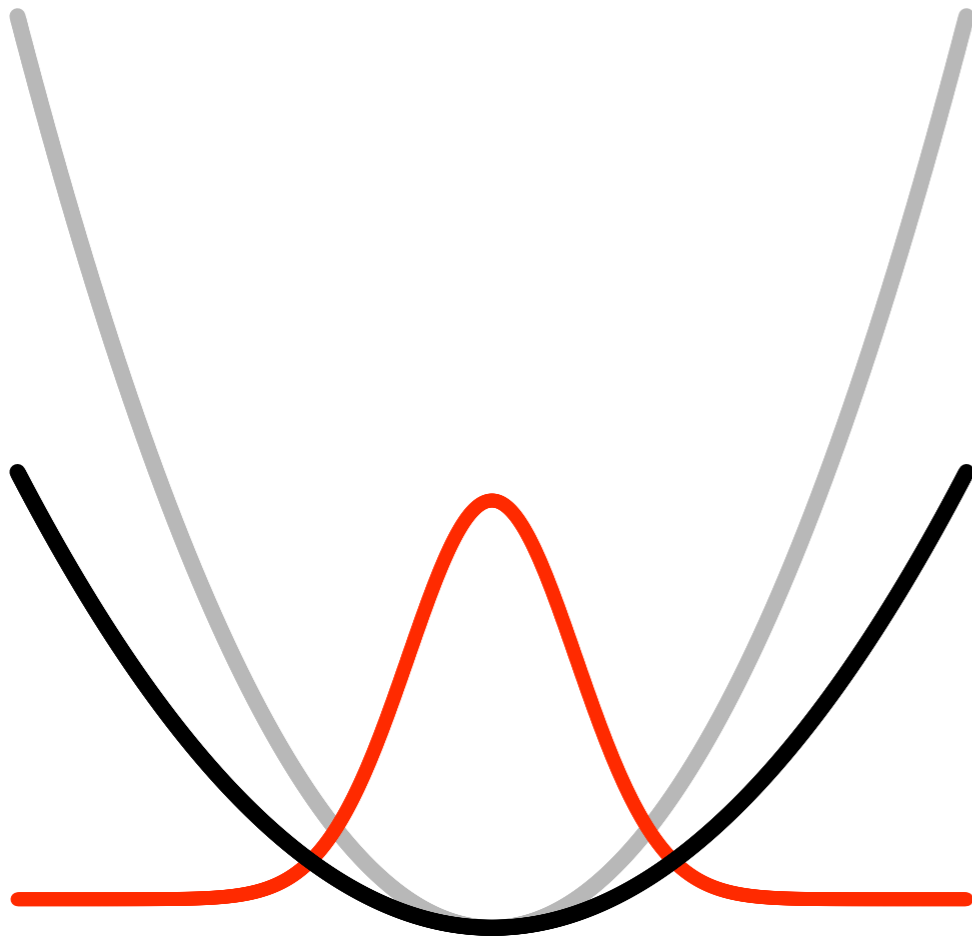
Squeezing the mechanical oscillator state





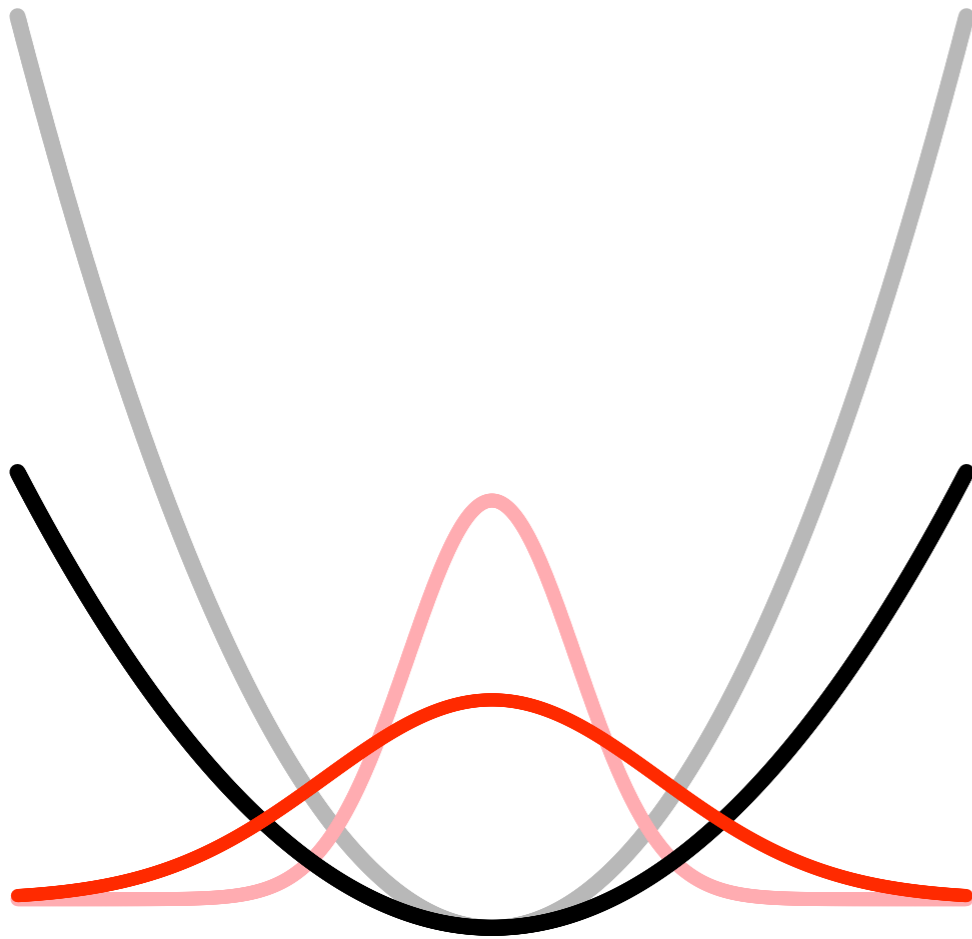
# Squeezed states

Squeezing the mechanical oscillator state



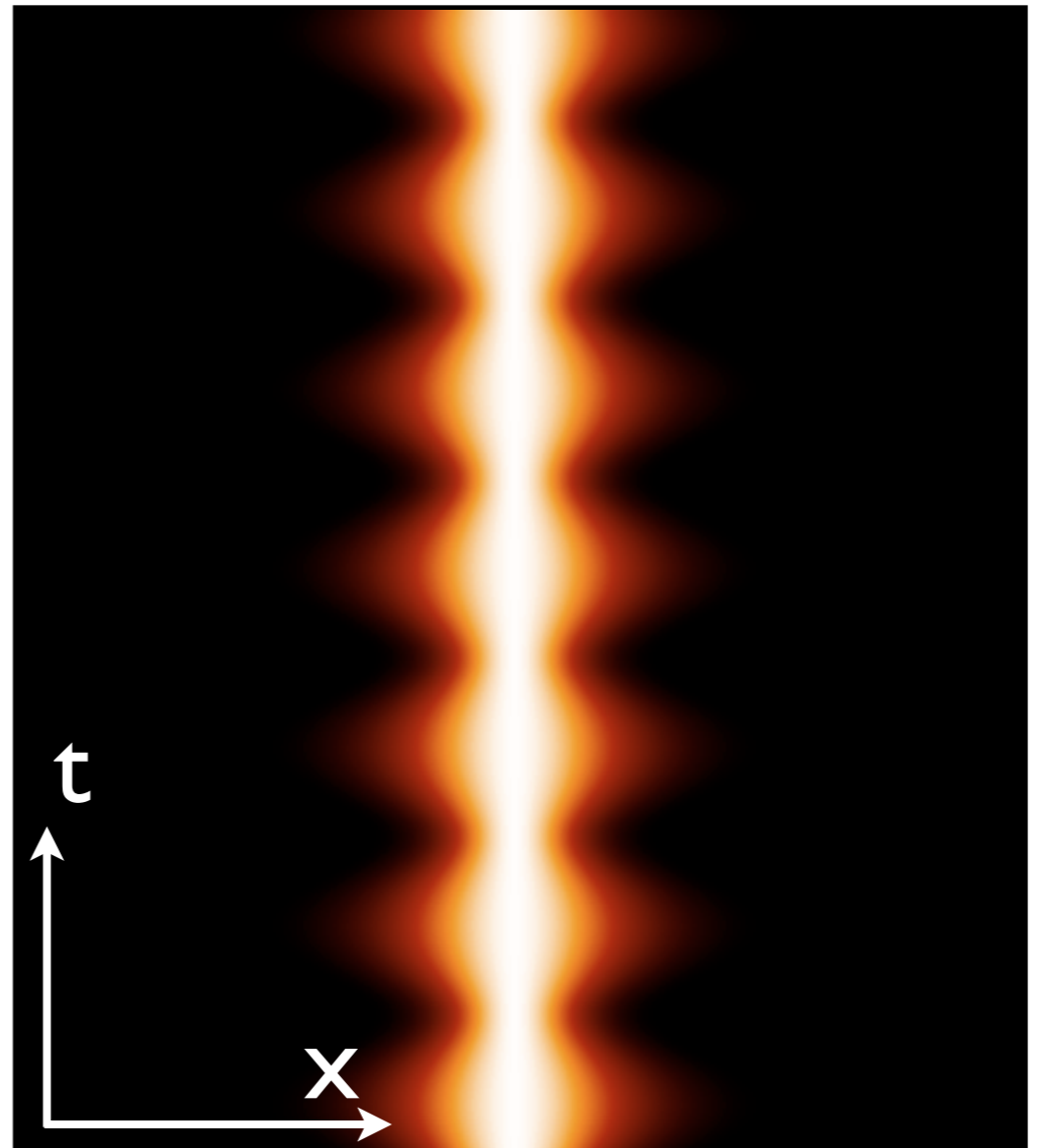
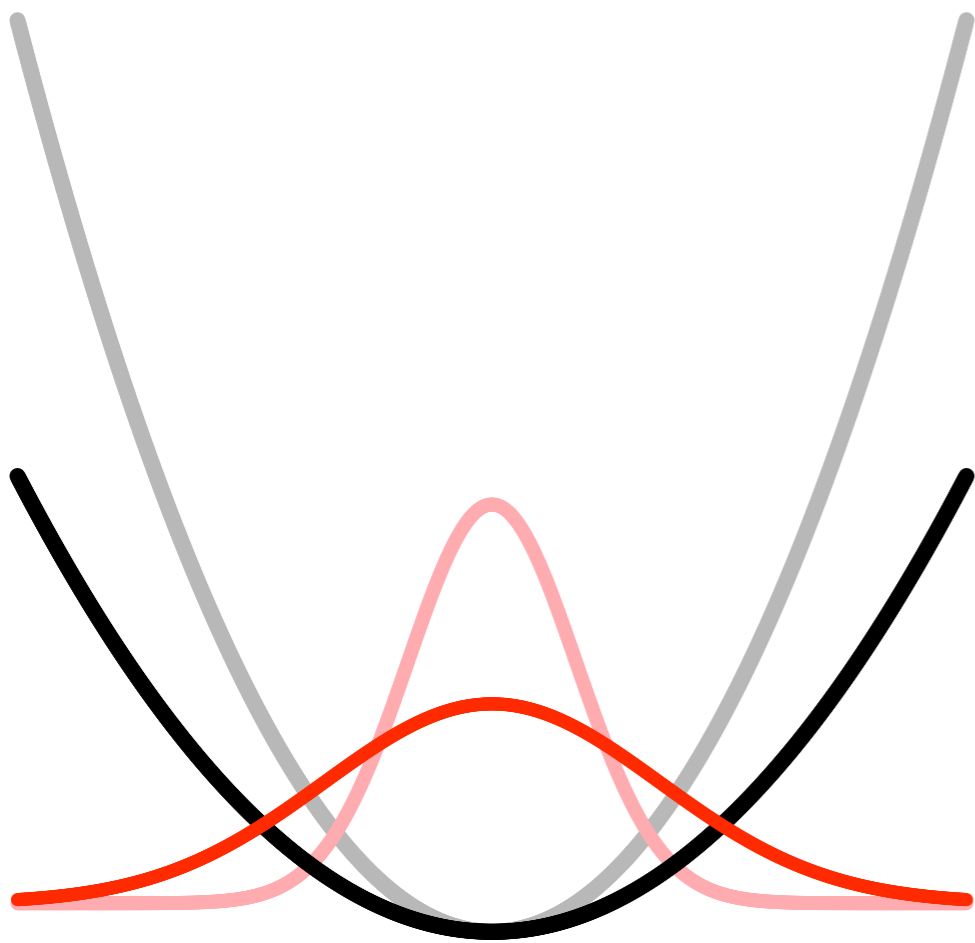
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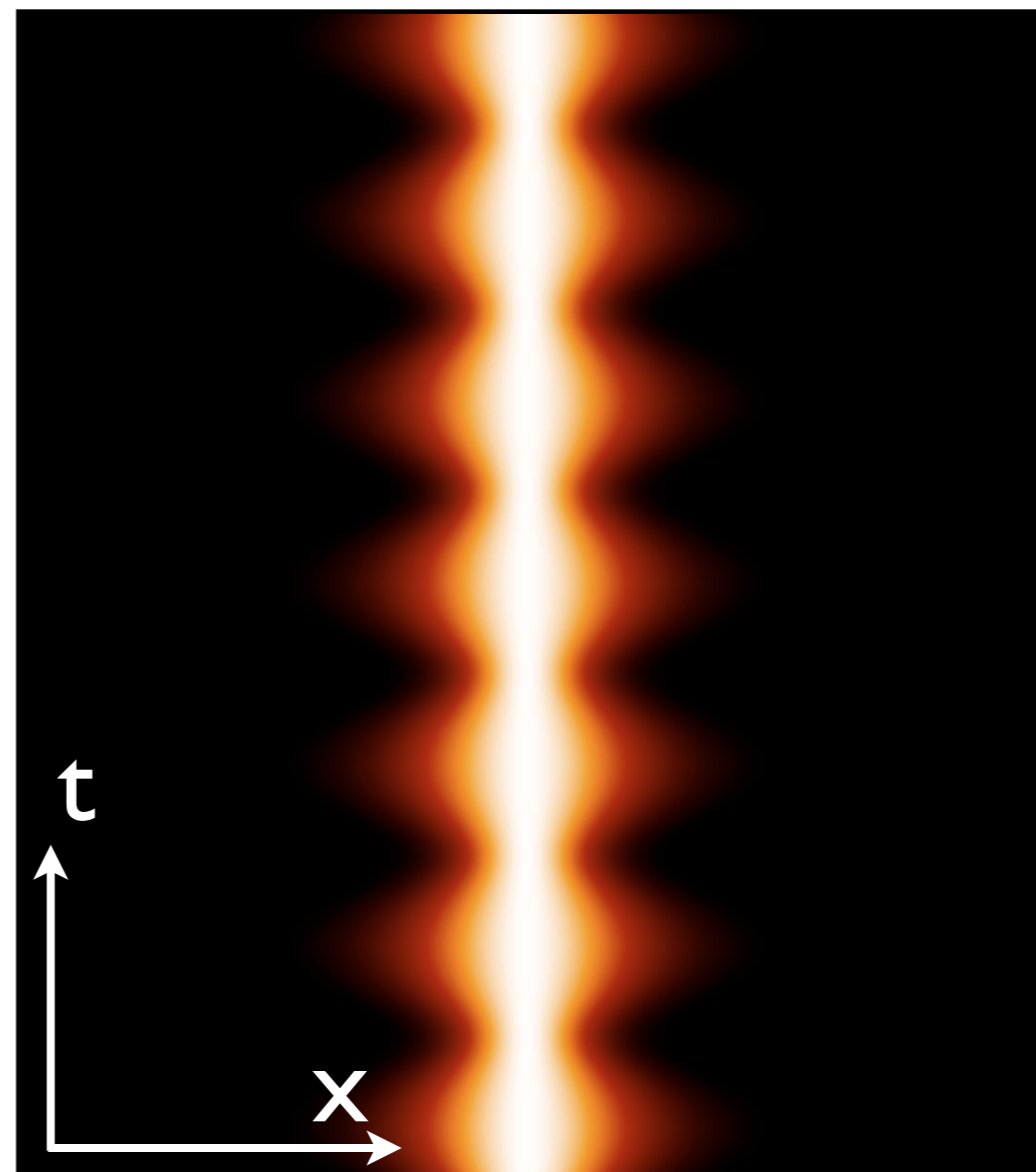
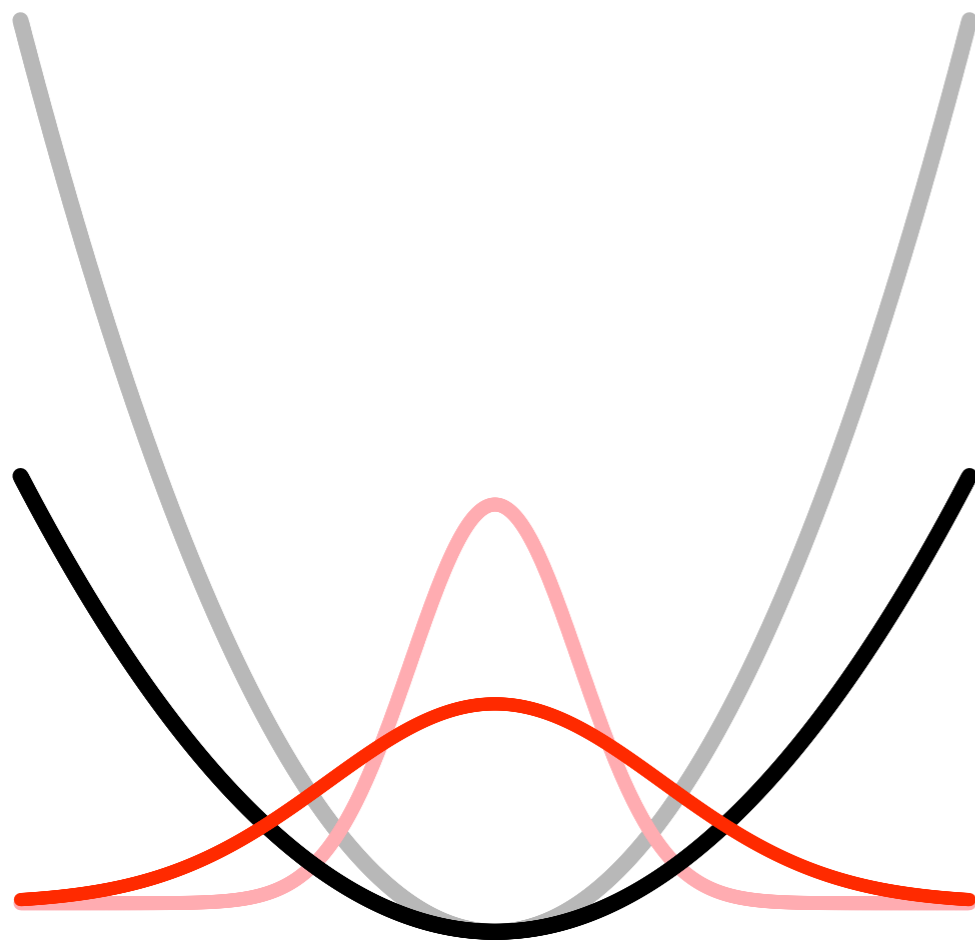
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# Squeezed states

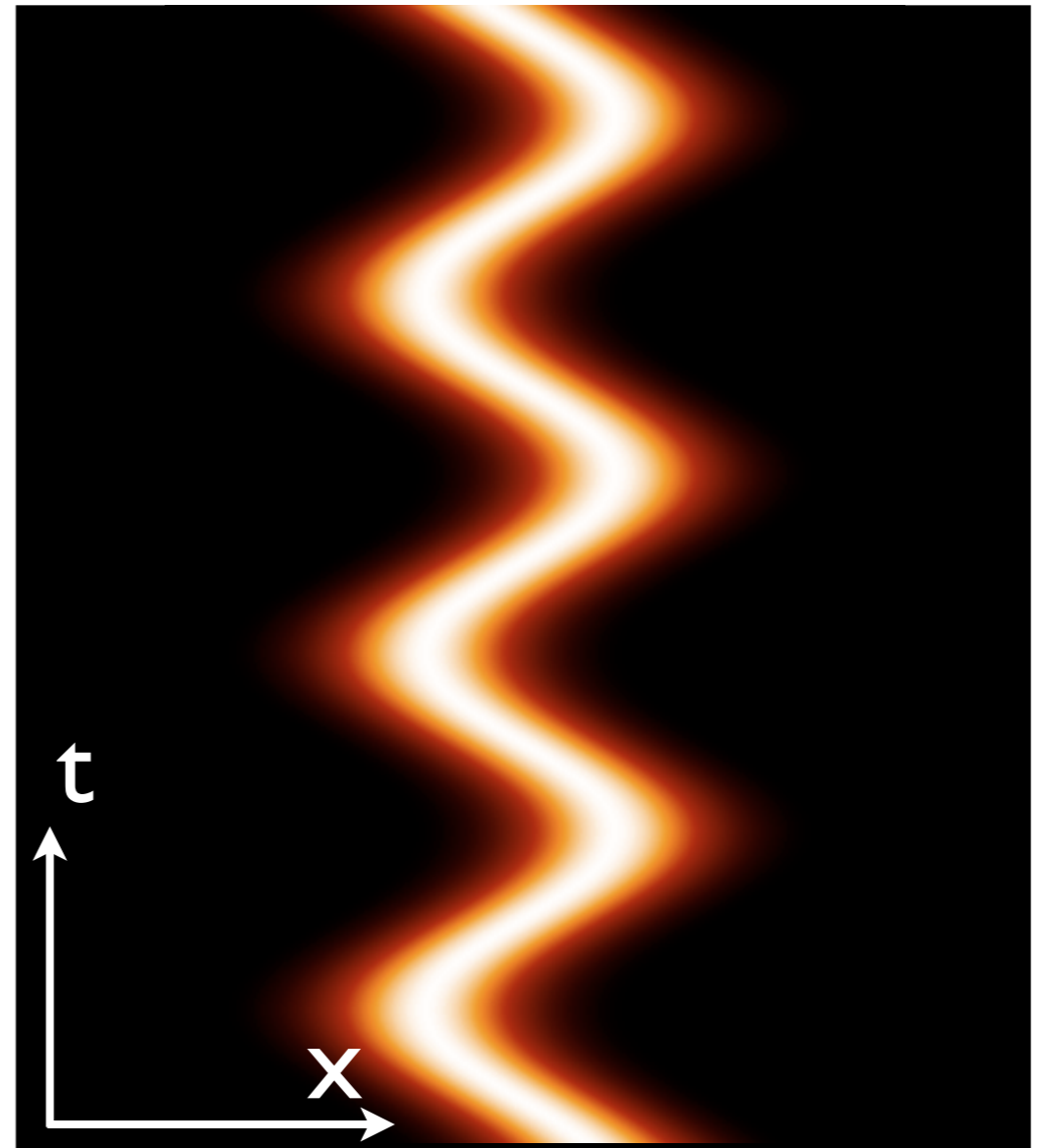
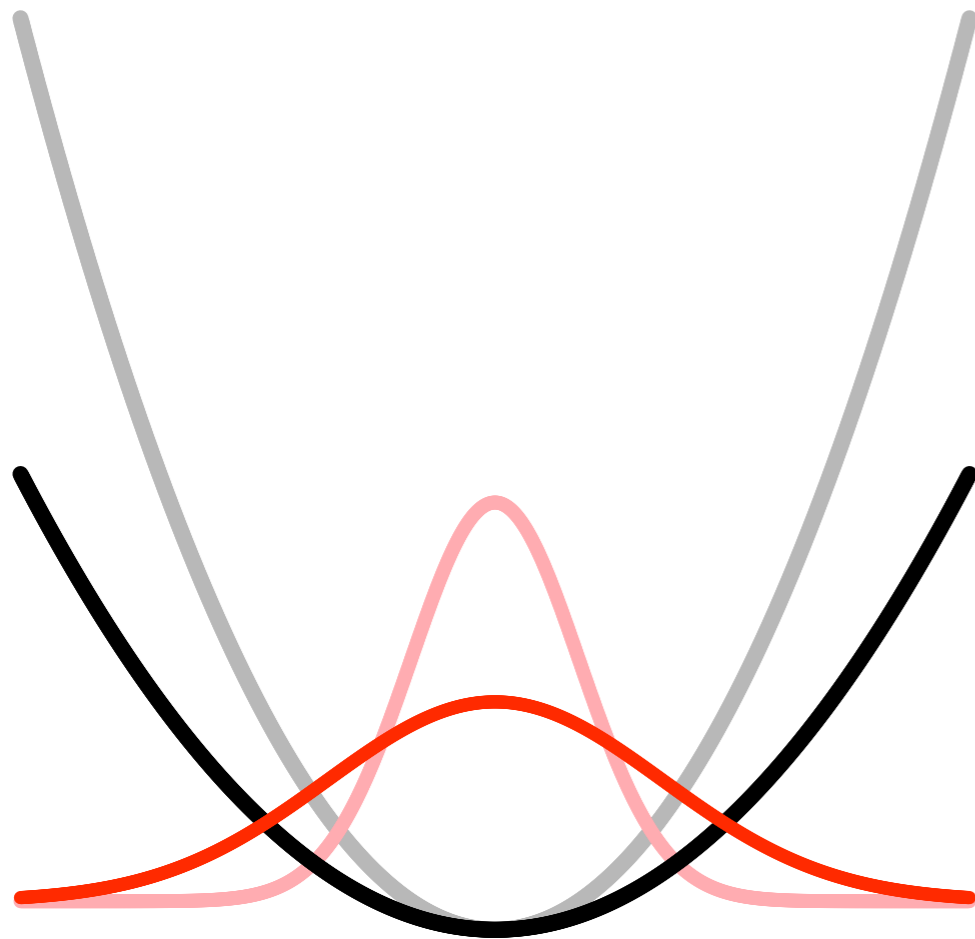
Squeezing the mechanical oscillator state



Periodic modulation of spring constant:  $\delta\omega_M^2(t) \propto \cos(2\omega_M t)$   
Parametric amplification

# Squeezed states

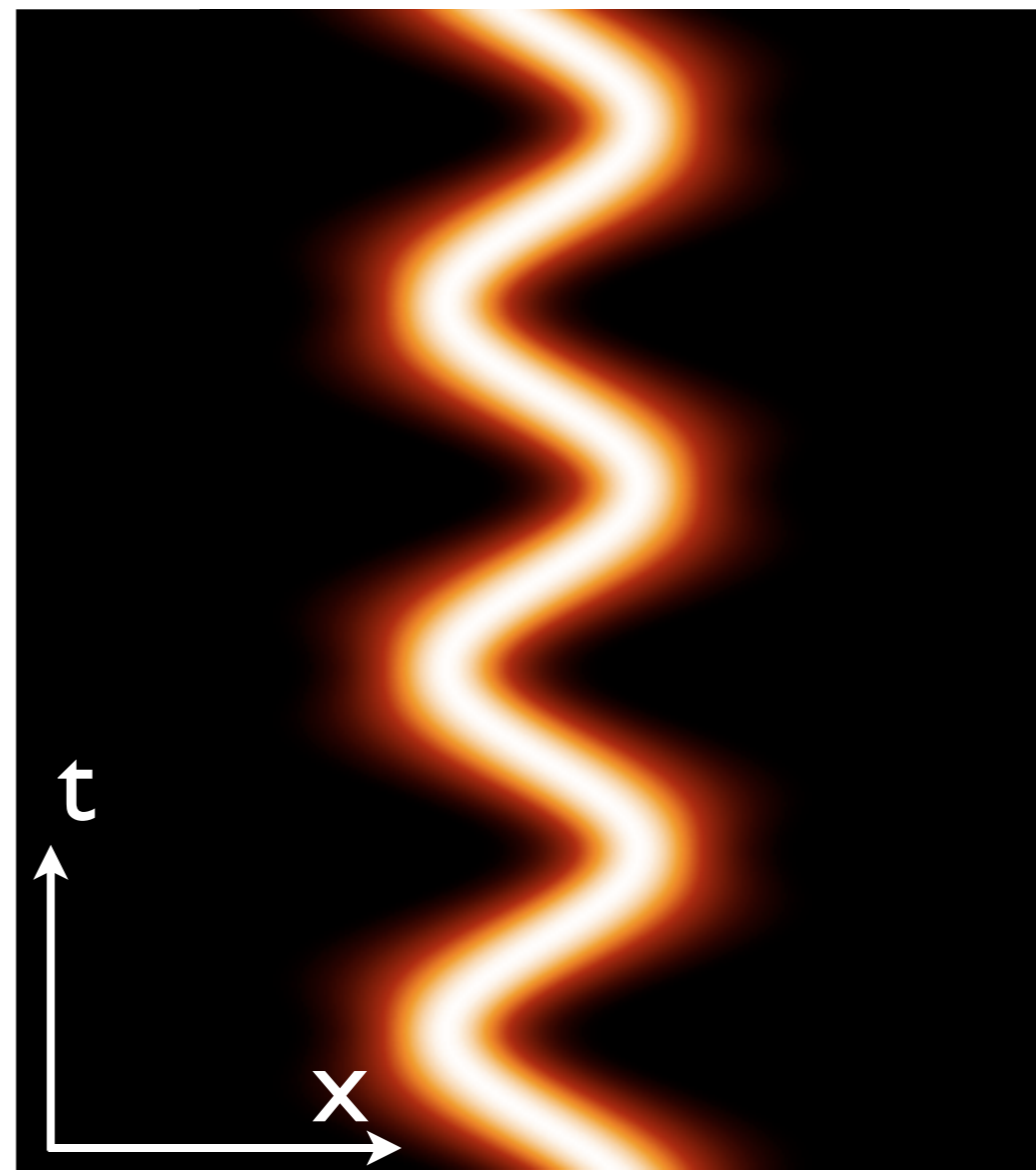
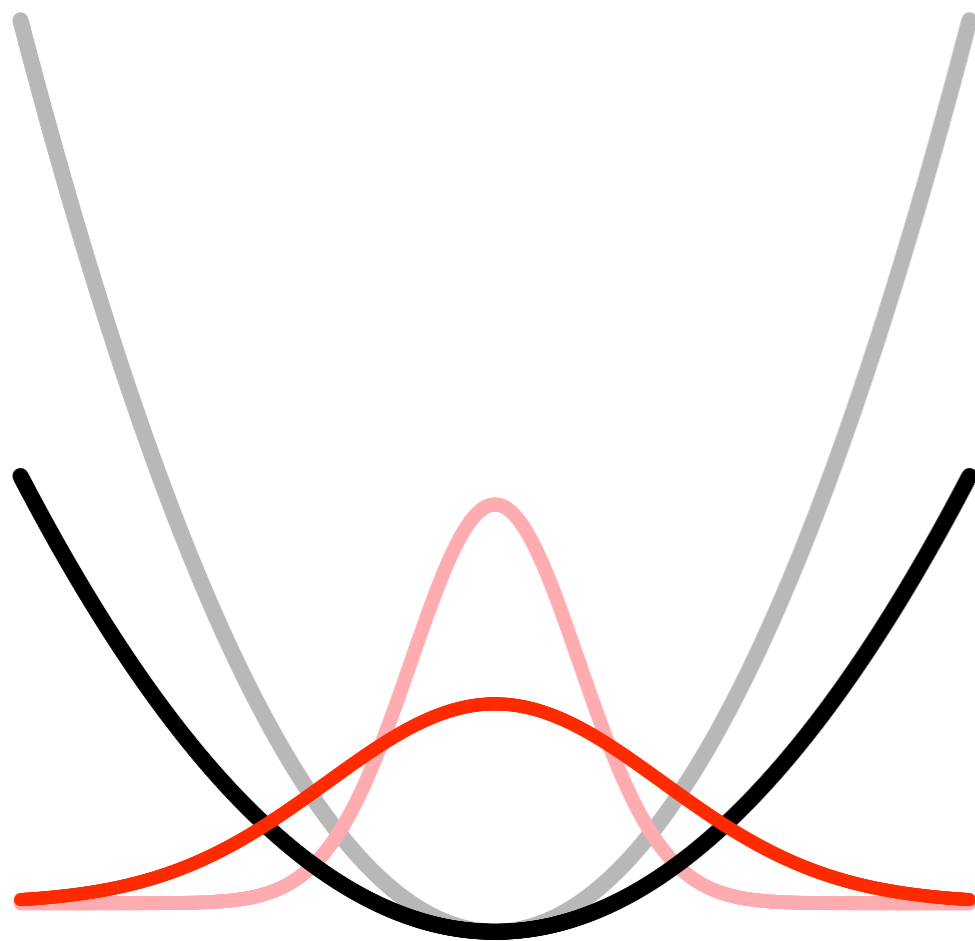
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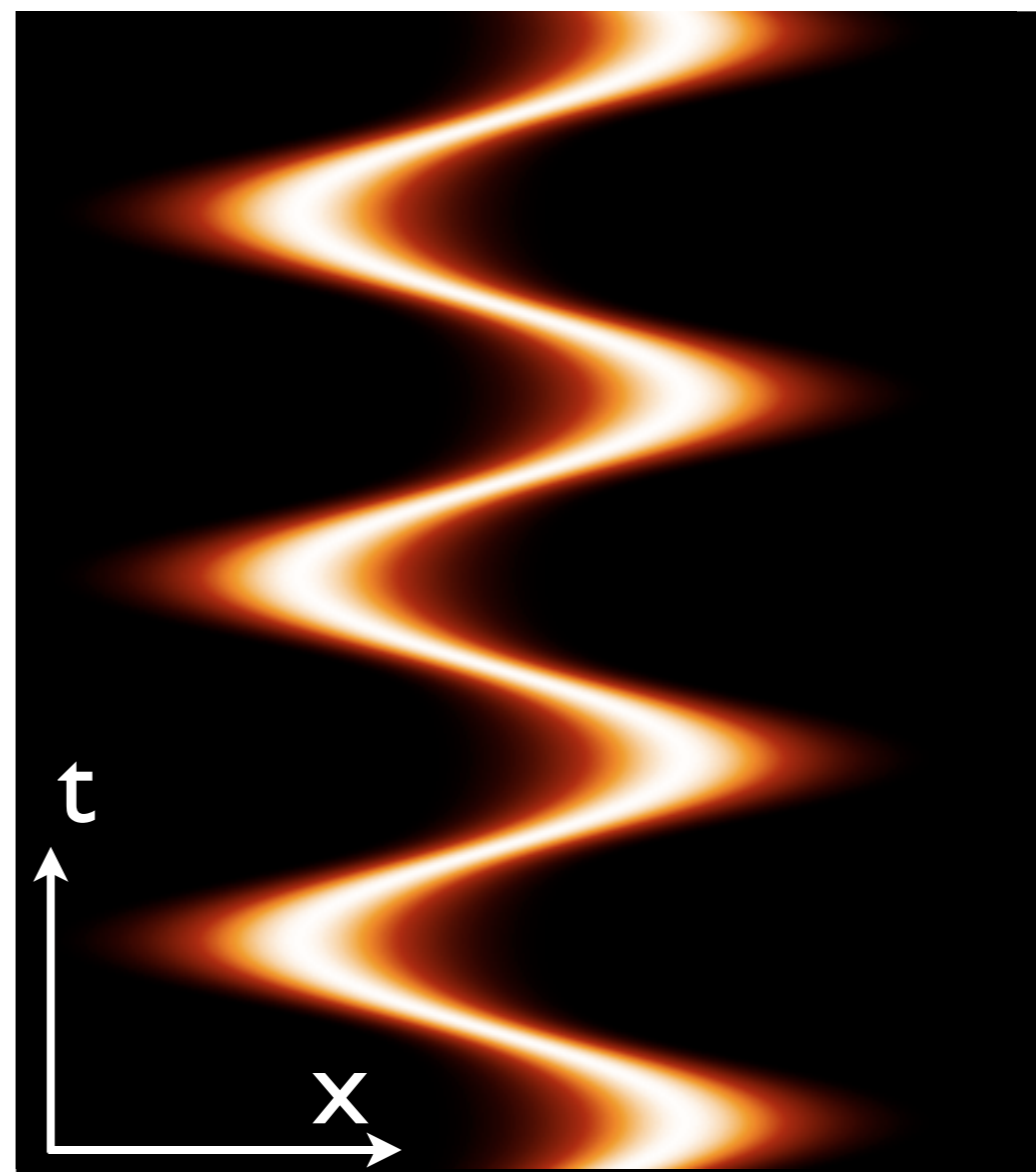
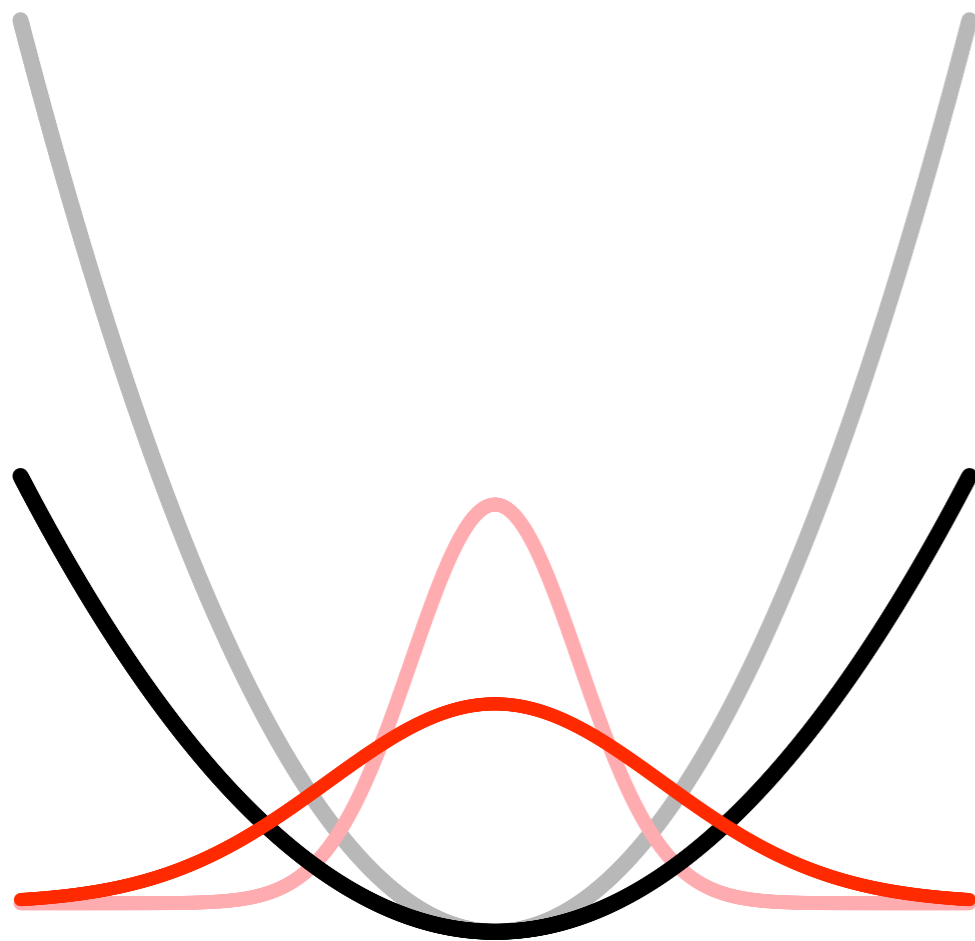
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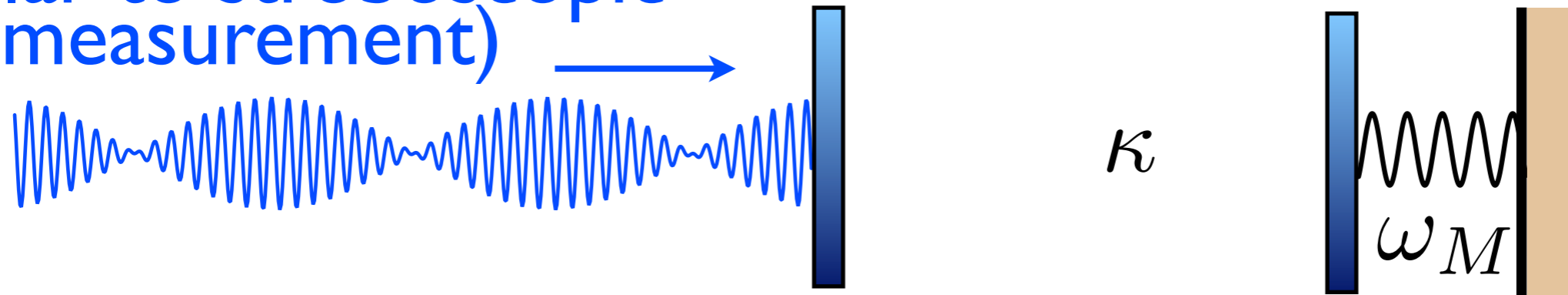


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Parametric amplification

# Measuring quadratures ("beating the SQL")

Clerk, Marquardt, Jacobs; NJP **10**, 095010 (2008)

amplitude-modulated input field  
(similar to stroboscopic  
measurement)



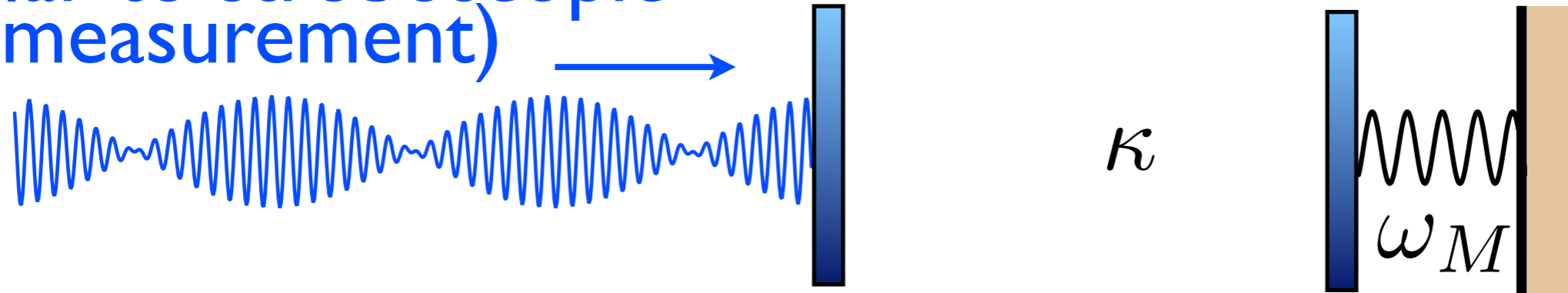
measure only one quadrature, back-action noise affects  
only the other one...need:  $\kappa \ll \omega_M$



# Measuring quadratures ("beating the SQL")

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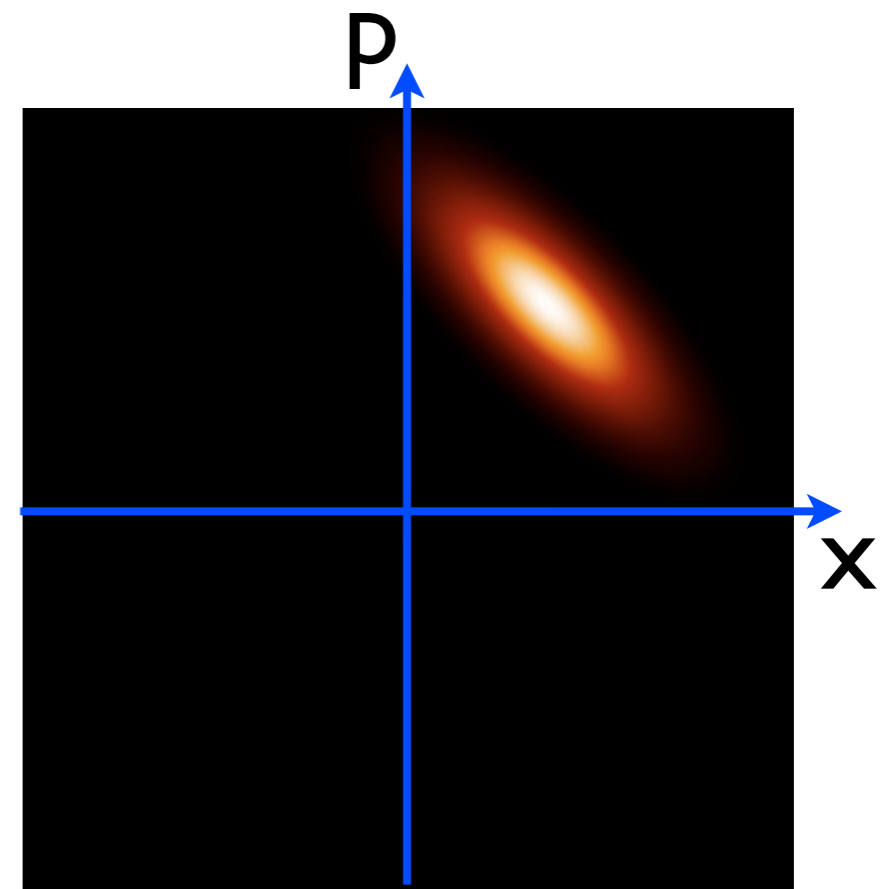


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**reconstruct  
mechanical  
Wigner density**

(quantum state tomography)

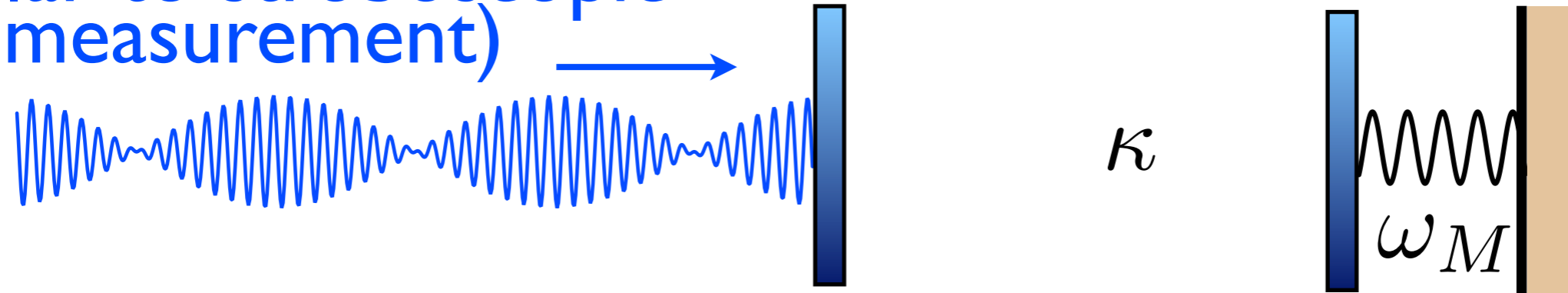
$$W(x, p) \propto \int dy e^{ipy/\hbar} \rho(x - y/2, x + y/2)$$



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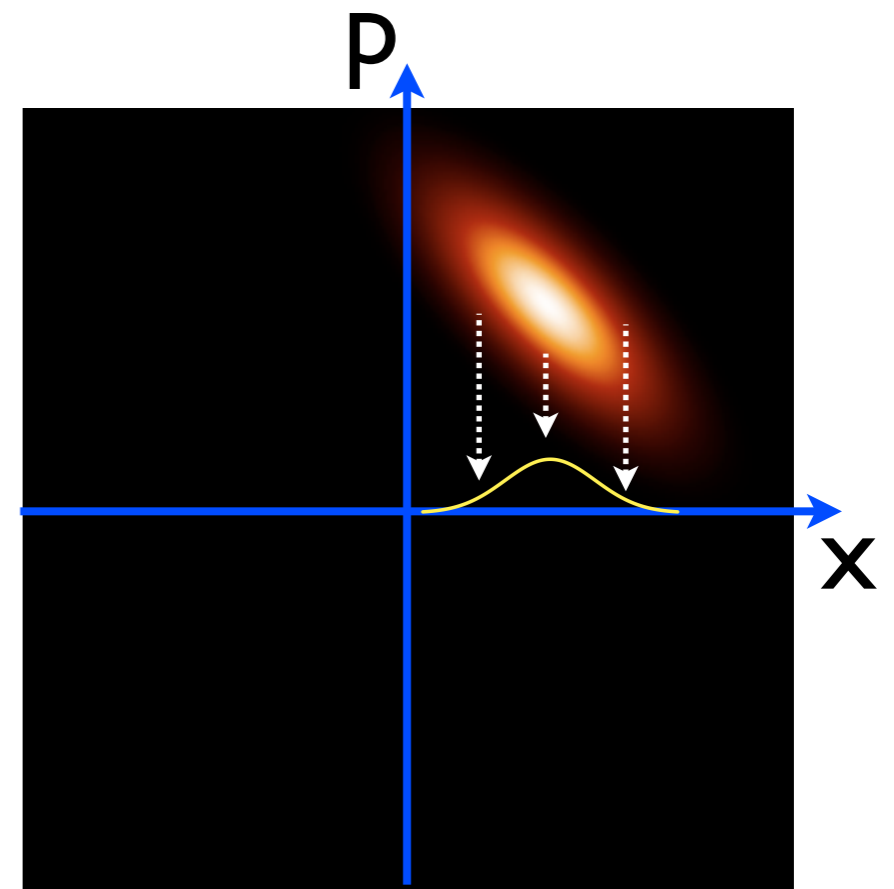


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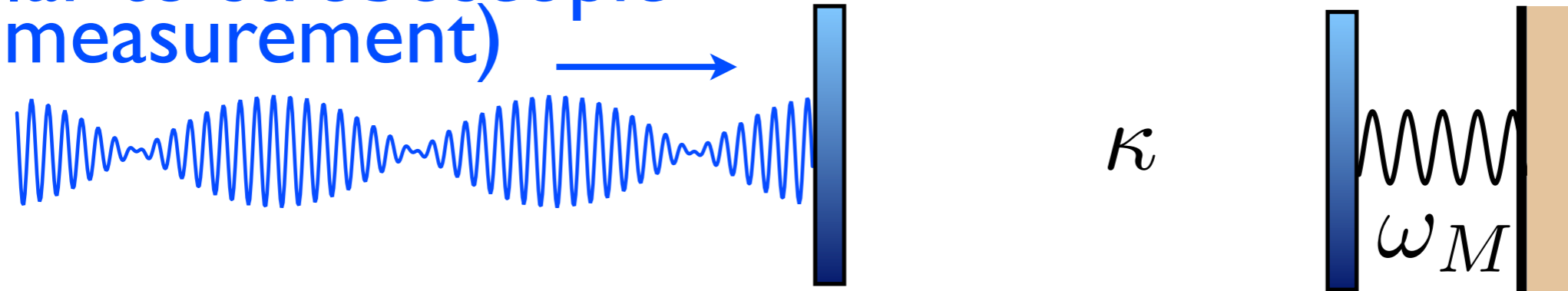
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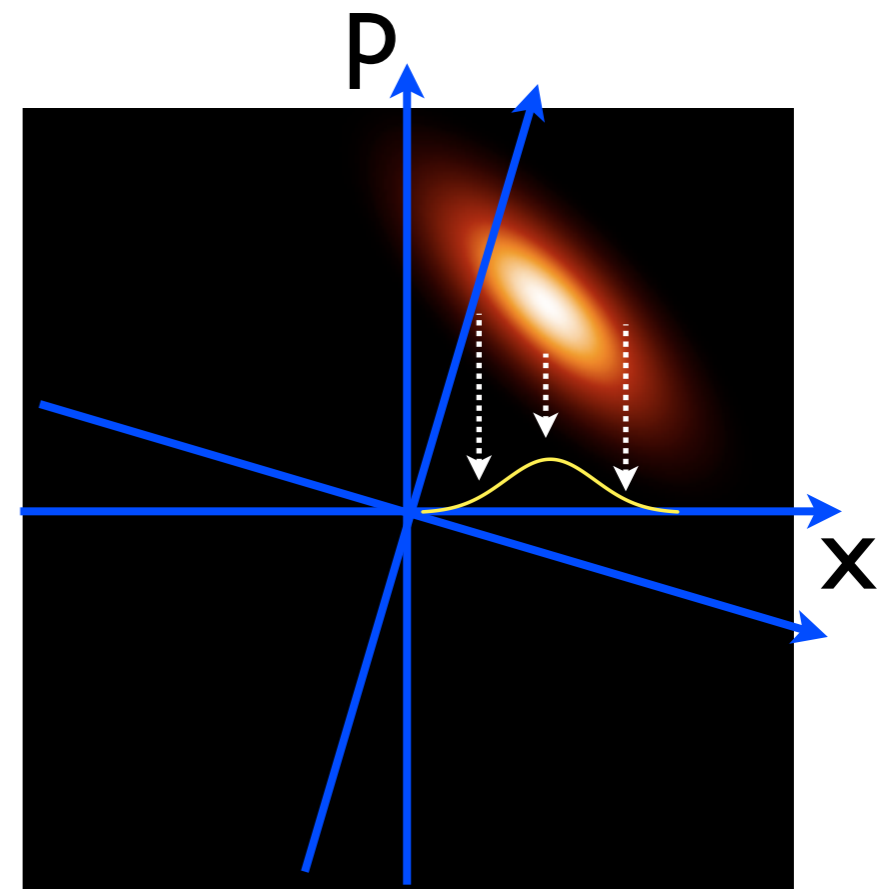


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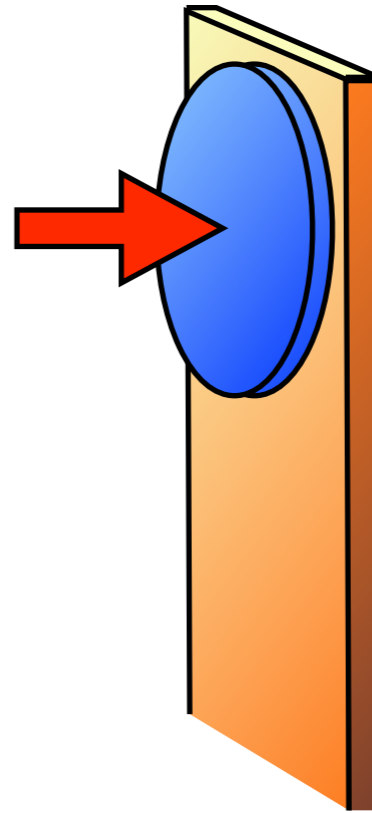
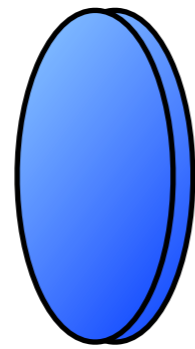
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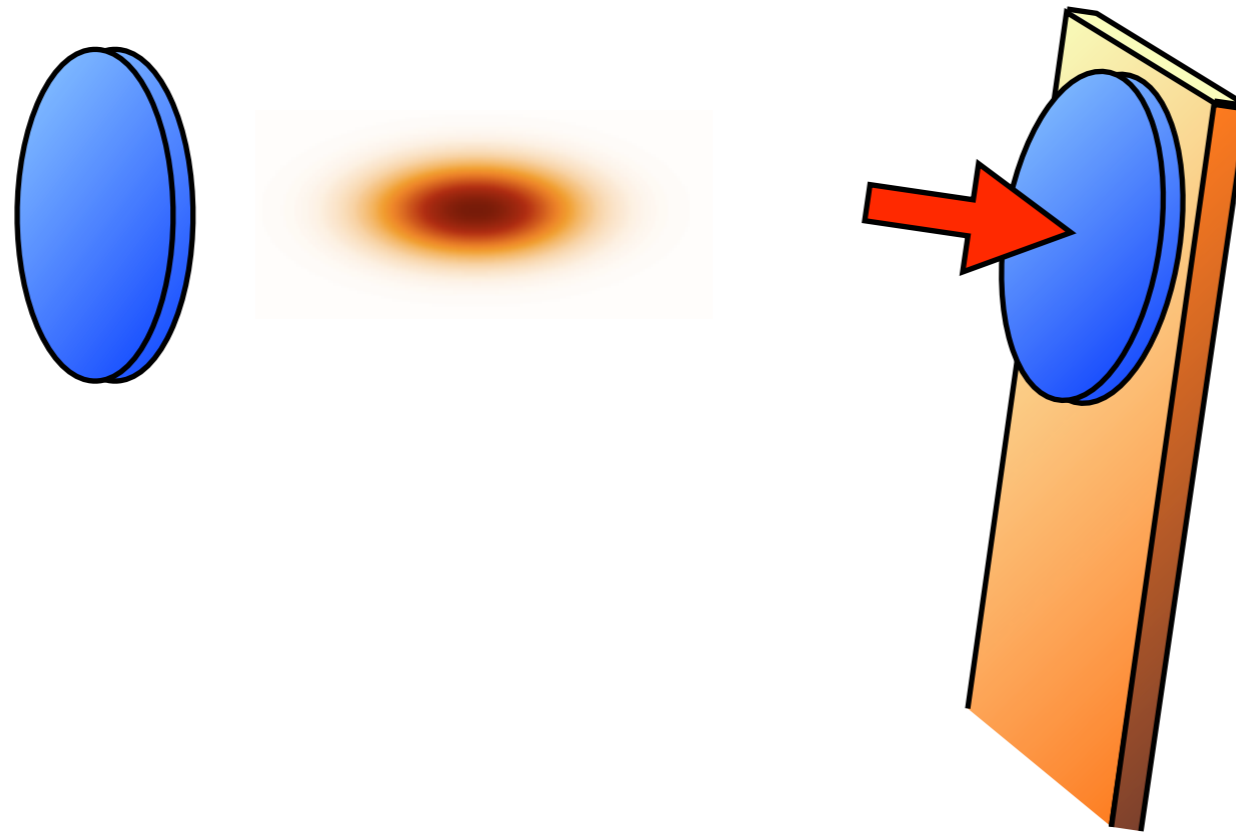
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# Optomechanical entanglement

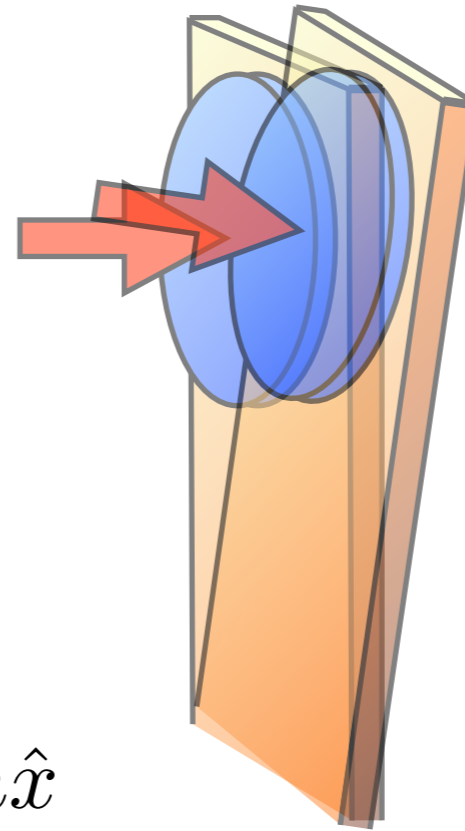
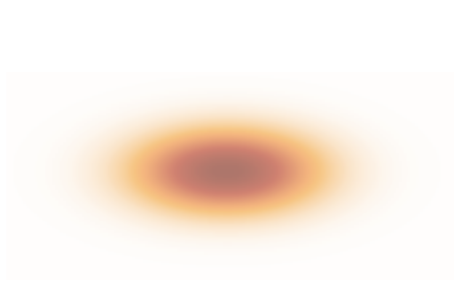
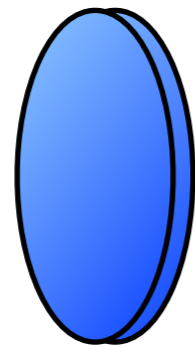


# Optomechanical entanglement



Bose, Jacobs, Knight 1997; Mancini et al. 1997

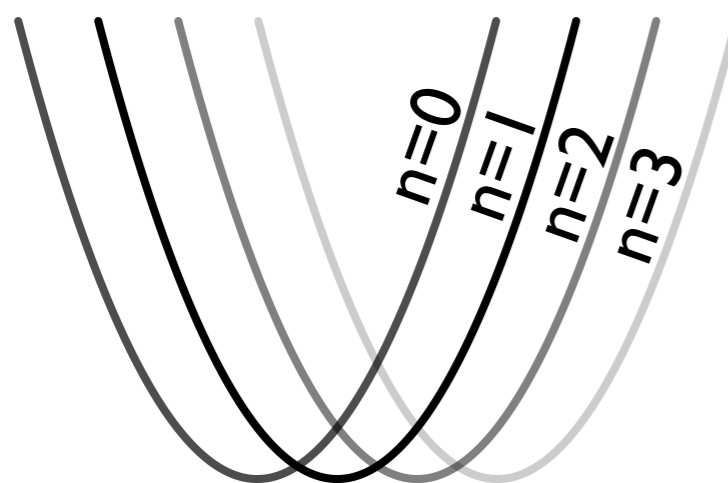
# Optomechanical entanglement



$$\hat{H} = \dots + g_0 \hat{n} \hat{x}$$

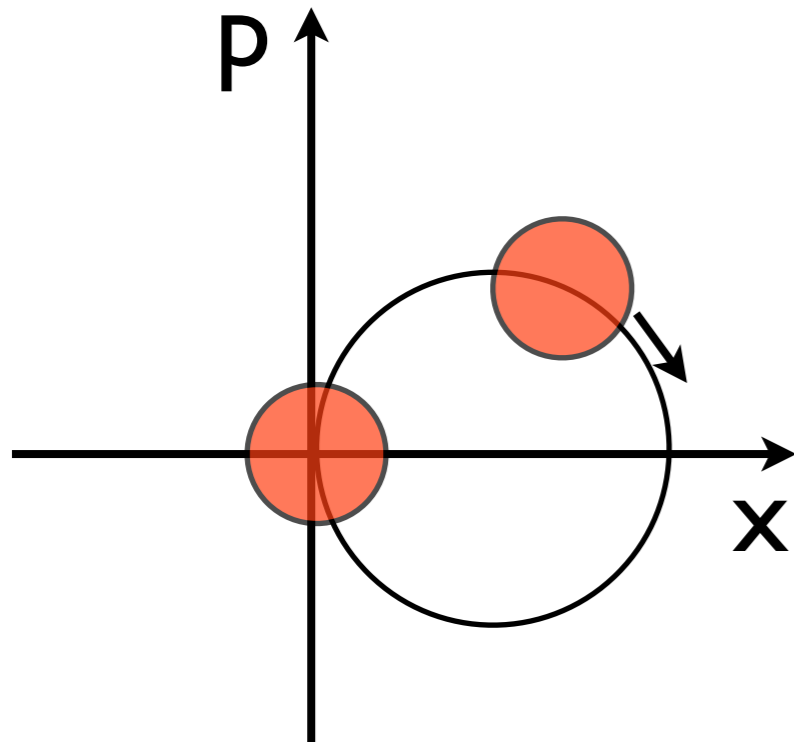
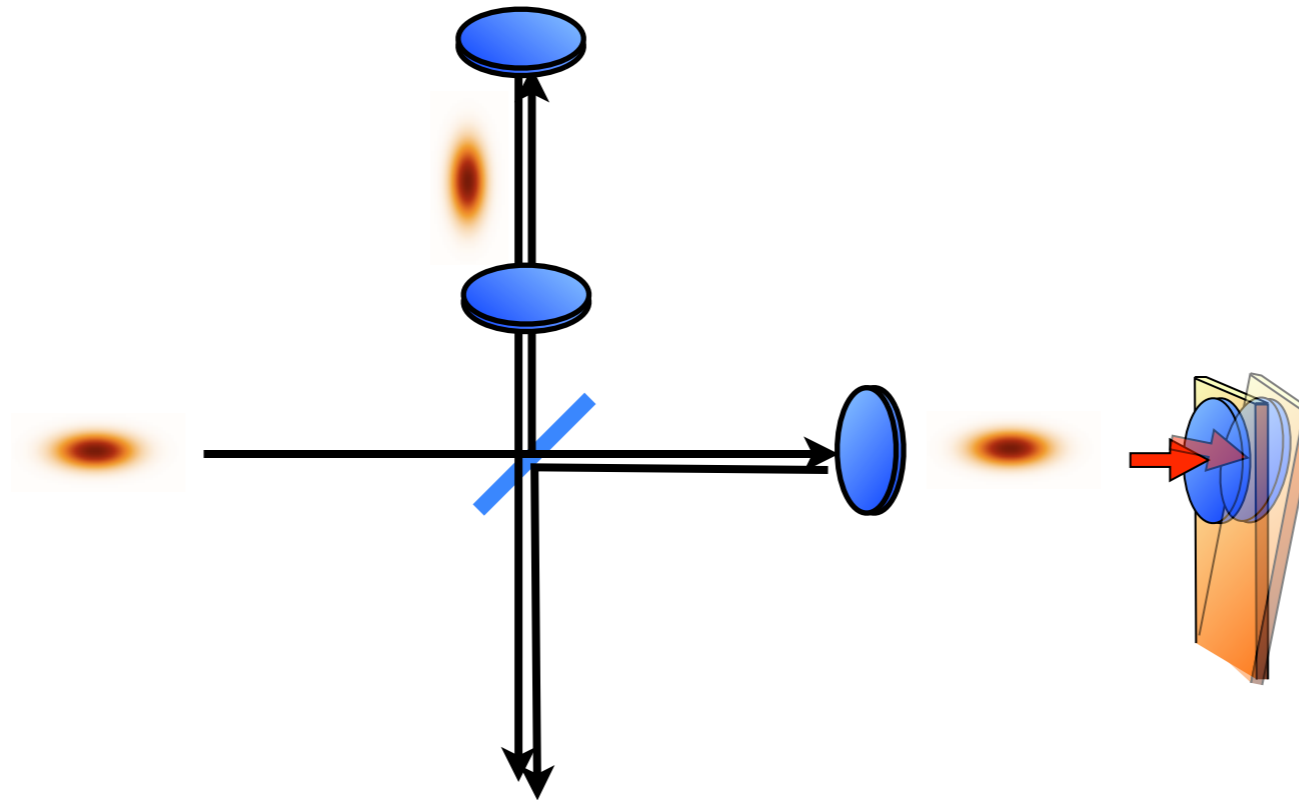
$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n e^{-i\varphi_n(t)} |n\rangle \otimes |\alpha = \alpha_n(t)\rangle$$

coherent mechanical state



entangled state  
(light field/mechanics)

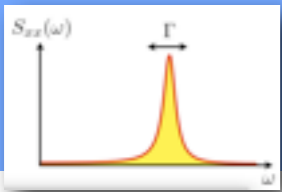
# Proposed optomechanical which-path experiment and quantum eraser



Recover photon coherence if interaction time equals a multiple of the mechanical period!

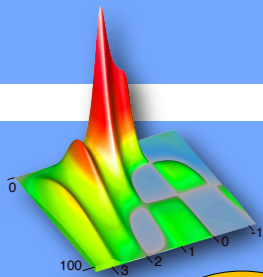
cf. Haroche experiments in 90s

# Optomechanics (Outline)

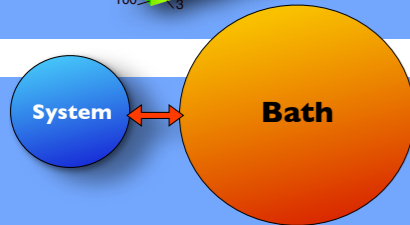


Displacement detection

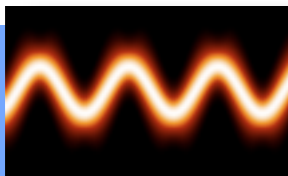
Linear optomechanics



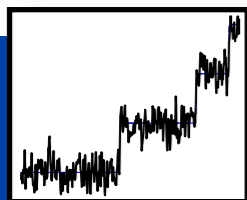
Nonlinear dynamics



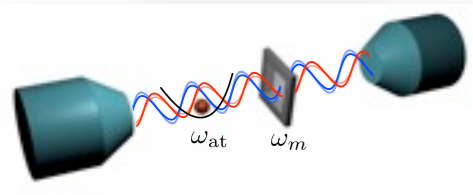
Quantum theory of cooling



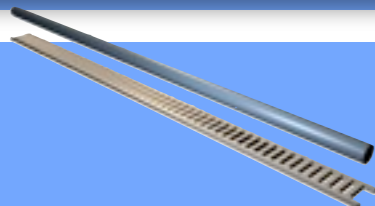
Interesting quantum states



Towards Fock state detection



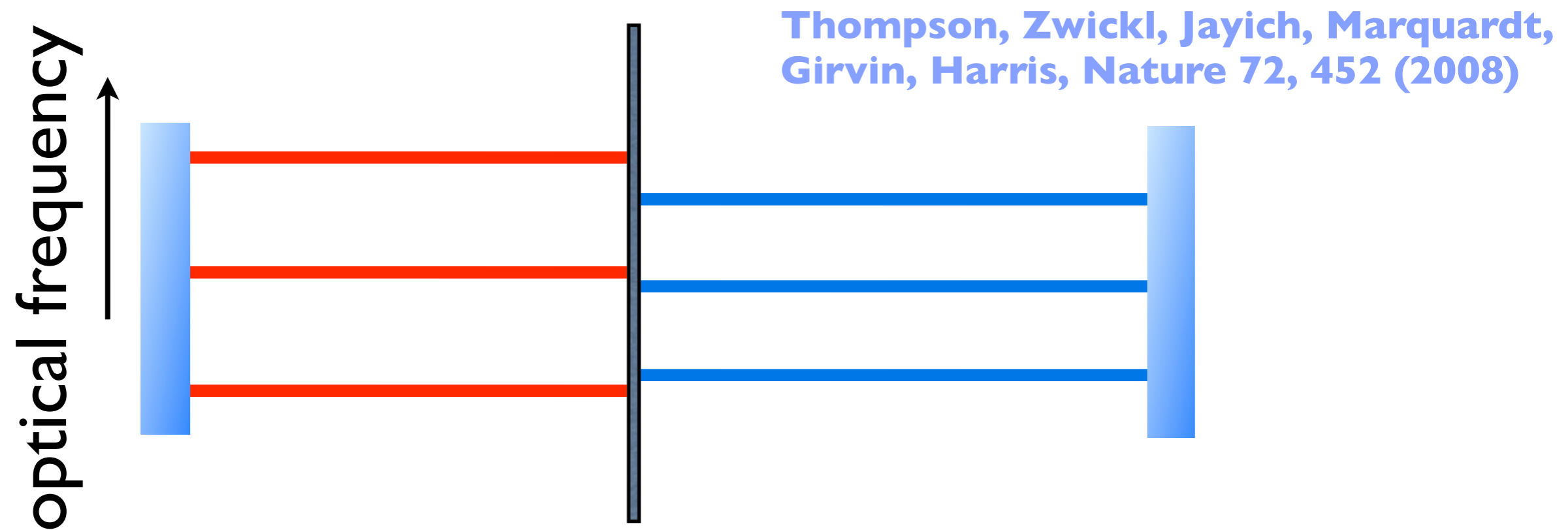
Hybrid systems: coupling to atoms



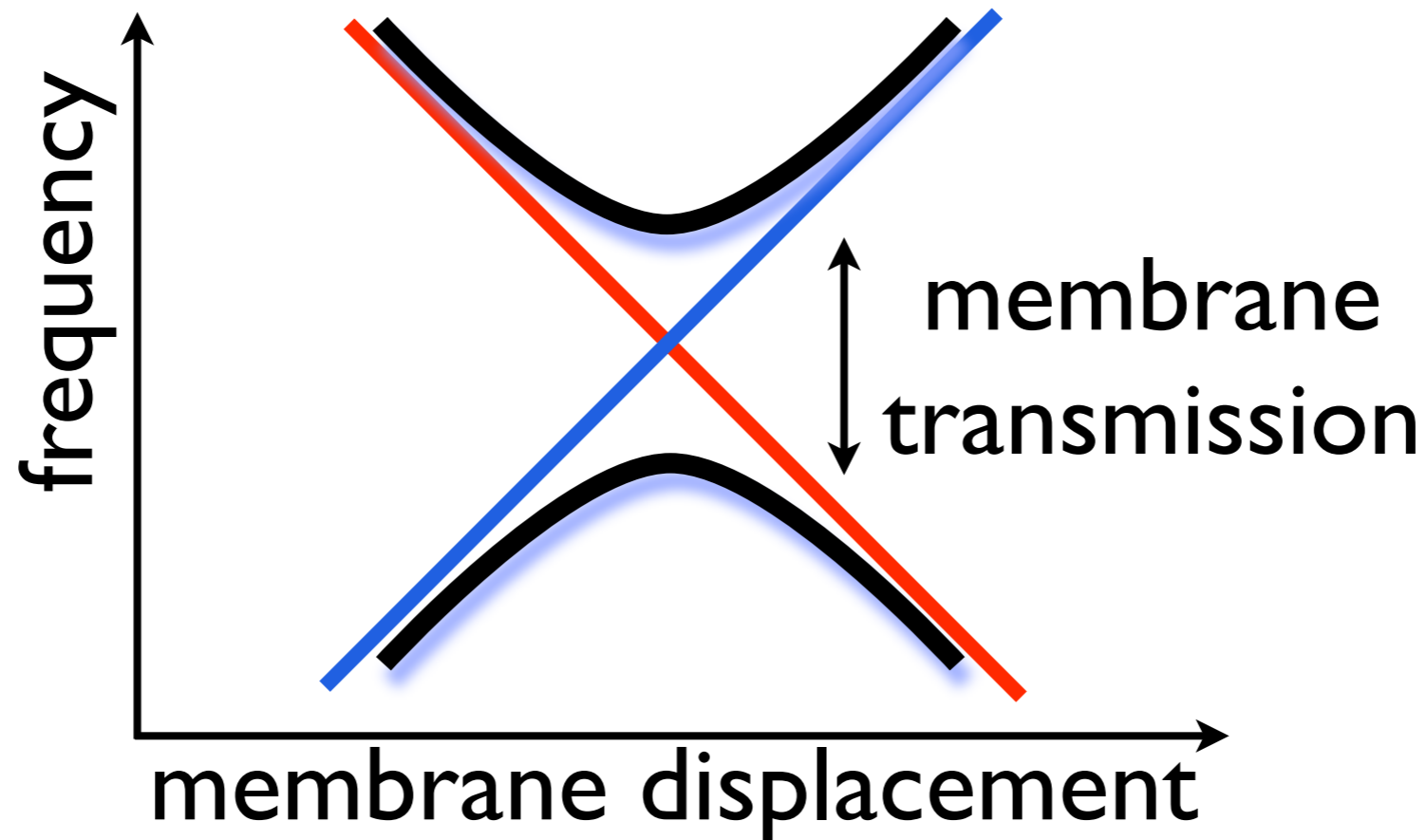
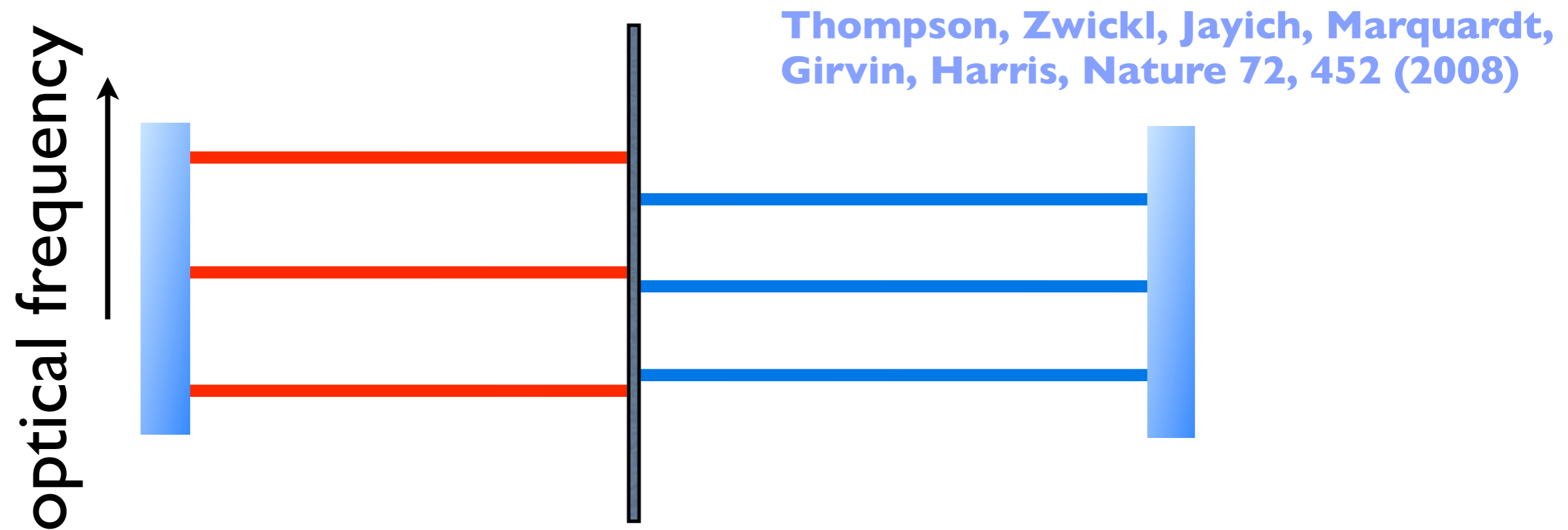
Optomechanical crystals & arrays



# “Membrane in the middle” setup



# “Membrane in the middle” setup



# Experiment (Harris group, Yale)

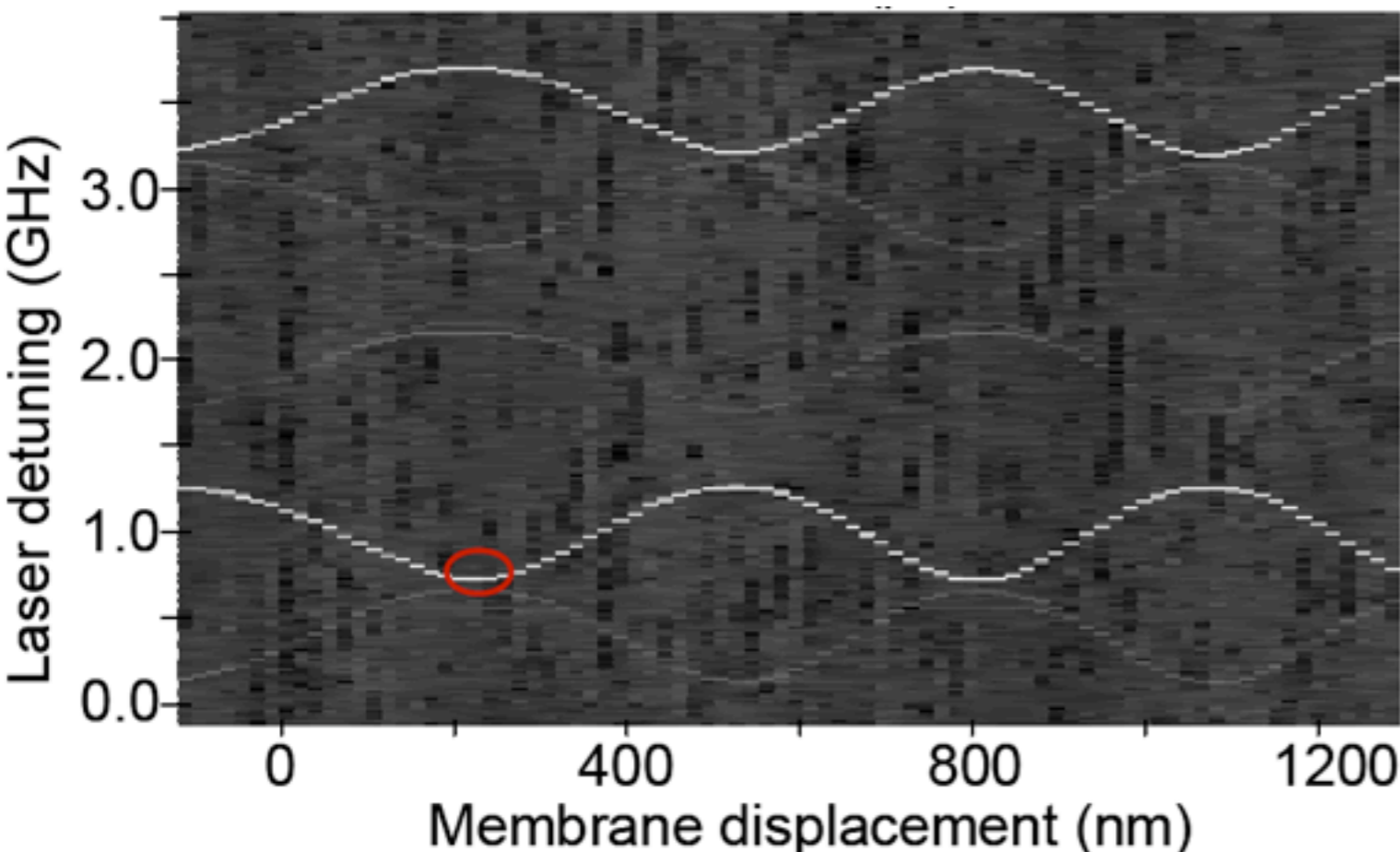


Mechanical frequency:

$$\omega_M = 2\pi \cdot 134 \text{ kHz}$$

Mechanical quality factor:

$$Q = 10^6 \div 10^7$$



Optomechanical cooling  
from **300K** to **7mK**

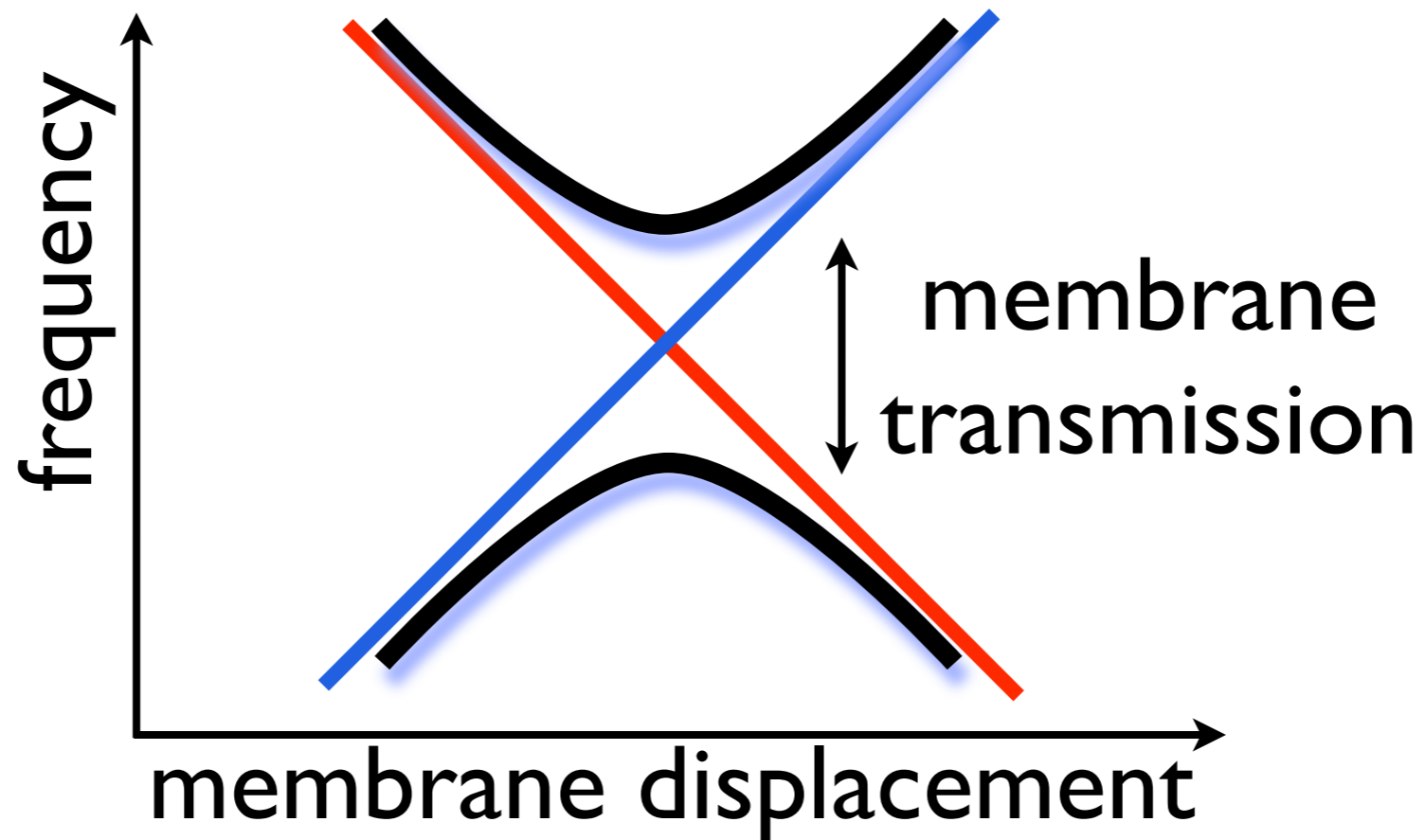
Thompson, Zwickl, Jayich, Marquardt,  
Girvin, Harris, *Nature* 72, 452 (2008)

# Towards Fock state detection of a macroscopic object

Detection of displacement  $x$ : *not* what we need!

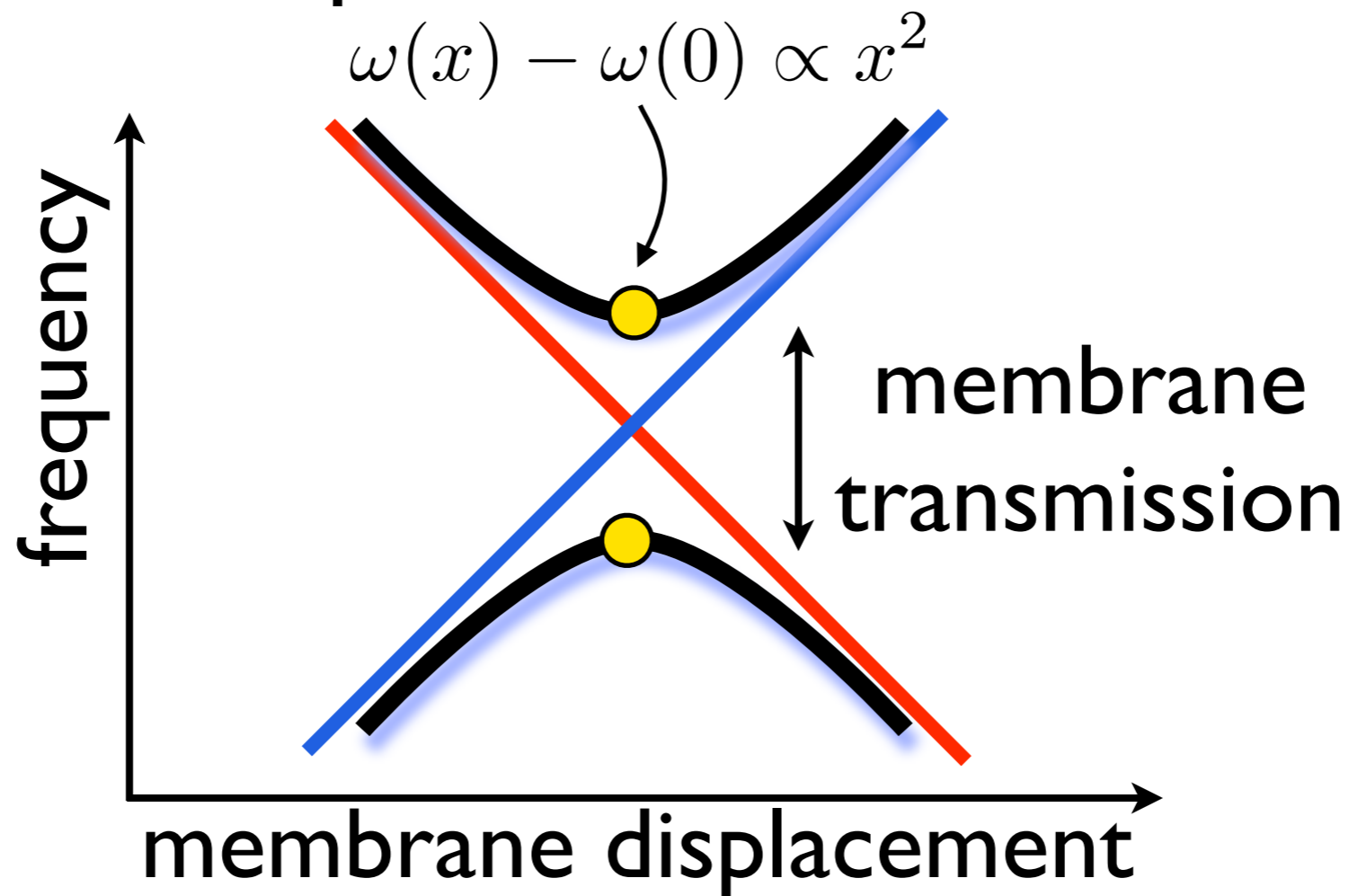
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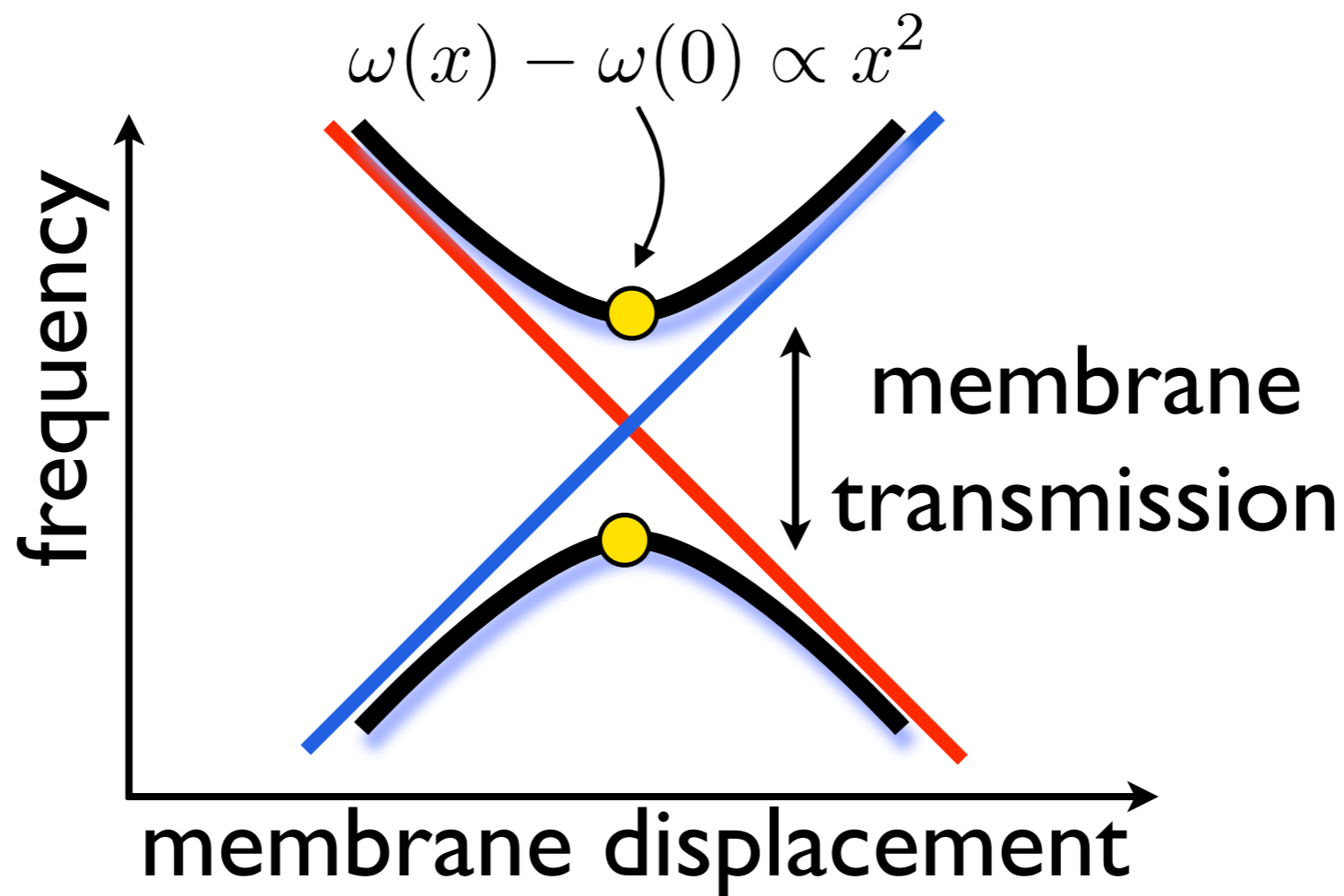


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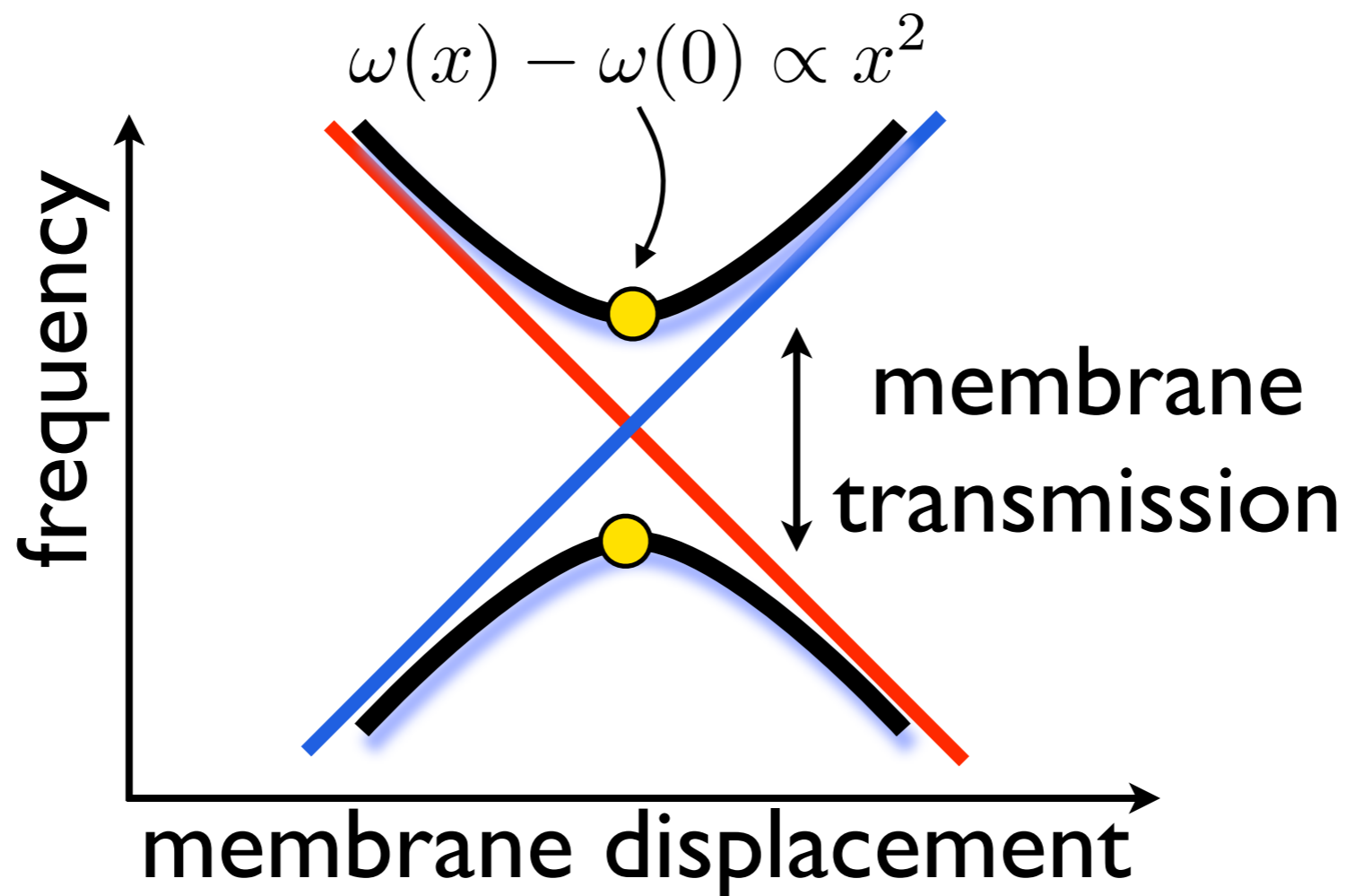
# Towards Fock state detection of a macroscopic object



phase shift of measurement beam:

$$\hat{\theta} \propto \hat{x}(t)^2 \propto (\hat{b}(t) + \hat{b}^\dagger(t))^2 = \hat{b}^2 e^{-i2\omega_M t} + \hat{b}^{\dagger 2} e^{+i2\omega_M t} + \hat{b}^\dagger \hat{b} + \hat{b} \hat{b}^\dagger$$

# Towards Fock state detection of a macroscopic object



phase shift of measurement beam:

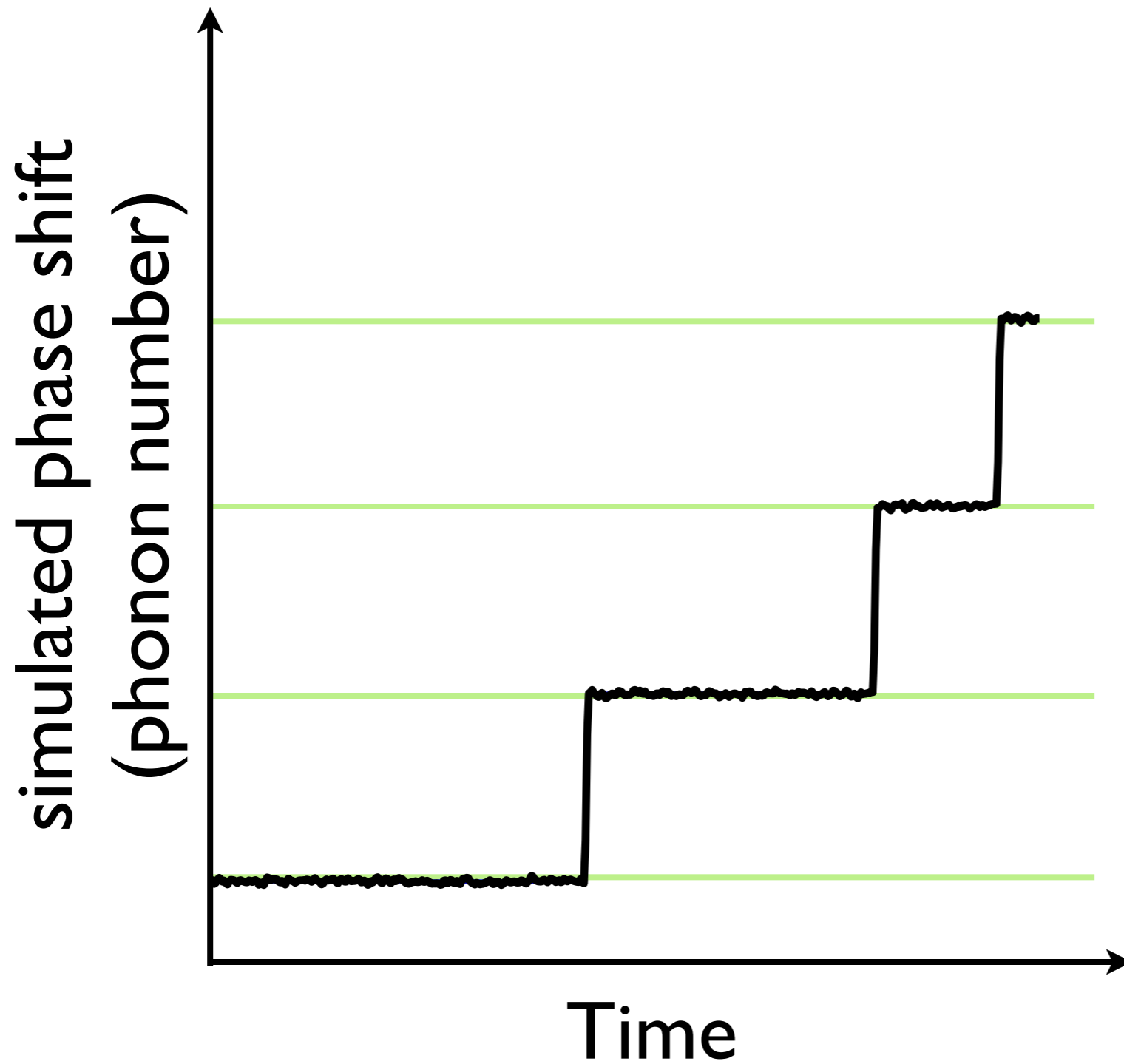
$$\overline{\hat{\theta}} \propto \overline{\hat{x}(t)^2} \propto \overline{(\hat{b}(t) + \hat{b}^\dagger(t))^2} \approx \underline{2\hat{b}^\dagger\hat{b}} + 1$$

(Time-average over  
cavity ring-down time)

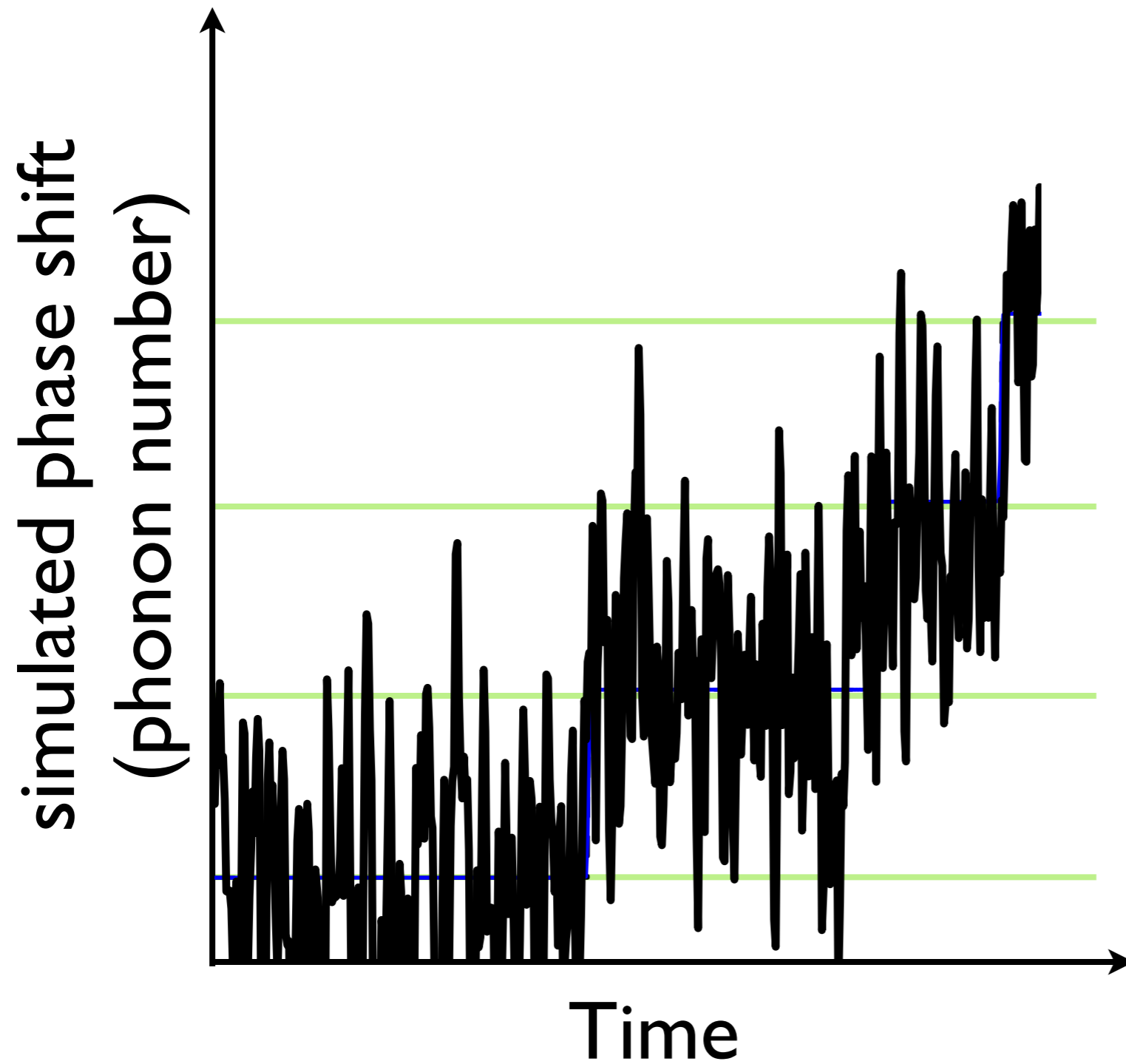
**QND measurement  
of phonon number!**



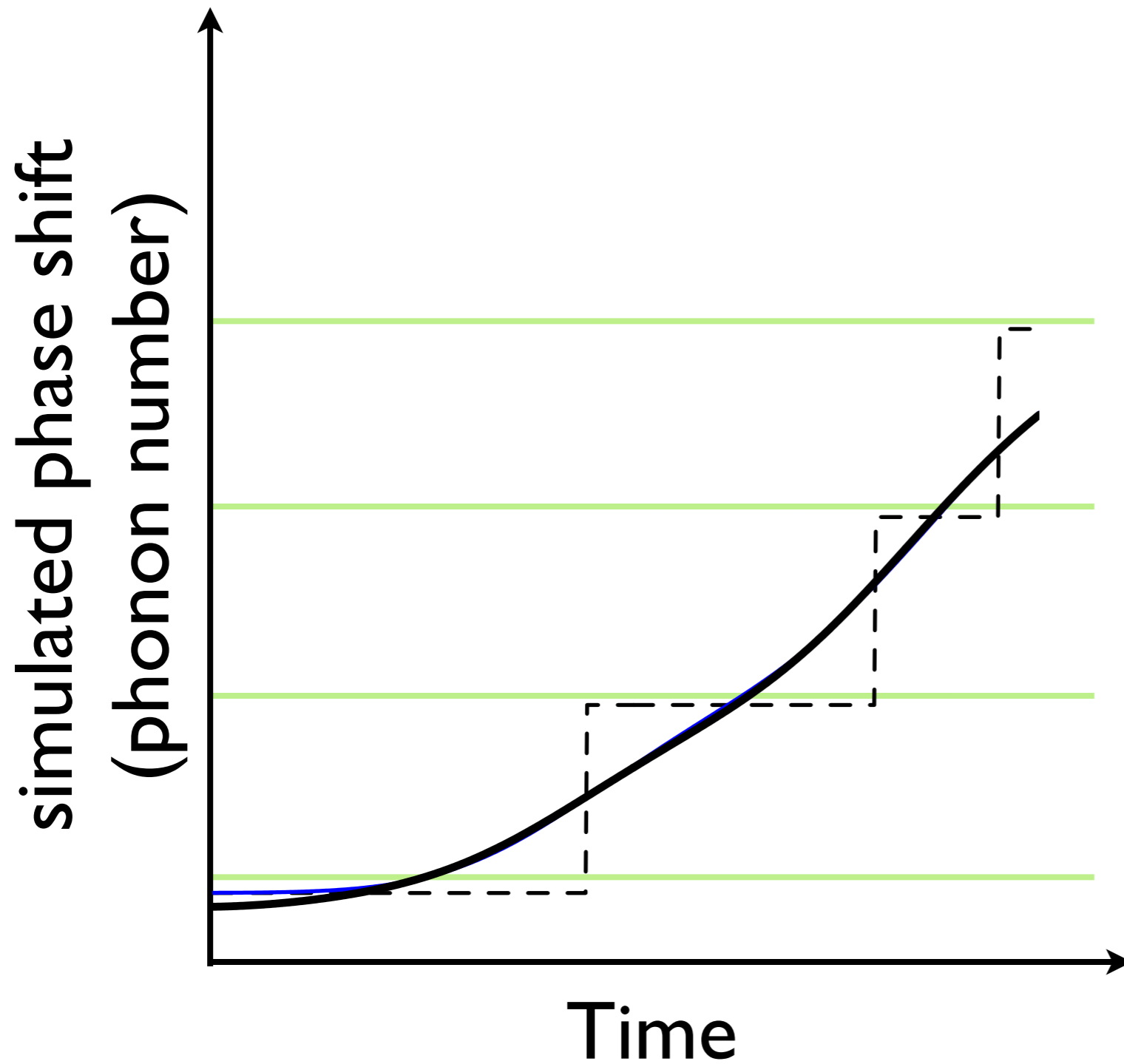
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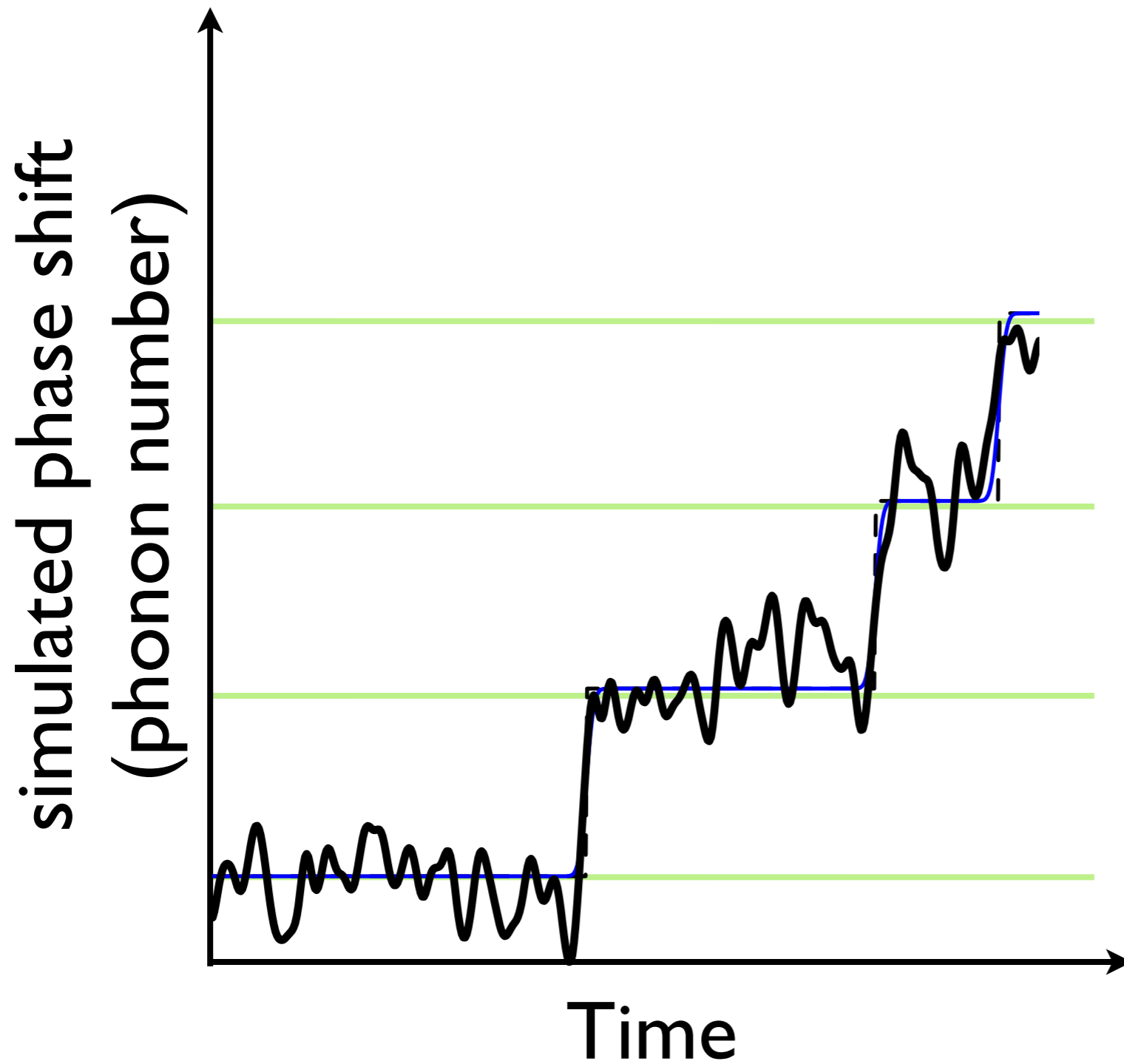
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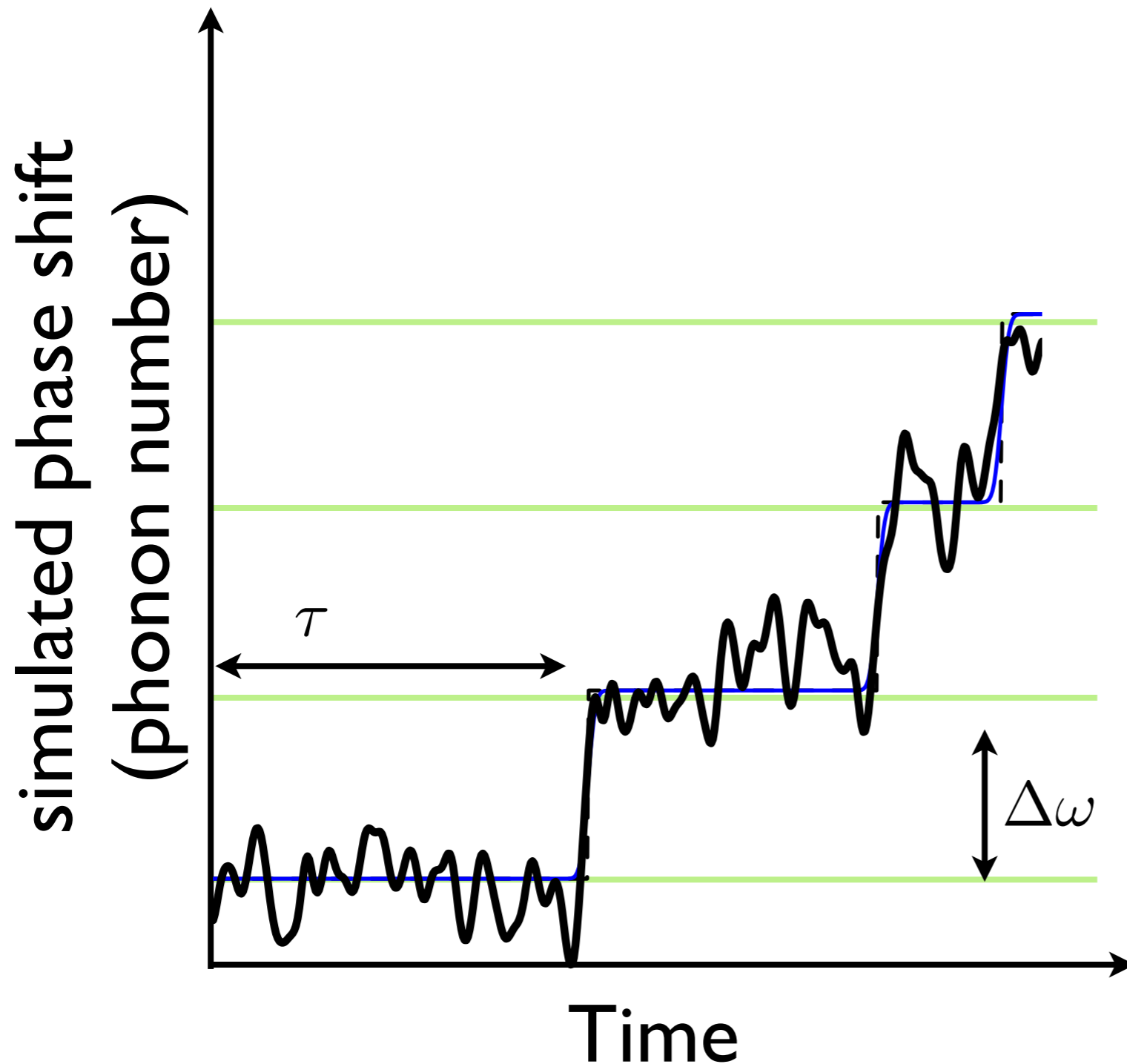
# Towards Fock state detection of a macroscopic object



# Towards Fock state detection of a macroscopic object



# Towards Fock state detection of a macroscopic object



**Signal-to-noise**

**ratio:** 
$$\frac{\tau \Delta\omega^2}{S_\omega}$$

Optical freq. shift  
per phonon:

$$\Delta\omega = x_{\text{ZPF}}^2 \omega''$$

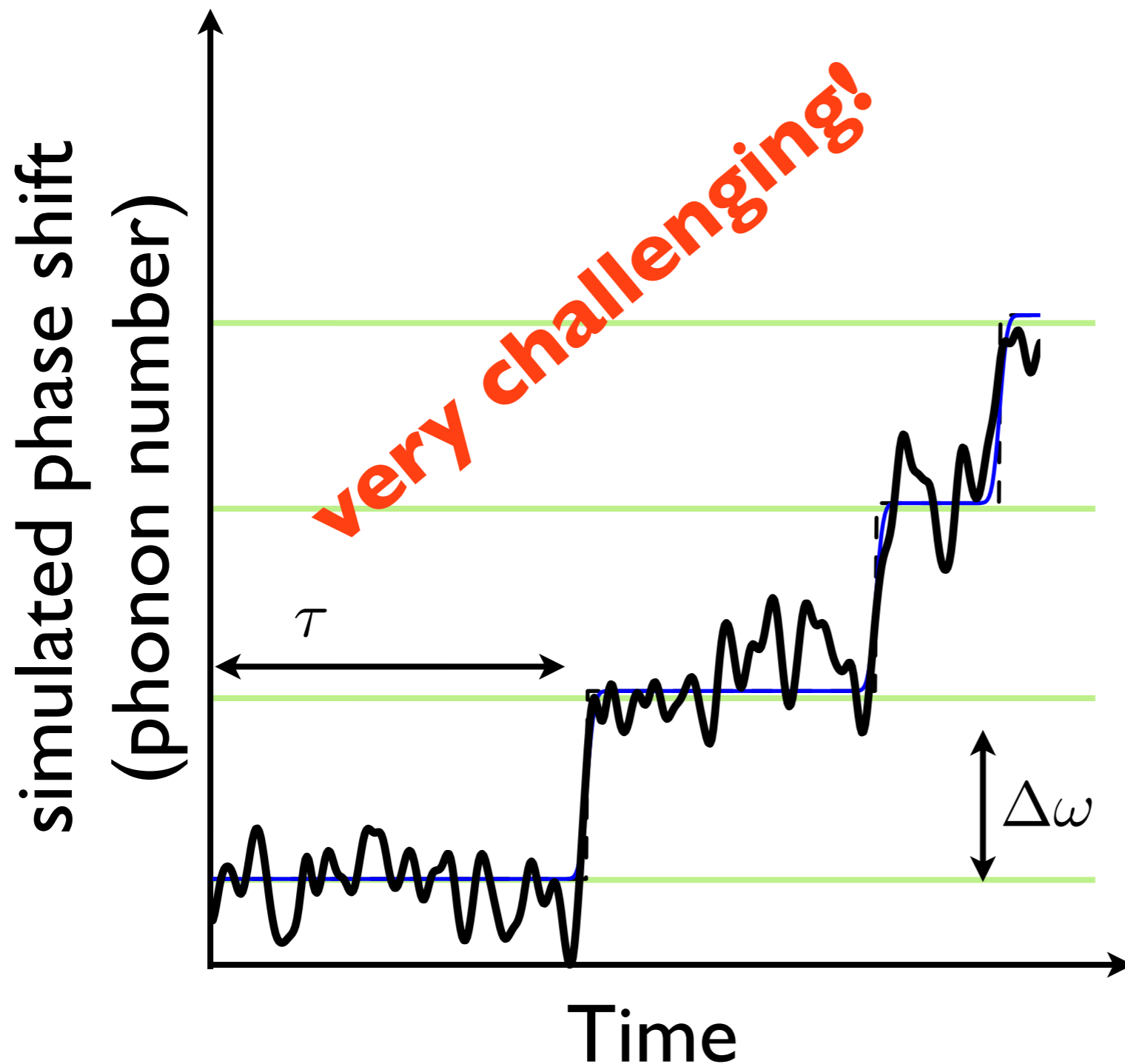
Noise power of  
freq. measurement:

$$S_\omega = \frac{\kappa}{16\bar{n}_{\text{cavity}}}$$

Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\text{thermal}}$$

# Towards Fock state detection of a macroscopic object



**Signal-to-noise**

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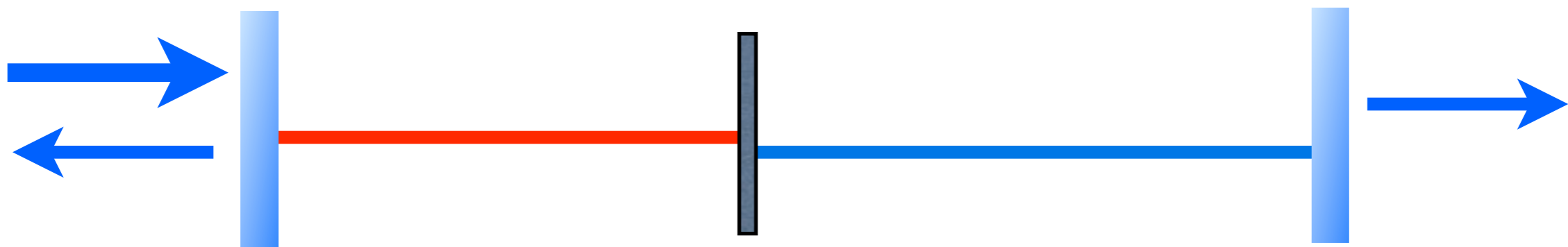
Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\text{thermal}}$$

# Towards Fock state detection of a macroscopic object

Ideal single-sided cavity: Can observe **only** phase of reflected light, i.e.  $x^2$ : good

Two-sided cavity: Can **also** observe transmitted vs. reflected intensity: **linear** in  $x$ !



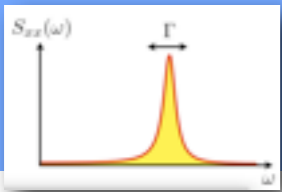
- need to go back to two-mode Hamiltonian!
- transitions between Fock states!

Single-sided cavity, but with losses: same story

Detailed analysis (Yanbei Chen's group, PRL 2009) shows: need

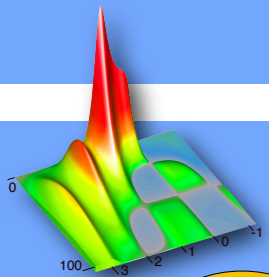
$$g_0 > \kappa_{\text{abs}} \quad \text{absorptive part of photon decay (or 2nd mirror)}$$

# Optomechanics (Outline)

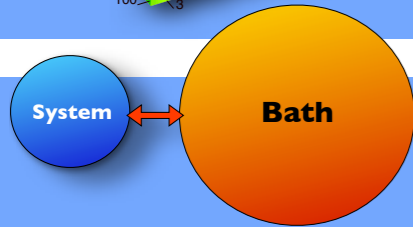


Displacement detection

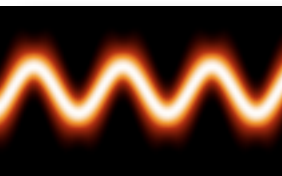
Linear optomechanics



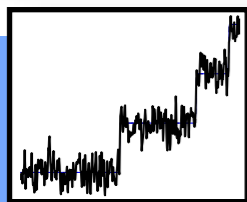
Nonlinear dynamics



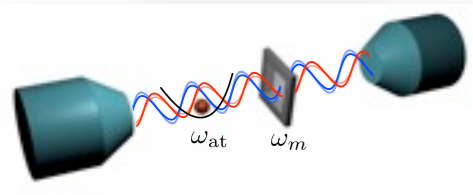
Quantum theory of cooling



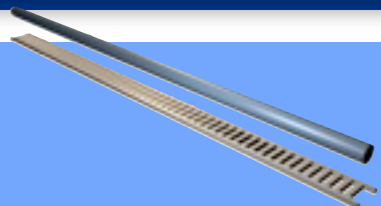
Interesting quantum states



Towards Fock state detection



Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

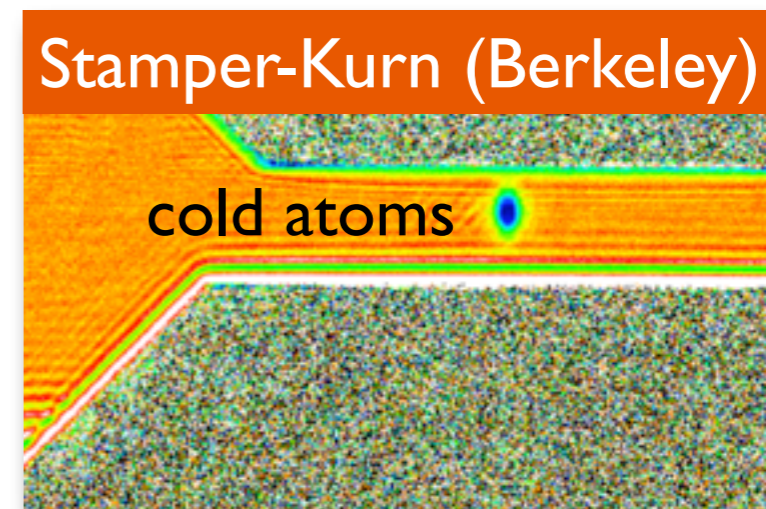


# Atom-membrane coupling

Note: Existing works simulate optomechanical effects using cold atoms

K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, *Nature Phys.* **4**, 561 (2008).

F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, *Science* **322**, 235 (2008).

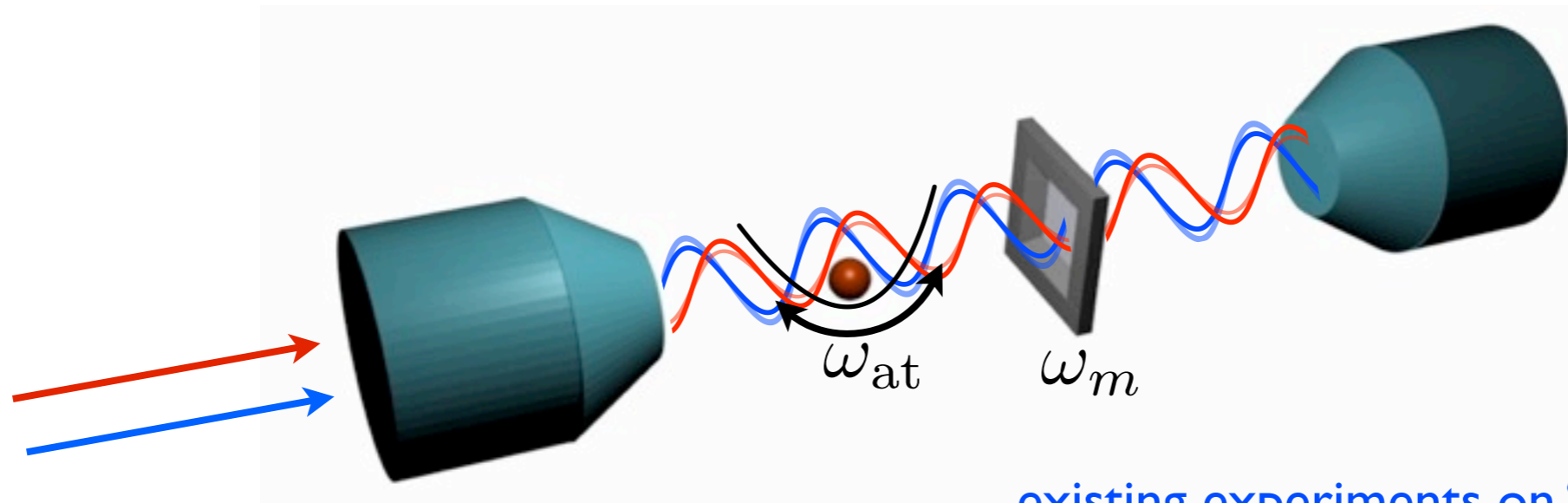


...profit from small mass of atomic cloud

Here: Coupling a single atom to a macroscopic mechanical object

Challenge: huge mass ratio

# Strong atom-membrane coupling via the light field



existing experiments on “optomechanics with cold atoms”: labs of Dan-Stamper Kurn (Berkeley) and Tilman Esslinger (ETH)

collaboration:

LMU (M. Ludwig, FM, P. Treutlein),

Innsbruck (K. Hammerer, C. Genes, M. Wallquist, P. Zoller),

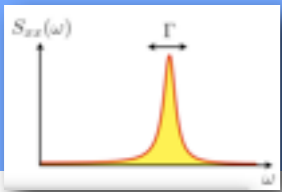
Boulder (J. Ye), Caltech (H. J. Kimble)

[Hammerer et al., PRL 2009](#)

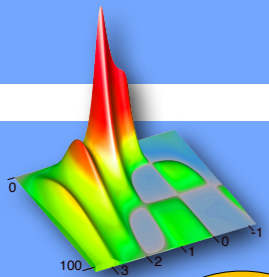
Goal:

$$\hat{H} = \underbrace{\hbar\omega_{\text{at}}\hat{a}^\dagger\hat{a}}_{\text{atom}} + \underbrace{\hbar\omega_m\hat{b}^\dagger\hat{b}}_{\text{membrane}} + \underbrace{\hbar G_{\text{eff}}(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b})}_{\text{atom-membrane coupling}}$$

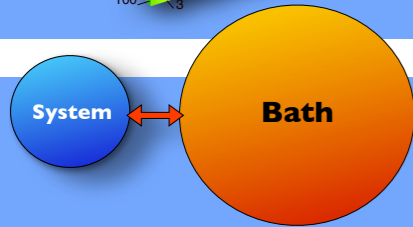
# Optomechanics (Outline)



Displacement detection

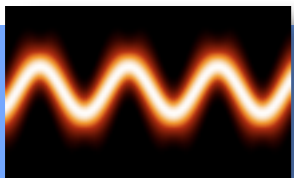


Linear optomechanics

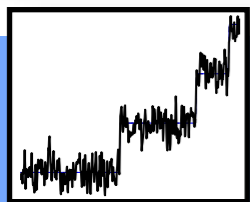


Nonlinear dynamics

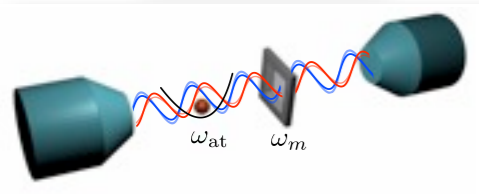
Quantum theory of cooling



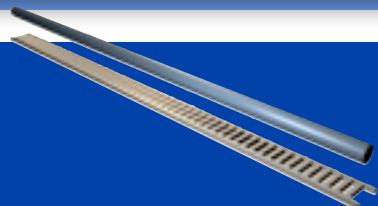
Interesting quantum states



Towards Fock state detection

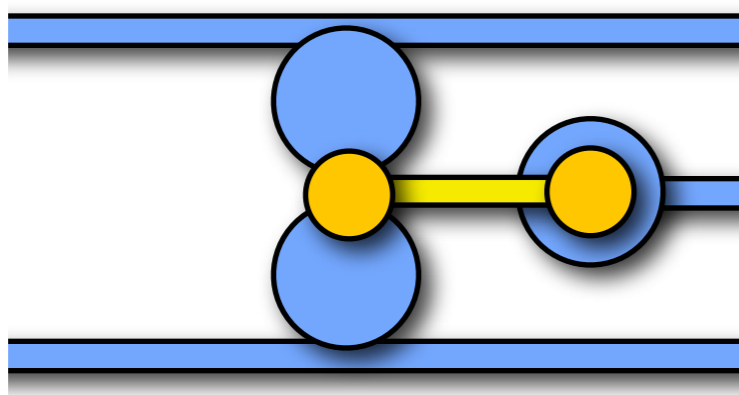
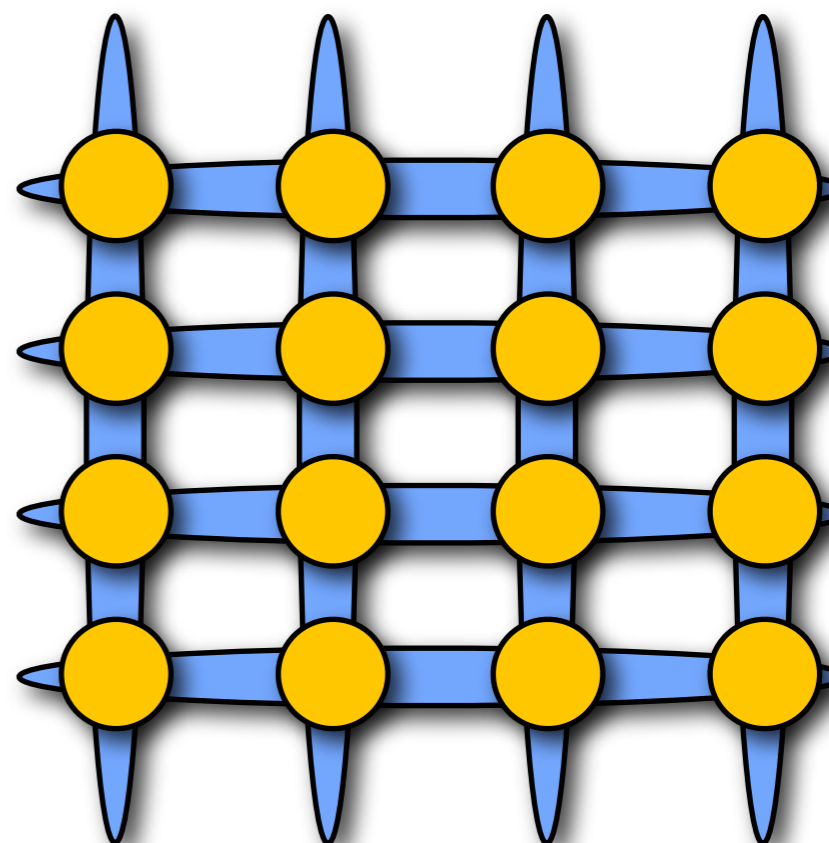
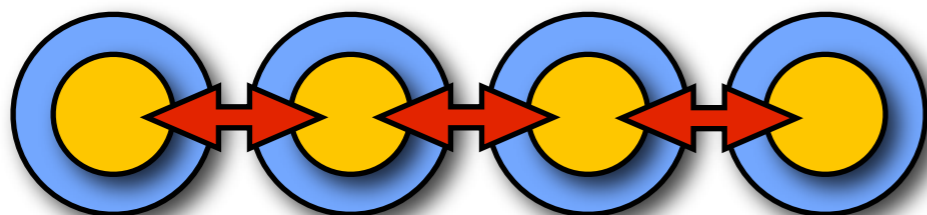
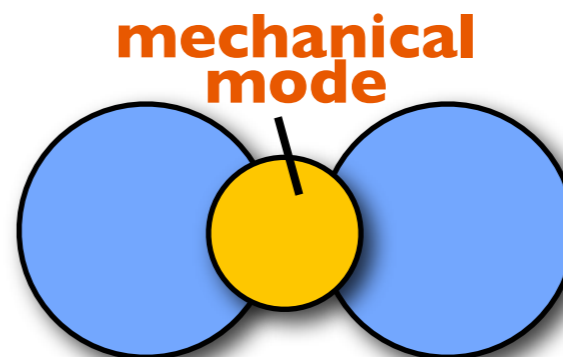
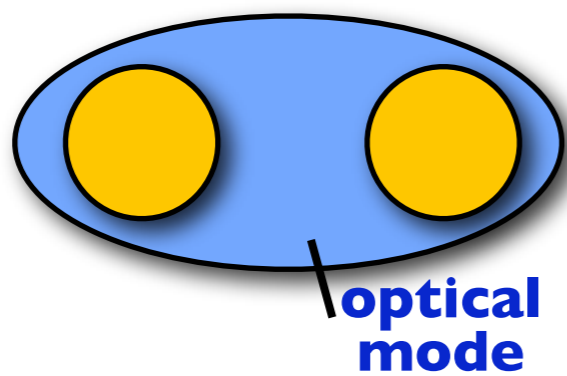


Hybrid systems: coupling to atoms

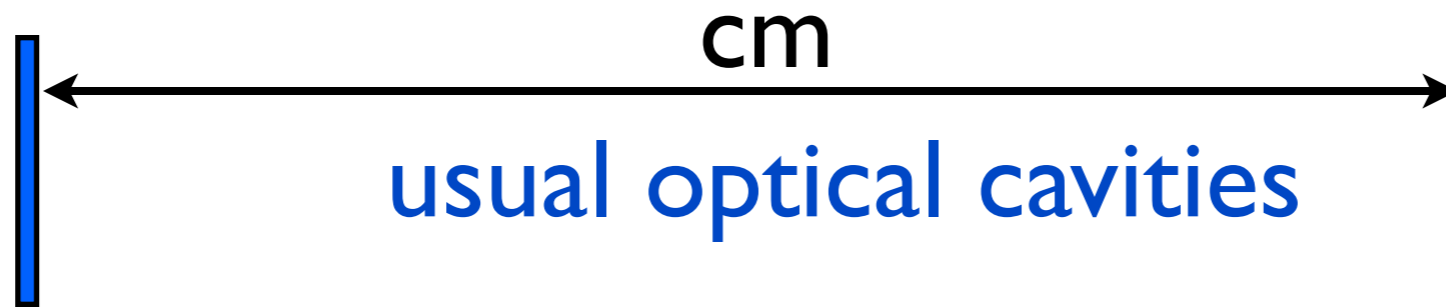


Optomechanical crystals & arrays

# Many modes



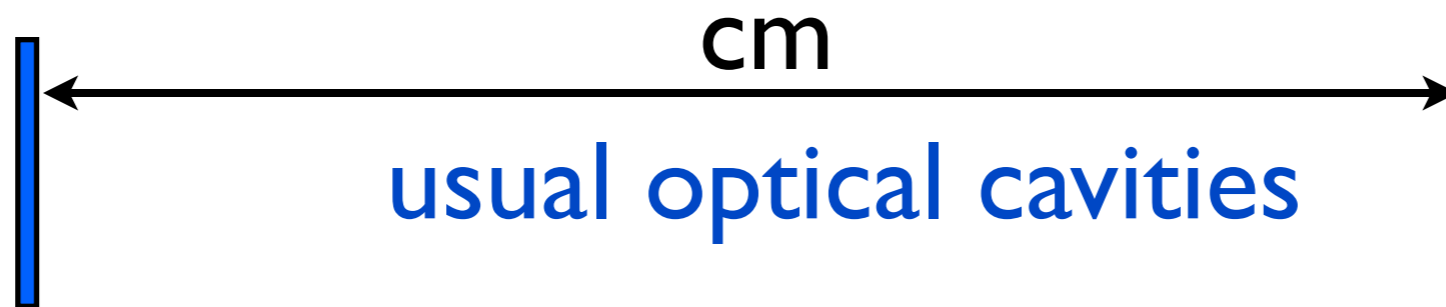
# Scaling down



10  $\mu\text{m}$

LKB, Aspelmeyer, Harris,  
Bouwmeester, ....

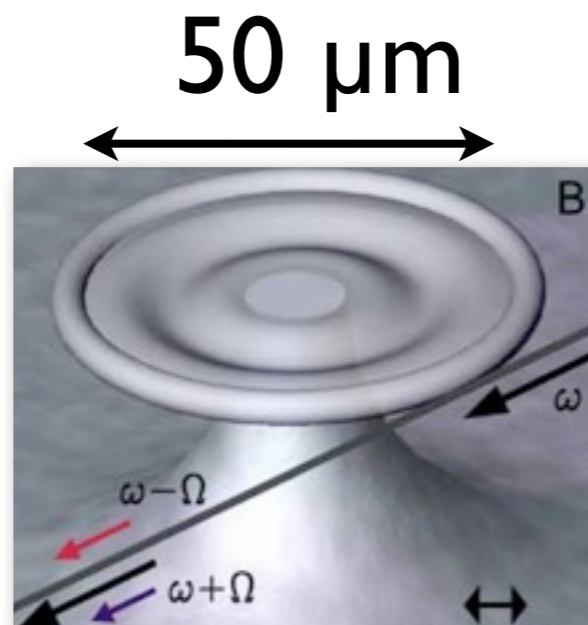
# Scaling down



10  $\mu\text{m}$

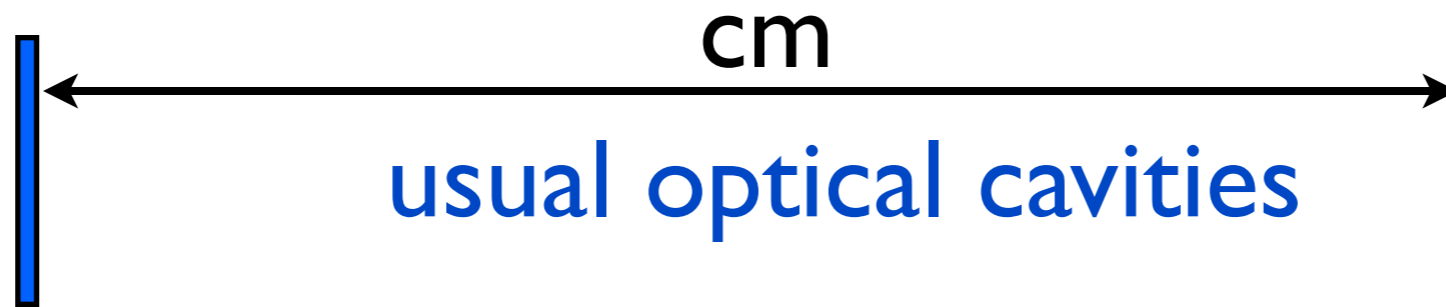
LKB, Aspelmeyer, Harris,  
Bouwmeester, ....

microtoroids



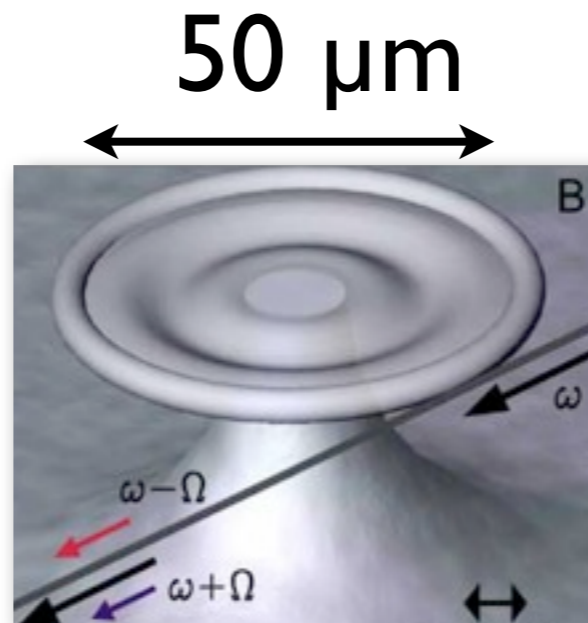
Vahala, Kippenberg, Carmon, ...

# Scaling down



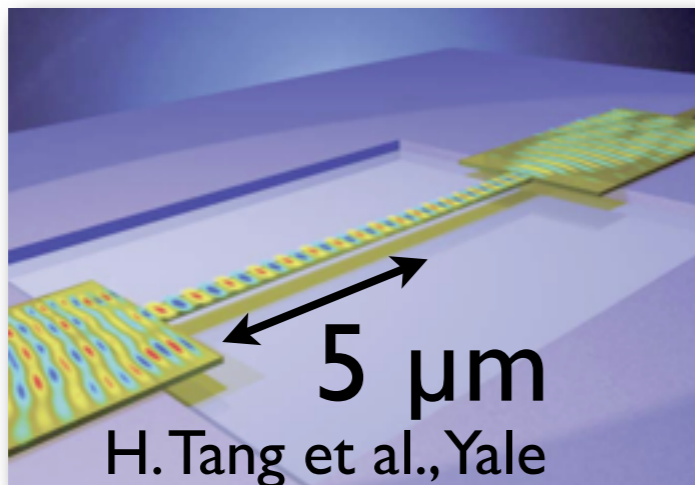
LKB, Aspelmeyer, Harris, Bouwmeester, ...

microtoroids

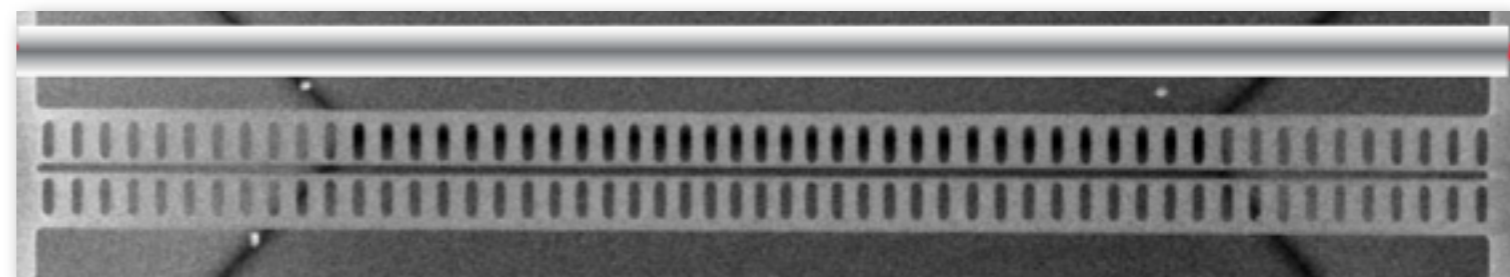


Vahala, Kippenberg, Carmon, ...

optomechanics in photonic circuits



optomechanical crystals

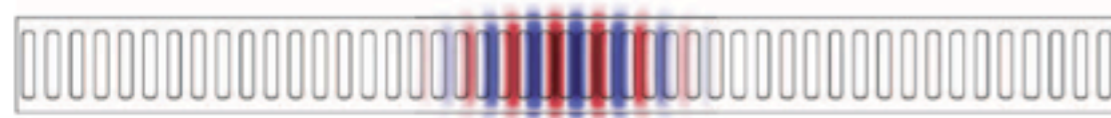


O. Painter et al., Caltech

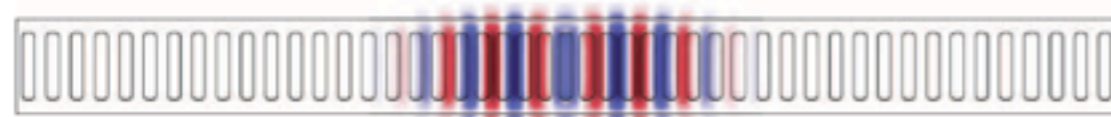
# Optomechanical crystals

## free-standing photonic crystal structures

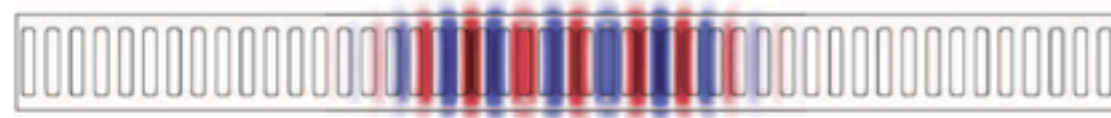
### optical modes



Fundamental 202 THz  $V_{\text{eff}} = 1.38 (\lambda_0/n)^3$



Second Order 195 THz  $V_{\text{eff}} = 1.72 (\lambda_0/n)^3$



Third Order 189 THz  $V_{\text{eff}} = 1.89 (\lambda_0/n)^3$

### vibrational modes



Breathing Mode 2.24 GHz  $m_{\text{eff}} = 334 \text{ fg}$



Accordian Mode 1.53 GHz  $m_{\text{eff}} = 681 \text{ fg}$



Pinch Mode 898 MHz  $m_{\text{eff}} = 68 \text{ fg}$

### advantages:

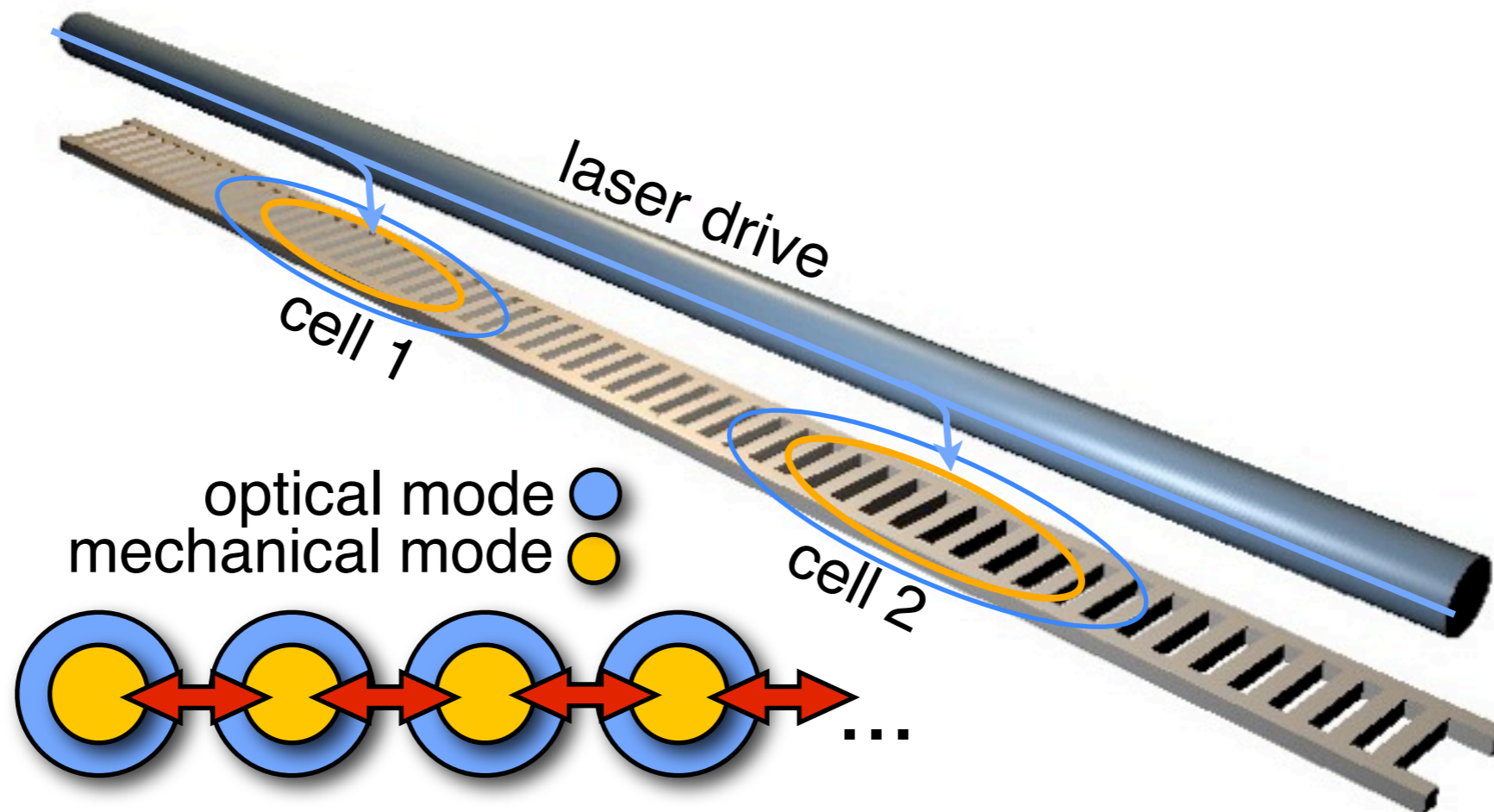
tight vibrational confinement:  
high frequencies, small mass  
(stronger quantum effects)

tight optical confinement:  
large optomechanical coupling  
(100 GHz/nm)

integrated on a chip



# Optomechanical arrays



collective nonlinear dynamics:  
classical / quantum

cf. Josephson arrays

# Dynamics in optomechanical arrays

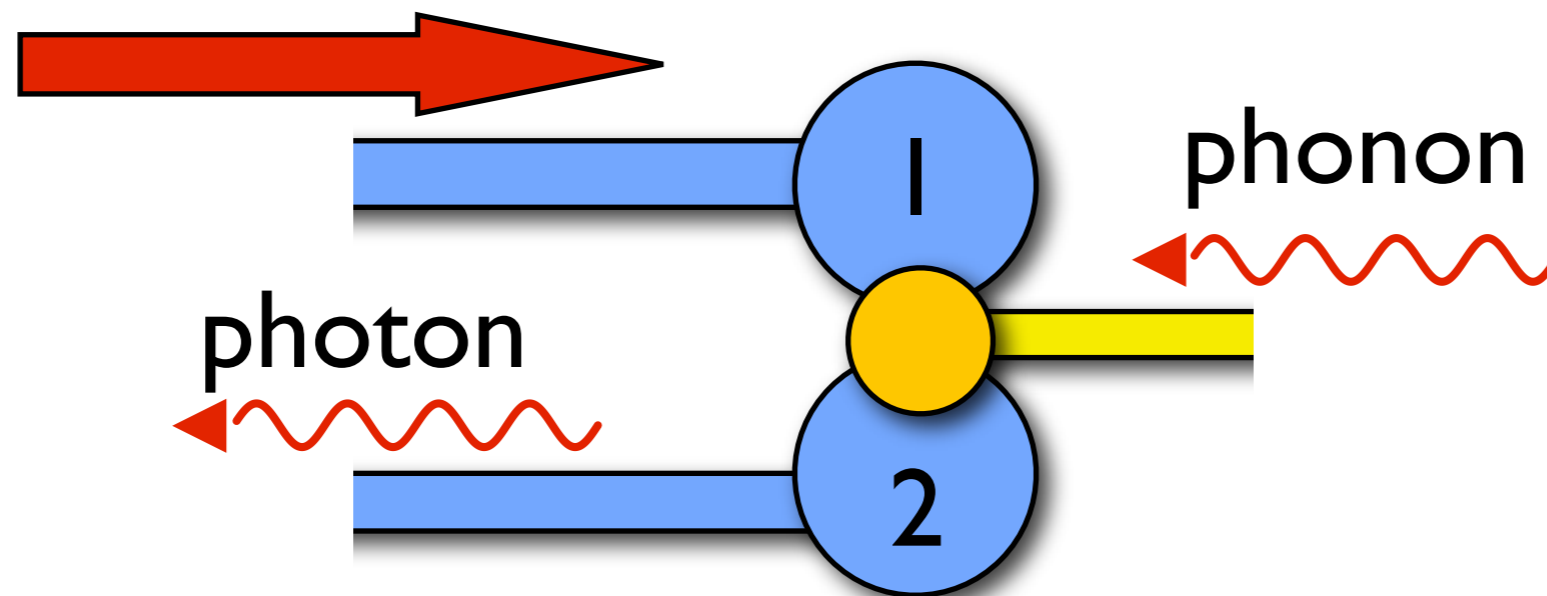
## Outlook

- 2D geometries
- Quantum or classical information processing and storage (continuous variables)
- Dissipative quantum many-body dynamics (quantum simulations)
- Hybrid devices: interfacing GHz qubits with light

# Photon-phonon translator

(concept: Painter group, Caltech)

strong optical “pump”

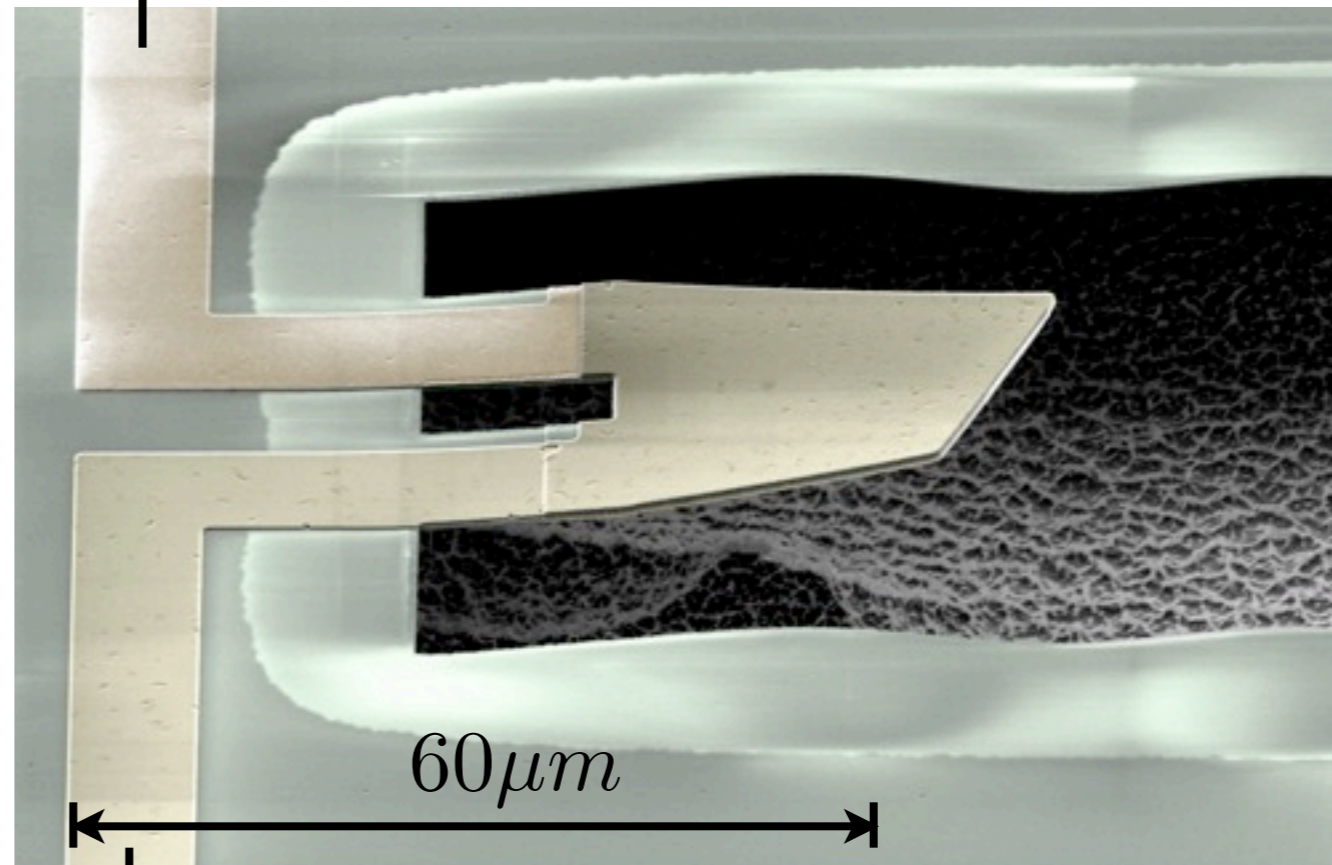


$$\hat{H} = \dots + \hbar g_0 (\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2) (\hat{b} + \hat{b}^\dagger)$$

# Superconducting qubit coupled to nanomechanical resonator

2010: Celand & Martinis labs

Josephson  
phase  
qubit

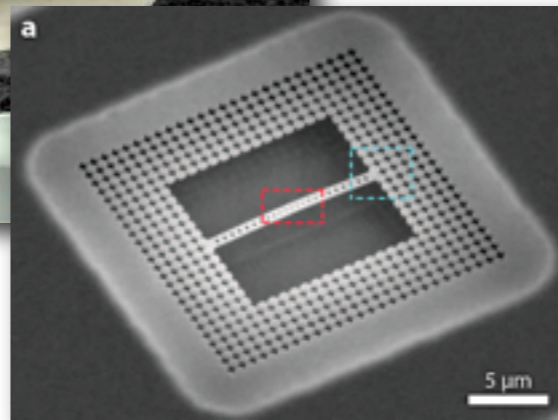
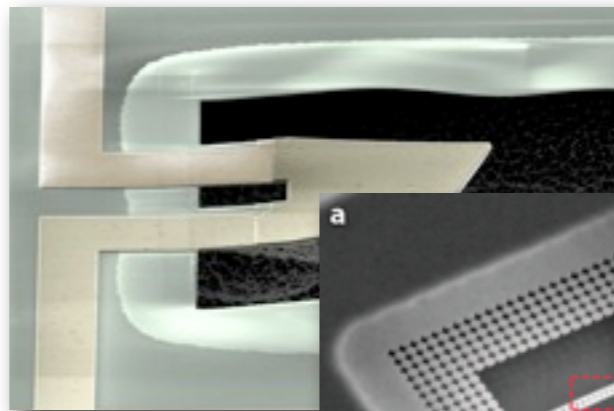
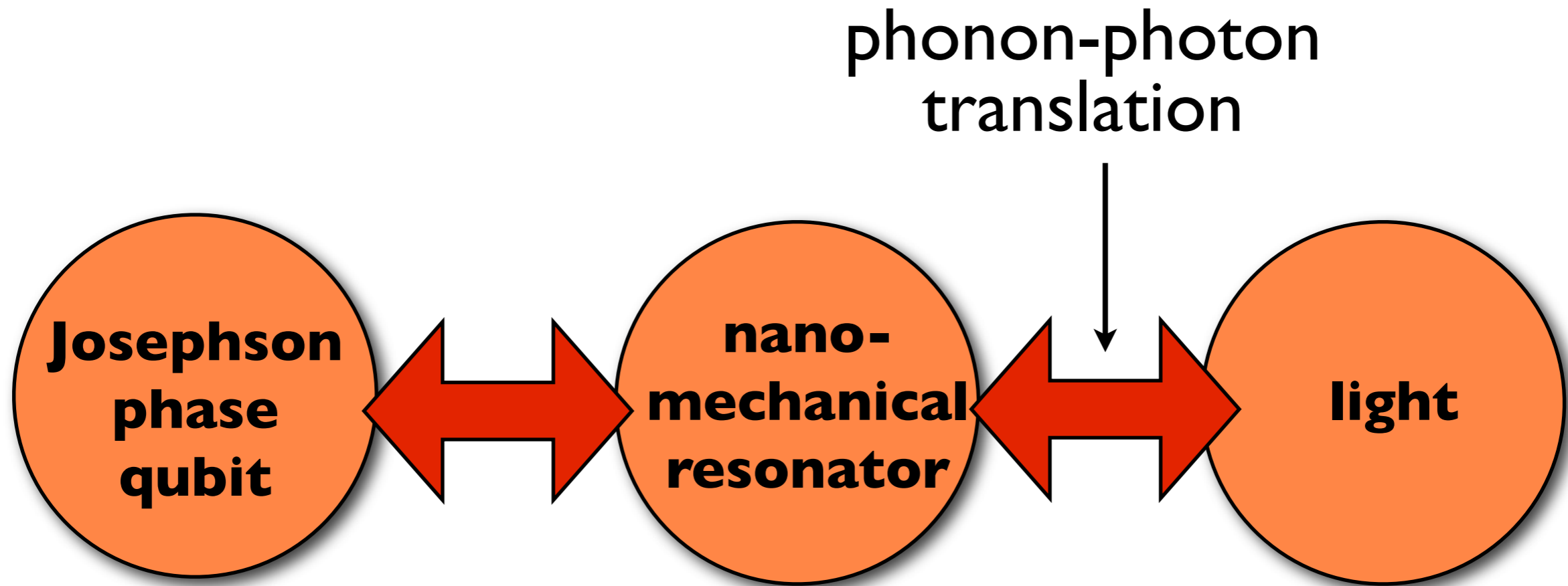


piezoelectric nanomechanical resonator

(GHz @ 20 mK: ground state!)

swap excitation between qubit and  
mechanical resonator in a few ns!

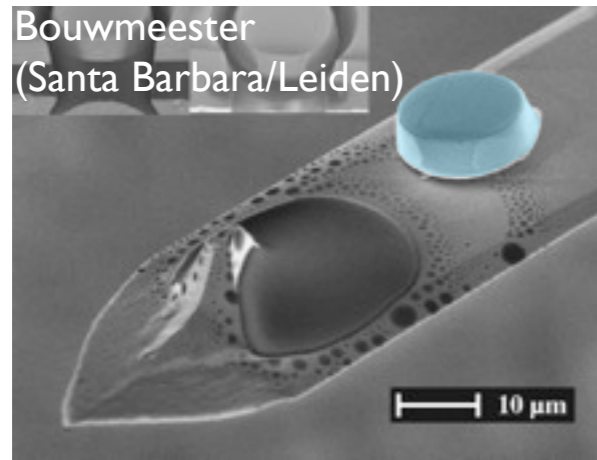
# Conversion of quantum information



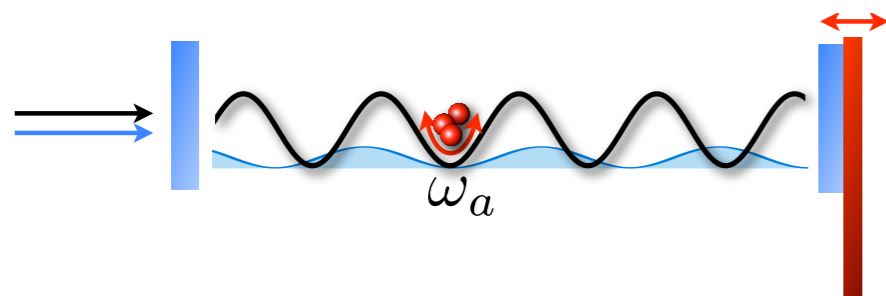
# Recent trends

- Ground-state cooling: success! (spring 2011)  
[Teufel et al. in microwave circuit;  
Painter group in optical regime]
- Optomechanical (photonic) crystals
- Multiple mechanical/optical modes
- Option: build arrays or 'optomechanical circuits'
- Strong improvements in coupling
- Possibly soon: ultrastrong coupling (resolve single photon-phonon coupling)
- Hybrid systems: Convert GHz quantum information (superconducting qubit) to photons
- Hybrid systems: atom/mechanics [e.g. Treutlein group]
- Levitating spheres: weak decoherence!  
[Barker/ Chang et al./ Romero-Isart et al.]

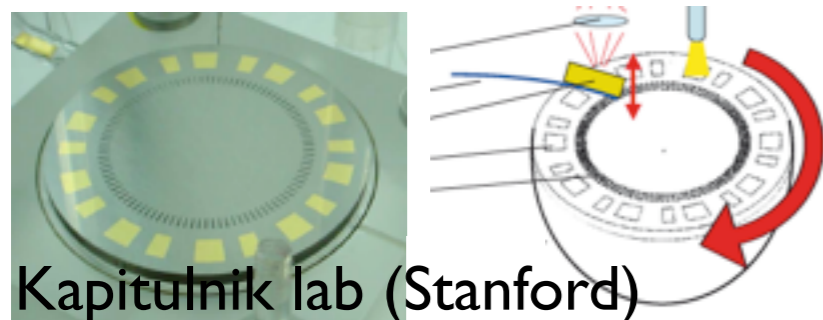
# Optomechanics: general outlook



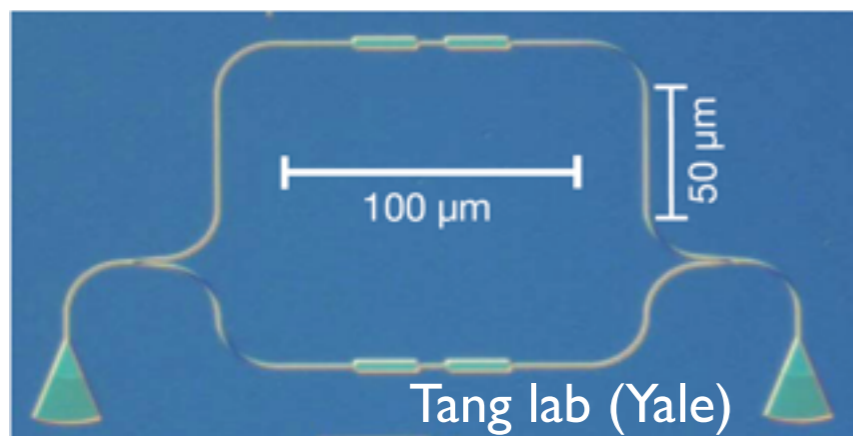
**Fundamental tests of quantum mechanics in a new regime:** entanglement with ‘macroscopic’ objects, unconventional decoherence? [e.g.: gravitationally induced?]



**Mechanics as a ‘bus’ for connecting hybrid components:** superconducting qubits, spins, photons, cold atoms, ....



**Precision measurements** [e.g. testing deviations from Newtonian gravity due to extra dimensions]



**Optomechanical circuits & arrays**  
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

# **Parameters of Optomechanical Systems**

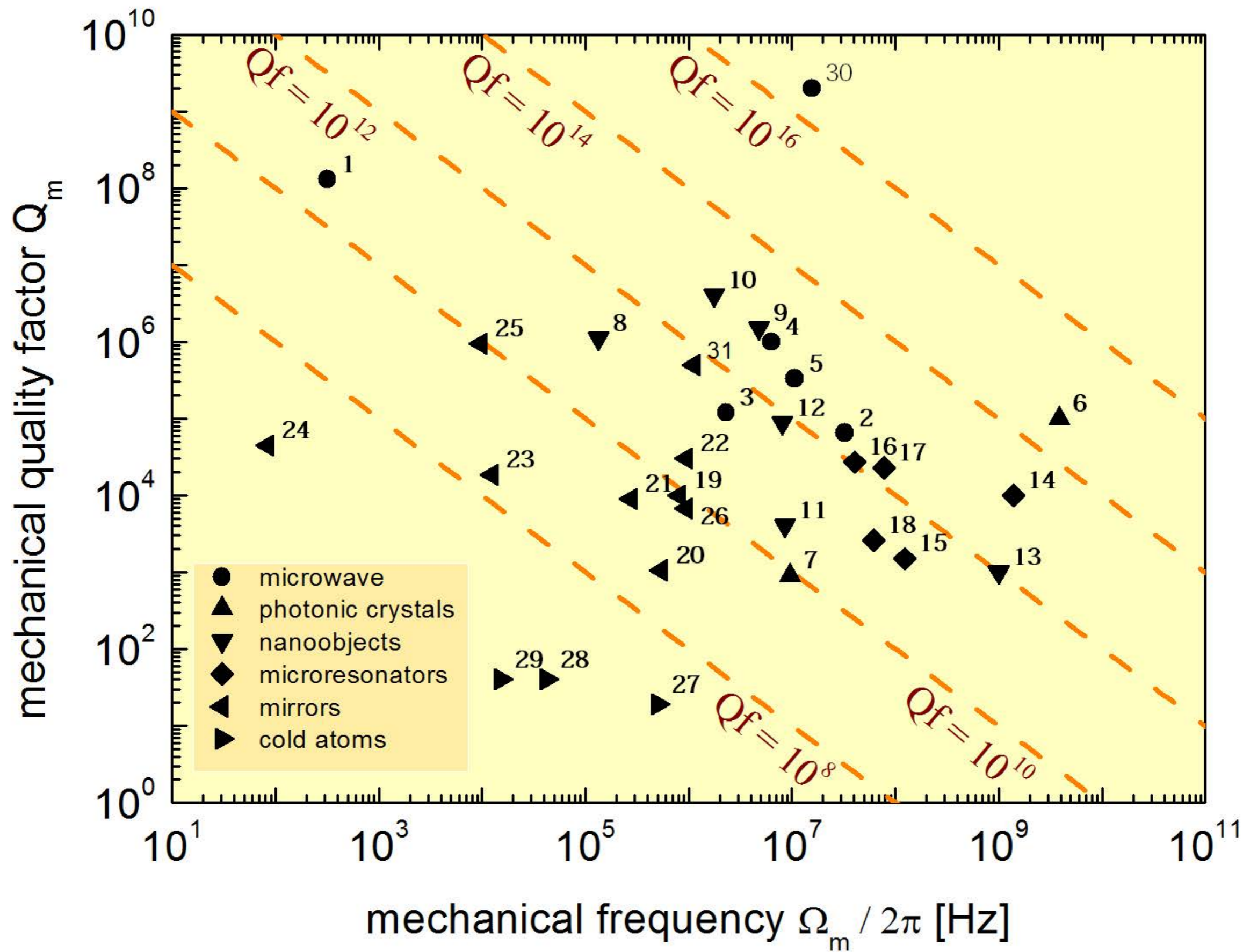


# Mechanical damping rate

$\Gamma_m$  rate of energy loss,  
linewidth in mechanical spectrum

$\Gamma_m \bar{n}_{th}$  rate of re-thermalization,  
ground state decoherence rate

$Q = \frac{\Omega_m}{\Gamma_m}$  Mechanical quality factor,  
number of oscillations during damping  
time



# Optomechanical coupling strength

$$g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger) \mapsto \underbrace{g_0 \alpha}_{g} (\delta \hat{a} + \delta \hat{a}^\dagger) (\hat{b} + \hat{b}^\dagger)$$

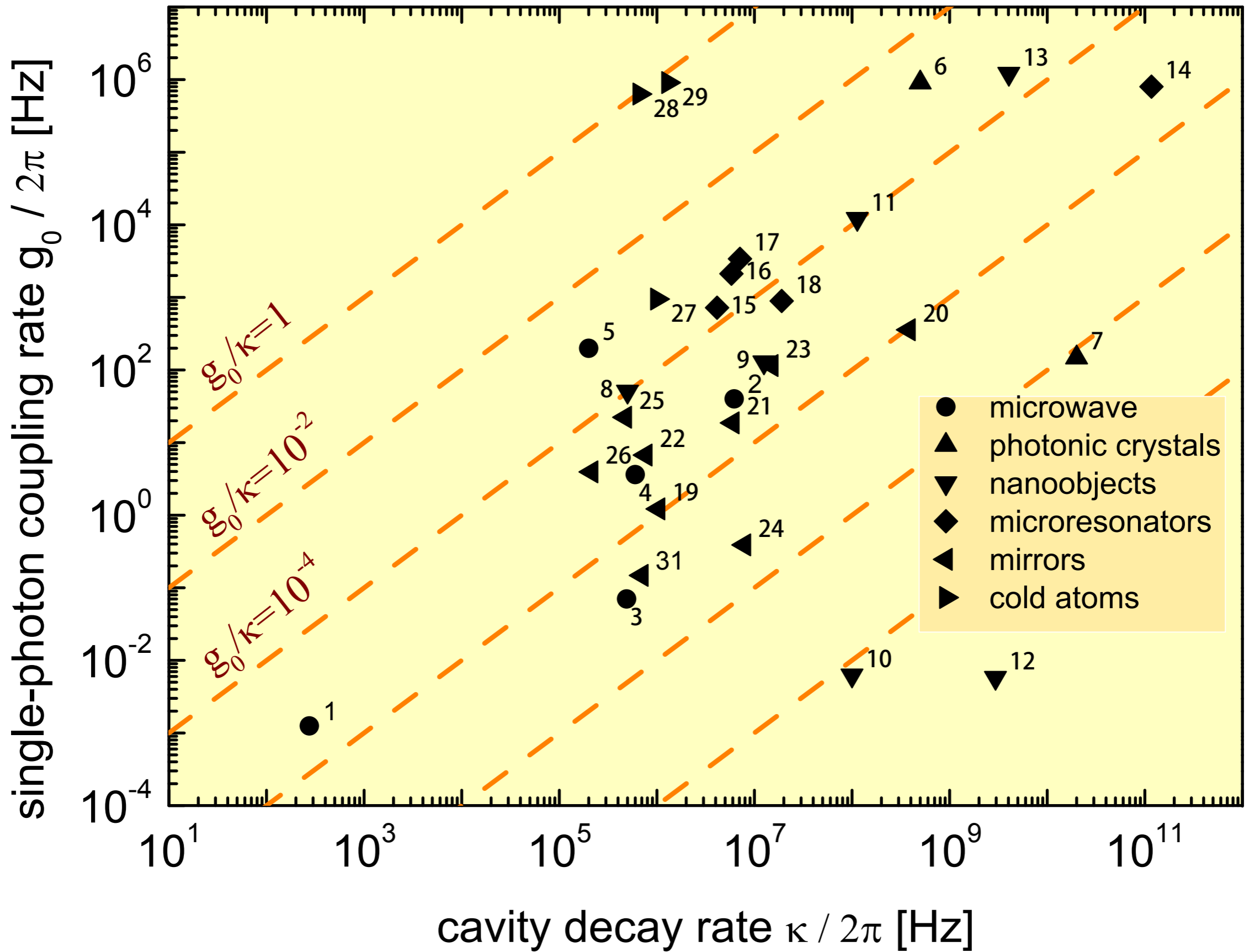
$g$  bilinear interaction  
tunable coupling!

$g_0$  Single-photon optomechanical coupling rate

nonclassical mechanical quantum states, ....

$g$  Linearized (driving-enhanced)  
optomechanical coupling rate

optomechanical damping rate, state transfer rate, ...



# Cooperativities

## Linearized (driving-enhanced) cooperativity

$$C = \frac{g_0^2 \bar{n}_{\text{cav}}}{\Gamma_m \kappa}$$

Optomechanically induced transparency,  
instability towards optomech. oscillations

## Linearized (driving-enhanced) quantum cooperativity

$$C_{\text{th}} = C = \frac{g_0^2 \bar{n}_{\text{cav}}}{\Gamma_m \bar{n}_{\text{th}} \kappa}$$

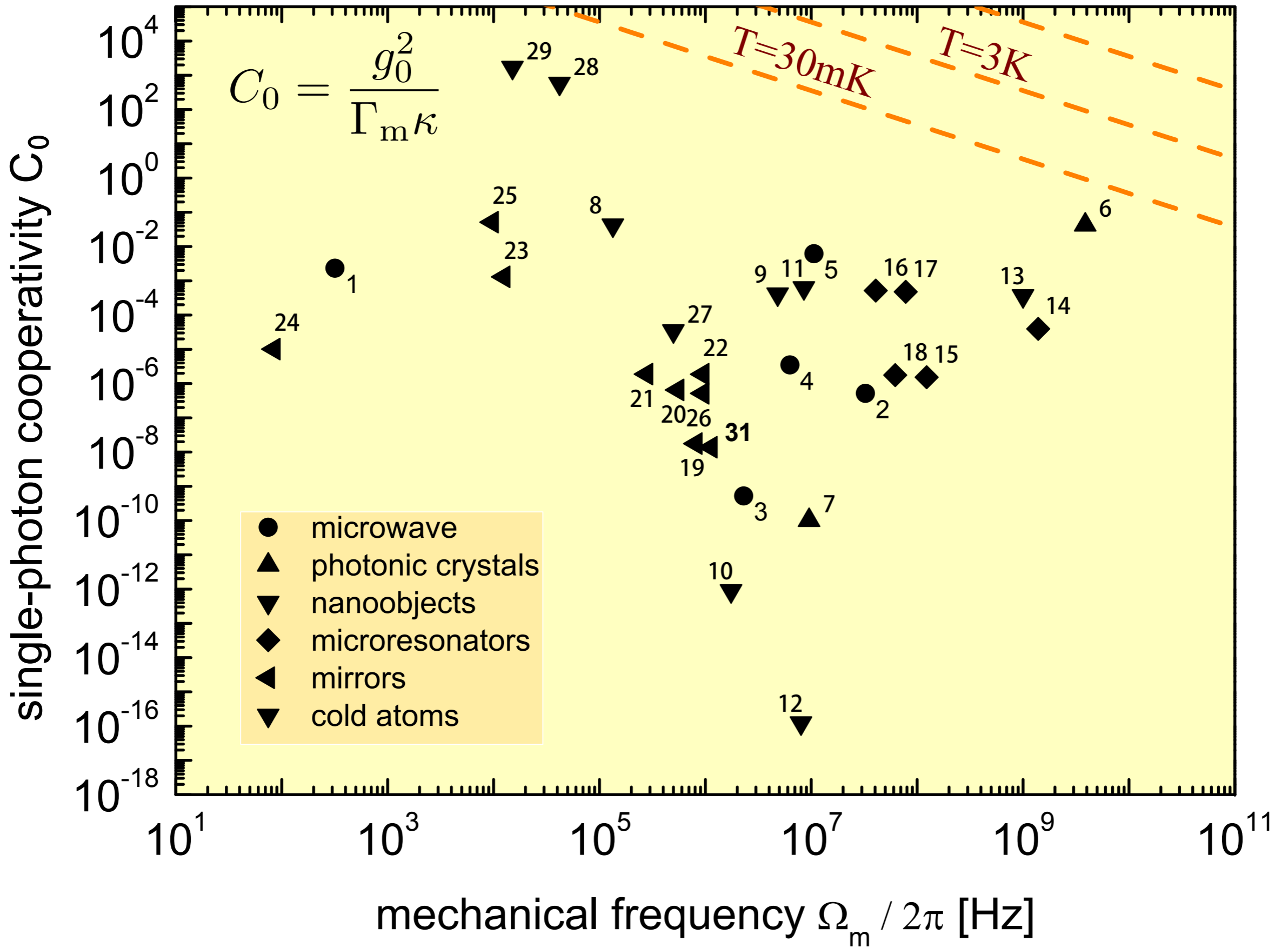
ground state cooling, state transfer,  
entanglement, squeezing of light, ...

## Single-photon cooperativity

$$C_0 = \frac{g_0^2}{\Gamma_m \kappa}$$

## Single-photon “quantum” cooperativity

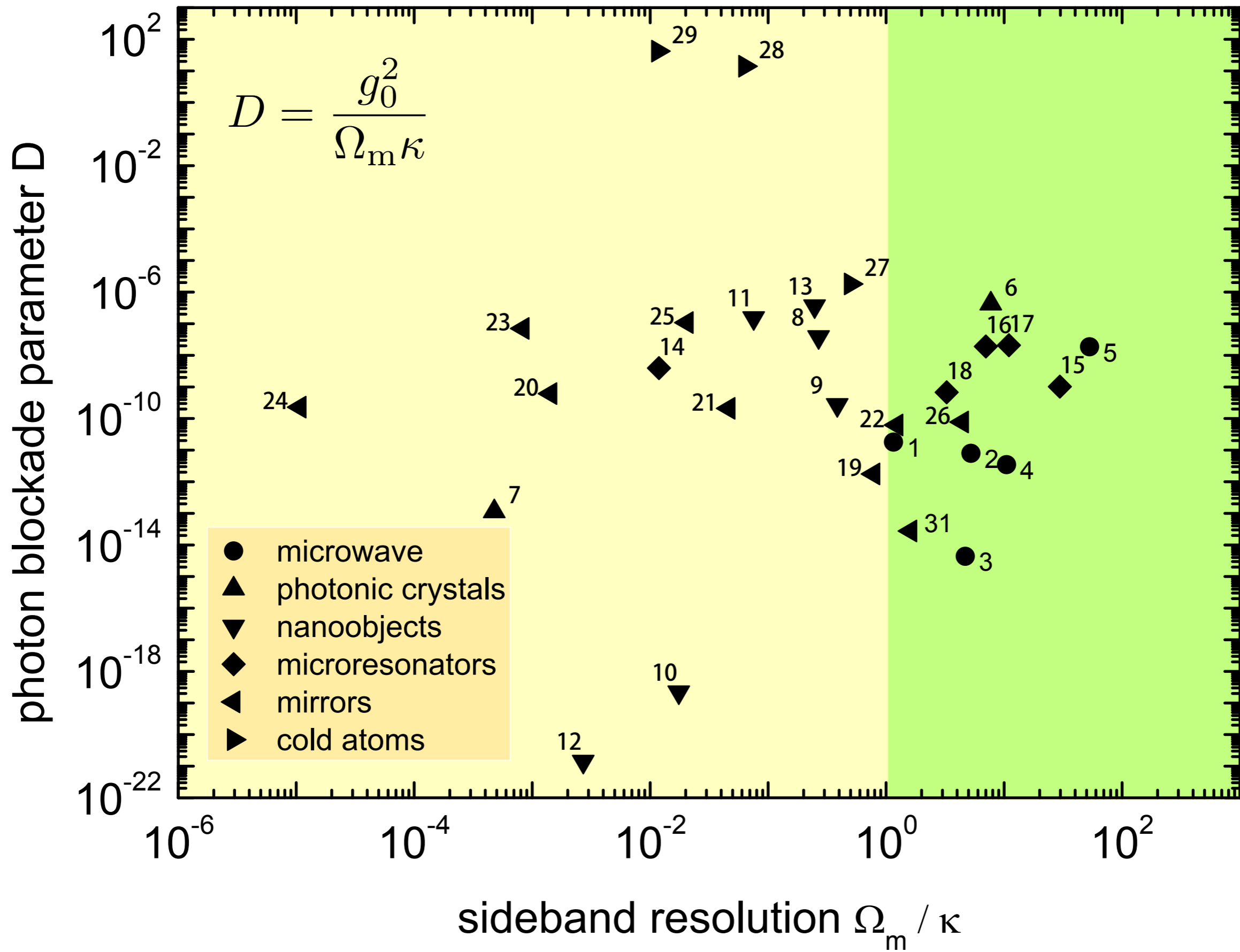
$$C_{0,\text{th}} = C = \frac{g_0^2}{\Gamma_m \bar{n}_{\text{th}} \kappa}$$



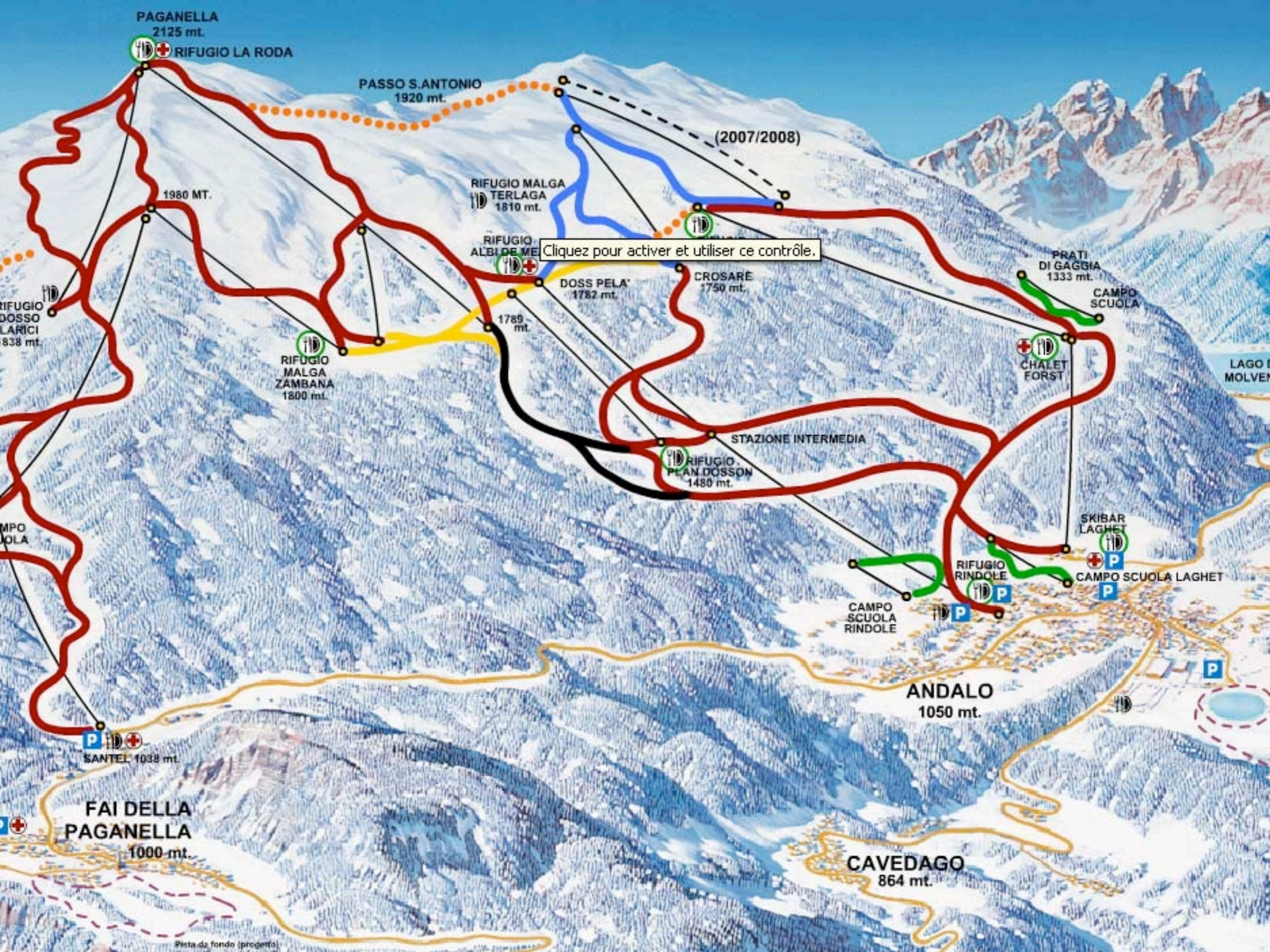
# Photon interaction

$$g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger) \quad \mapsto \quad -\frac{g_0^2}{\Omega_m} (\hat{a}^\dagger \hat{a})^2$$

photon blockade, photon QND measurement, ...







PAGANELLA  
2125 mt.  
RIFUGIO LA RODA

PASSO S. ANTONIO  
1920 mt.

RIFUGIO MALGA  
TERLAGA  
1810 mt.

RIFUGIO  
ALBI DE ME

DOSS PELLA  
1782 mt.

CROSARE  
1750 mt.

RIFUGIO  
MALGA  
ZAMBANA  
1800 mt.

1789  
mt.

RIFUGIO  
PLAN DOSSON  
1480 mt.

STAZIONE INTERMEDIA

PRATI  
DI GAGGIA  
1333 mt.

CAMPO  
SCUOLA

CHALET  
FORST

SKIBAR  
LAGHET

CAMPO SCUOLA LAGHET

RIFUGIO  
RINDOLE

CAMPO  
SCUOLA  
RINDOLE

ANDALO  
1050 mt.

SANTEL  
1038 mt.

FAI DELLA  
PAGANELLA  
1000 mt.

CAVEDAGO  
864 mt.

Cliquez pour activer et utiliser ce contrôle.

(2007/2008)

## Linear Optomechanics

- Displacement detection
- Optical Spring
- Cooling & Amplification
- Two-tone drive: “Optomechanically induced transparency”
- State transfer, pulsed operation
- Wavelength conversion
- Radiation Pressure Shot Noise
- Squeezing of Light
- Squeezing of Mechanics
- Entanglement
- Precision measurements

## Optomechanical Circuits

- Bandstructure in arrays
- Synchronization/patterns in arrays
- Transport & pulses in arrays

## Nonlinear Optomechanics

- Self-induced mechanical oscillations
- Synchronization of oscillations
- Chaos

## Nonlinear Quantum Optomechanics

- Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical “which-way” experiment
- Nonclassical mechanical q. states
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to other two-level systems

Note:

Yesterday's red is today's green!



# Linear Optomechanics

- Displacement detection
- Optical Spring
- Cooling & Amplification
- Two-tone drive: “Optomechanically induced transparency”
- Ground state cooling
- State transfer, pulsed operation
- Wavelength conversion
- Radiation Pressure Shot Noise
- Squeezing of Light
- Squeezing of Mechanics
- Light-Mechanics Entanglement
- Accelerometers
- Single-quadrature detection, Wigner density
- Optomechanics with an active medium
- Measure gravity or other small forces
- Mechanics-Mechanics entanglement
- Pulsed measurement
- Quantum Feedback
- Rotational Optomechanics

# Multimode

- Mechanical information processing
- Bandstructure in arrays
- Synchronization/patterns in arrays
- Transport & pulses in arrays

# Nonlinear Optomechanics

- Self-induced mechanical oscillations
- Attractor diagram?
- Synchronization of oscillations
- Chaos

○ White: yet unknown challenges/goals

# Nonlinear Quantum Optomechanics

- QND Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical “which-way” experiment
- Nonclassical mechanical q. states
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to other two-level systems
- Optomechanical Matter-Wave Interferometry

# Optomechanics

Review “Cavity Optomechanics”:  
M.Aspelmeyer, T. Kippenberg, FM  
arXiv:1303.0733